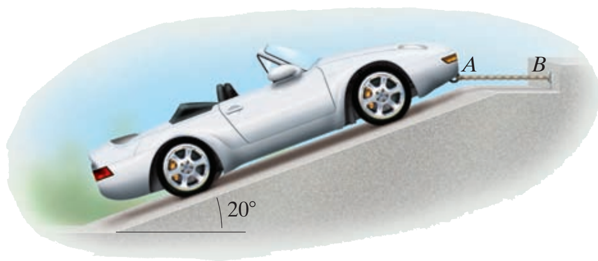
Problem 3.1 In Active Example 3.1, suppose that the angle between the ramp supporting the car is increased from 20° to 30°. Draw the free-body diagram of the car showing the new geometry. Suppose that the cable from A to B must exert a 1900-lb horizontal force on the car to hold it in place. Determine the car's weight in pounds.



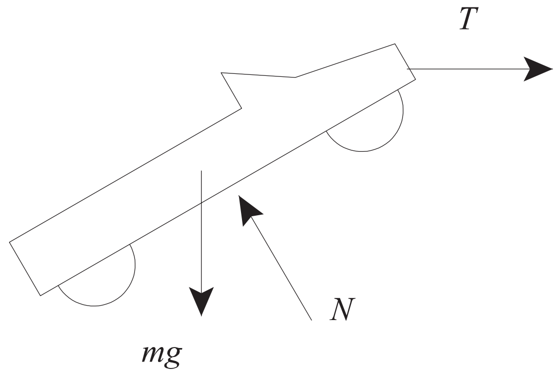
Solution: The free-body diagram is shown to the right. Applying the equilibrium equations

$$\sum F_x : T - N \sin 30^\circ = 0,$$

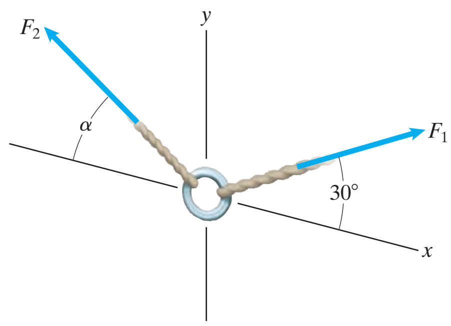
$$\sum F_y : N \cos 30^\circ - mg = 0$$

Setting $T = 1900$ lb and solving yields

$$N = 3800 \text{ lb}, \quad \boxed{mg = 3290 \text{ lb}}$$



Problem 3.2 The ring weighs 5 lb and is in equilibrium. The force $F_1 = 4.5$ lb. Determine the force F_2 and the angle α.



Solution: The free-body diagram is shown below the drawing. The equilibrium equations are

$$\sum F_x : F_1 \cos 30^\circ - F_2 \cos \alpha = 0$$

$$\sum F_y : F_1 \sin 30^\circ + F_2 \sin \alpha - 5 \text{ lb} = 0$$

We can write these equations as

$$F_2 \sin \alpha = 5 \text{ lb} - F_1 \sin 30^\circ$$

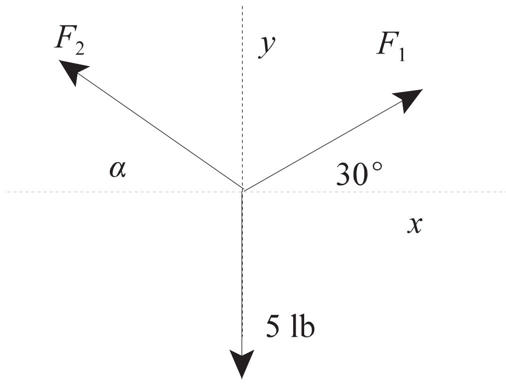
$$F_2 \cos \alpha = F_1 \cos 30^\circ$$

Dividing these equations and using the known value for F_1 we have.

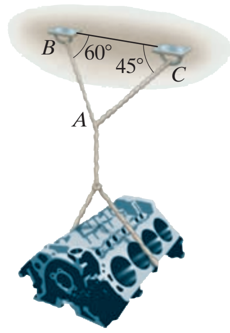
$$\tan \alpha = \frac{5 \text{ lb} - (4.5 \text{ lb}) \sin 30^\circ}{(4.5 \text{ lb}) \cos 30^\circ} = 0.706 \Rightarrow \alpha = 35.2^\circ$$

$$F_2 = \frac{(4.5 \text{ lb}) \cos 30^\circ}{\cos \alpha} = 4.77 \text{ lb}$$

$$\boxed{F_2 = 4.77 \text{ lb}, \alpha = 35.2^\circ}$$



Problem 3.3 In Example 3.2, suppose that the attachment point C is moved to the right and cable AC is extended so that the angle between cable AC and the ceiling decreases from 45° to 35°. The angle between cable AB and the ceiling remains 60°. What are the tensions in cables AB and AC?



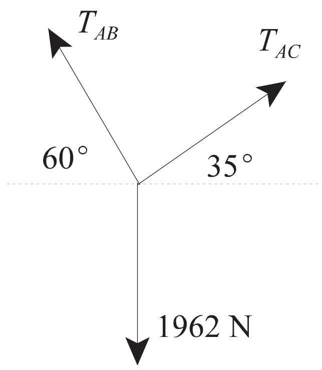
Solution: The free-body diagram is shown below the picture. The equilibrium equations are:

$$\sum F_x : T_{AC} \cos 35^\circ - T_{AB} \cos 60^\circ = 0$$

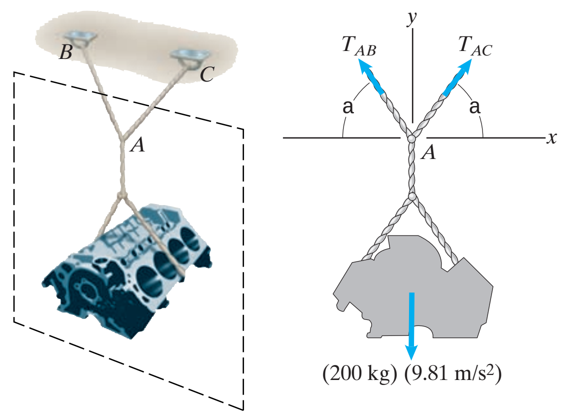
$$\sum F_y : T_{AC} \sin 35^\circ + T_{AB} \sin 60^\circ - 1962 \text{ N} = 0$$

Solving we find

$$\boxed{T_{AB} = 1610 \text{ N}, \; T_{AC} = 985 \text{ N}}$$



Problem 3.4 The 200-kg engine block is suspended by the cables AB and AC. The angle $\alpha = 40^\circ$. The free-body diagram obtained by isolating the part of the system within the dashed line is shown. Determine the forces T_{AB} and T_{AC}.



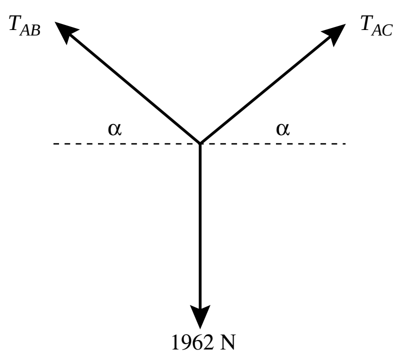
Solution:

$\alpha = 40^\circ$

$$\sum F_x : T_{AC} \cos\alpha - T_{AB} \cos\alpha = 0$$

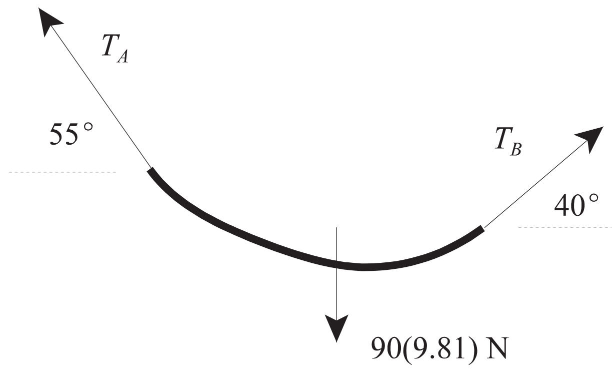
$$\sum F_y : T_{AC} \sin\alpha + T_{AB} \sin\alpha - 1962 \text{ N} = 0$$

Solving: $\boxed{T_{AB} = T_{AC} = 1.526 \text{ kN}}$



Problem 3.5 A heavy rope used as a mooring line for a cruise ship sags as shown. If the mass of the rope is 90 kg, what are the tensions in the rope at A and B?

Solution: The free-body diagram is shown.
The equilibrium equations are

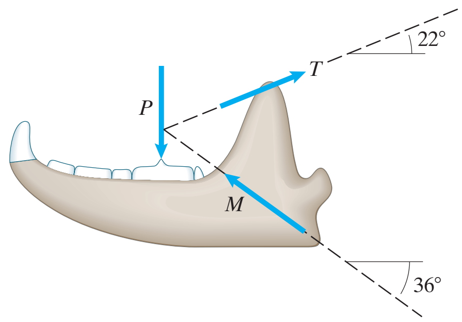


$$\sum F_x : T_B \cos 40^\circ - T_A \cos 55^\circ = 0$$

$$\sum F_y : T_B \sin 40^\circ + T_A \sin 55^\circ - 90(9.81)\ \text{N} \ = 0$$

Solving: $T_A = \boxed{679\ \text{N}, T_B = 508\ \text{N}}$

Problem 3.6 A physiologist estimates that the masseter muscle of a predator, *Martes*, is capable of exerting a force M as large as 900 N. Assume that the jaw is in equilibrium and determine the necessary force T that the temporalis muscle exerts and the force P exerted on the object being bitten.



Solution: The equilibrium equations are

$$\sum F_x : T \cos 22^\circ - M \cos 36^\circ = 0$$

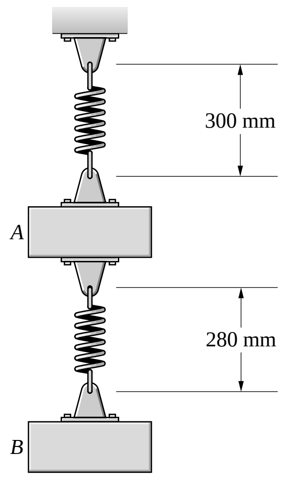
$$\sum F_y : T \sin 22^\circ + M \sin 36^\circ - P = 0$$

Setting $M = 900$ N, and solving, we find

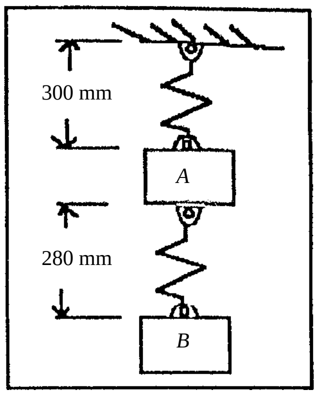
$$\boxed{T = 785\ \text{N}, P = 823\ \text{N}}$$

Problem 3.7 The two springs are identical, with unstretched lengths 250 mm and spring constants $k = 1200$ N/m.

(a) Draw the free-body diagram of block A.
(b) Draw the free-body diagram of block B.
(c) What are the masses of the two blocks?



Solution: The tension in the upper spring acts on block **A** in the positive ***Y*** direction, Solve the spring force-deflection equation for the tension in the upper spring. Apply the equilibrium conditions to block A. Repeat the steps for block B.



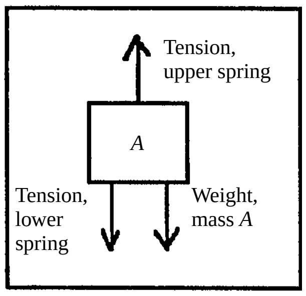
$$\mathbf{T}_{UA} = 0\mathbf{i} + \left(1200\ \frac{\text{N}}{\text{m}}\right)(0.3\text{ m} - 0.25\text{ m})\mathbf{j} = 0\mathbf{i} + 60\mathbf{j}\text{ N}$$

Similarly, the tension in the lower spring acts on block **A** in the negative ***Y*** direction

$$\mathbf{T}_{LA} = 0\mathbf{i} - \left(1200\ \frac{\text{N}}{\text{m}}\right)(0.28\text{ m} - 0.25\text{ m})\mathbf{j} = 0\mathbf{i} - 36\mathbf{j}\text{ N}$$

The weight is $\mathbf{W}_A = 0\mathbf{i} - |\mathbf{W}_A|\mathbf{j}$

The equilibrium conditions are



$$\sum \mathbf{F} = \sum \mathbf{F}_x + \sum \mathbf{F}_y = 0, \quad \sum \mathbf{F} = \mathbf{W}_A + \mathbf{T}_{UA} + \mathbf{T}_{LA} = 0$$

Collect and combine like terms in **i**, **j**

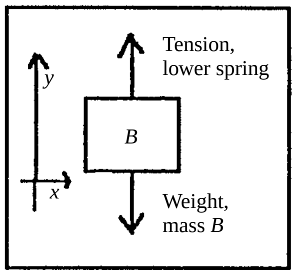
$$\sum \mathbf{F}_y = (-|\mathbf{W}_A| + 60 - 36)\mathbf{j} = 0$$

Solve $|\mathbf{W}_A| = (60 - 36) = 24$ N

The mass of A is

$$m_A = \frac{|\mathbf{W}_L|}{|\mathbf{g}|} = \frac{24\text{ N}}{9.81\text{ m/s}^2} = 2.45\text{ kg}$$

The free body diagram for block **B** is shown.



The tension in the lower spring $\mathbf{T}_{LB} = 0\mathbf{i} + 36\mathbf{j}$

The weight: $\mathbf{W}_B = 0\mathbf{i} - |\mathbf{W}_B|\mathbf{j}$
Apply the equilibrium conditions to block B.

$$\sum \mathbf{F} = \mathbf{W}_B + \mathbf{T}_{LB} = 0$$

Collect and combine like terms in **i**, **j**:

$$\sum \mathbf{F}_y = (-|\mathbf{W}_B| + 36)\mathbf{j} = 0$$

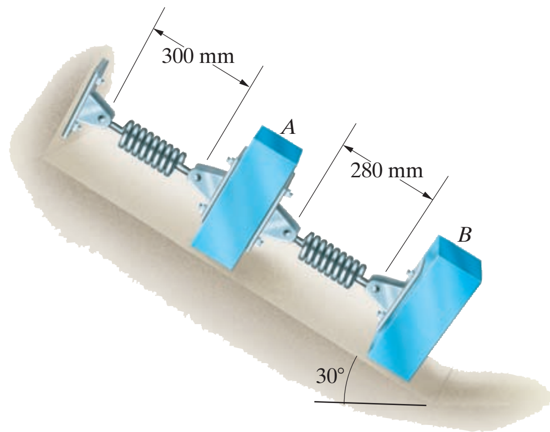
Solve: $|\mathbf{W}_B| = 36$ N

The mass of B is given by $m_B = \dfrac{|\mathbf{W}_B|}{|\mathbf{g}|} = \dfrac{36\text{ N}}{9.81\text{ m/s}^2} = 3.67\text{ kg}$

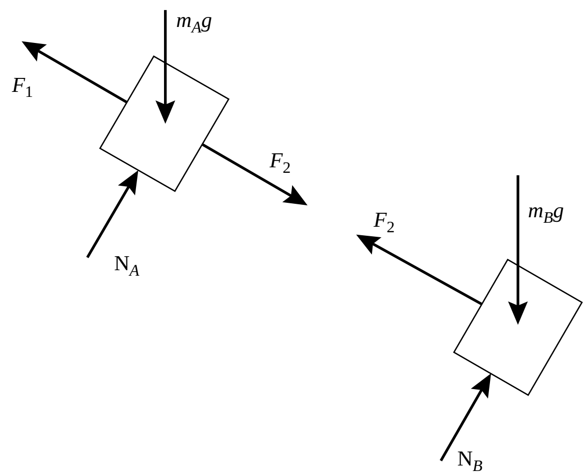
Problem 3.8 The two springs in Problem 3.7 are identical, with unstretched lengths of 250 mm. Suppose that their spring constant k is unknown and the sum of the masses of blocks A and B is 10 kg. Determine the value of k and the masses of the two blocks.

Solution: All of the forces are in the vertical direction so we will use scalar equations. First, consider the upper spring supporting both masses (10 kg total mass). The equation of equilibrium for block the entire assembly supported by the upper spring is A is $T_{UA} - (m_A + m_B)g = 0$, where $T_{UA} = k(\ell_U - 0.25)$ N. The equation of equilibrium for block B is $T_{UB} - m_B g = 0$, where $T_{UB} = k(\ell_L - 0.25)$ N. The equation of equilibrium for block A alone is $T_{UA} + T_{LA} - m_A g = 0$ where $T_{LA} = -T_{UB}$. Using $g = 9.81$ m/s^2, and solving simultaneously, we get $\boxed{k = 1962 \text{ N/m}, \ m_A = 4 \text{ kg, and } m_B = 6 \text{ kg}}$.

Problem 3.9 The inclined surface is smooth (Remember that "smooth" means that friction is negligible). The two springs are identical, with unstretched lengths of 250 mm and spring constants $k = 1200$ N/m. What are the masses of blocks A and B?



Solution:



$F_1 = (1200 \text{ N/m})(0.3 - 0.25)m = 60$ N

$F_2 = (1200 \text{ N/m})(0.28 - 0.25)m = 36$ N

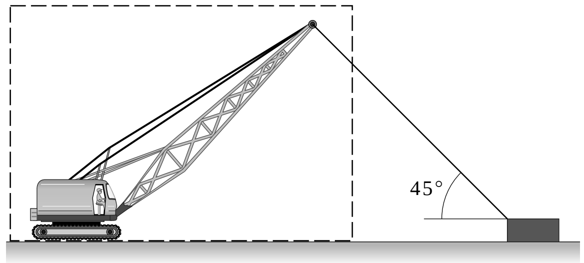
$\sum F_B \searrow: -F_2 + m_B g \sin 30^\circ = 0$

$\sum F_A \searrow: -F_1 + F_2 + m_A g \sin 30^\circ = 0$

Solving: $\boxed{m_A = 4.89 \text{ kg}, \ m_B = 7.34 \text{ kg}}$

Problem 3.10 The mass of the crane is 20,000 kg. The crane's cable is attached to a caisson whose mass is 400 kg. The tension in the cable is 1 kN.

(a) Determine the magnitudes of the normal and friction forces exerted on the crane by the level ground.
(b) Determine the magnitudes of the normal and friction forces exerted on the caisson by the level ground.



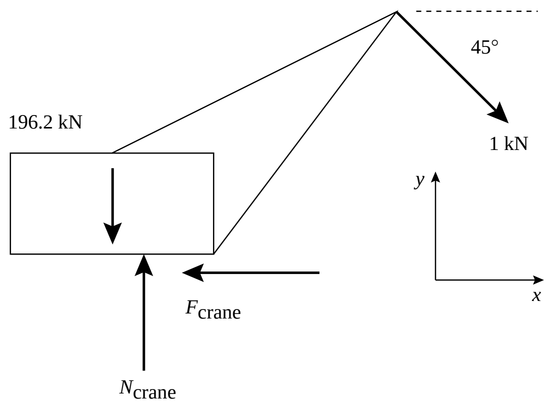
Strategy: To do part (a), draw the free-body diagram of the crane and the part of its cable within the dashed line.

Solution:

(a) $\sum F_y : N_{\text{crane}} - 196.2 \text{ kN} - 1 \text{ kN} \sin 45^\circ = 0$

$\sum F_x : -F_{\text{crane}} + 1 \text{ kN} \cos 45^\circ = 0$

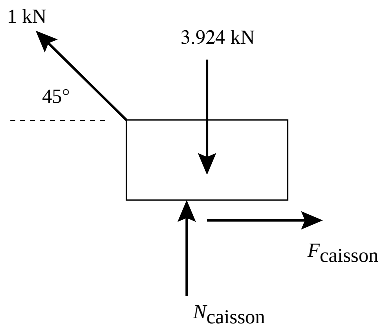
$\boxed{N_{\text{crane}} = 196.9 \text{ kN}, \ F_{\text{crane}} = 0.707 \text{ kN}}$



(b) $\sum F_y : N_{\text{caisson}} - 3.924 \text{ kN} + 1 \text{ kN} \sin 45^\circ = 0$

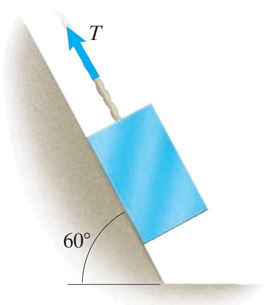
$\sum F_x : -1 \text{ kN} \cos 45^\circ + F_{\text{caisson}} = 0$

$\boxed{N_{\text{caisson}} = 3.22 \text{ kN}, \ F_{\text{caisson}} = 0.707 \text{ kN}}$



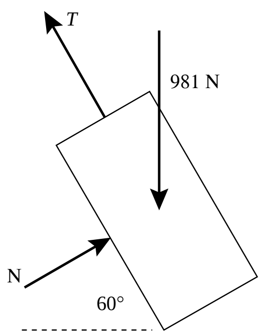
Problem 3.11 The inclined surface is smooth. The 100-kg crate is held stationary by a force T applied to the cable.

(a) Draw the free-body diagram of the crate.
(b) Determine the force T.



Solution:

(a) The FBD



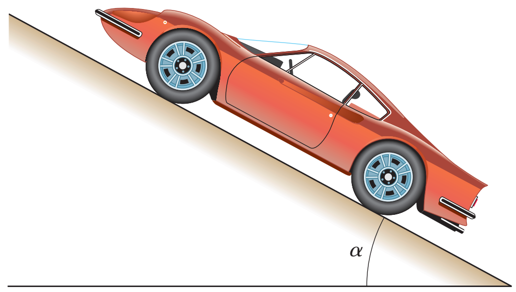
(b) $\sum F \nwarrow : T - 981 \text{ N} \sin 60^\circ = 0$

$\boxed{T = 850 \text{ N}}$

Problem 3.12 The 1200-kg car is stationary on the sloping road.

(a) If $\alpha = 20^\circ$, what are the magnitudes of the total normal and friction forces exerted on the car's tires by the road?

(b) The car can remain stationary only if the total friction force necessary for equilibrium is not greater than 0.6 times the total normal force. What is the largest angle α for which the car can remain stationary?



Solution:

(a) $\alpha = 20^\circ$

$$\sum F_{\nearrow} : N - 11.772 \text{ kN} \cos\alpha = 0$$

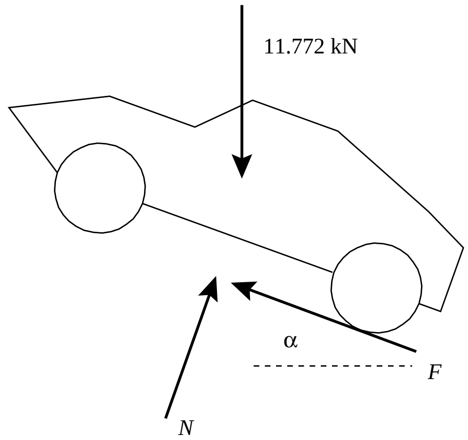
$$\sum F_{\nwarrow} : F - 11.772 \text{ kN} \sin\alpha = 0$$

$$\boxed{N = 11.06 \text{ kN}, \quad F = 4.03 \text{ kN}}$$

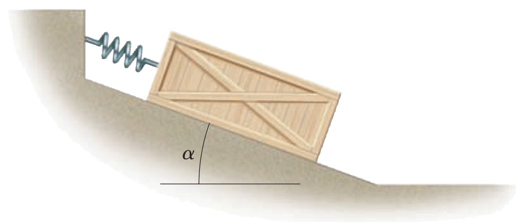
(b) $F = 0.6$ N

$$\sum F_{\nearrow} : N - 11.772 \text{ kN} \cos\alpha = 0 \qquad \Rightarrow \boxed{\alpha = 31.0^\circ}$$

$$\sum F_{\nwarrow} : F - 11.772 \text{ kN} \sin\alpha = 0$$



Problem 3.13 The 100-lb crate is in equilibrium on the smooth surface. The spring constant is $k = 400$ lb/ft. Let S be the stretch of the spring. Obtain an equation for S (in feet) as a function of the angle α.

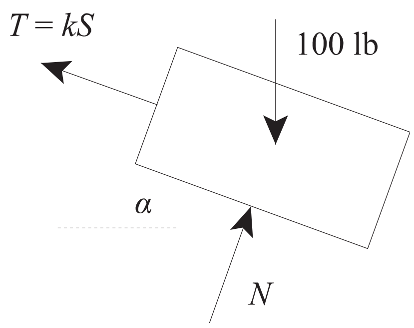


Solution: The free-body diagram is shown.
The equilibrium equation in the direction parallel to the inclined surface is

$$kS - (100 \text{ lb}) \sin\alpha = 0$$

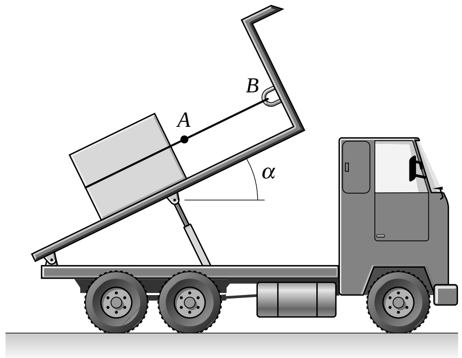
Solving for S and using the given value for k we find

$$\boxed{S = (0.25 \text{ ft}) \sin\alpha}$$

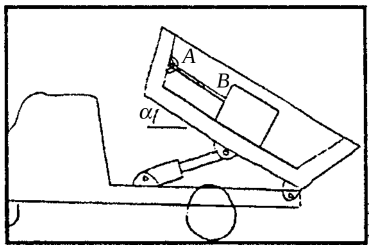


Problem 3.14 A 600-lb box is held in place on the smooth bed of the dump truck by the rope *AB*.

(a) If $\alpha = 25^\circ$, what is the tension in the rope?
(b) If the rope will safely support a tension of 400 lb, what is the maximum allowable value of α?

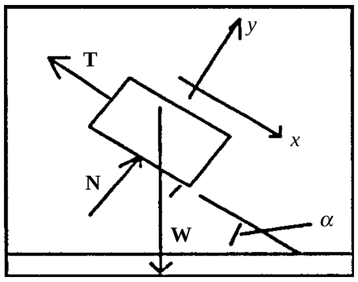


Solution: Isolate the box. Resolve the forces into scalar components, and solve the equilibrium equations.



The external forces are the weight, the tension in the rope, and the normal force exerted by the surface. The angle between the x axis and the weight vector is $-(90-\alpha)$ (or $270+\alpha$). The weight vector is

$$\mathbf{W} = |\mathbf{W}|(\mathbf{i}\sin\alpha - \mathbf{j}\cos\alpha) = (600)(\mathbf{i}\sin\alpha - \mathbf{j}\cos\alpha)$$



The projections of the rope tension and the normal force are

$$\mathbf{T} = -|\mathbf{T}_x|\mathbf{i} + 0\mathbf{j} \quad \mathbf{N} = 0\mathbf{i} + |\mathbf{N}_y|\mathbf{j}$$

The equilibrium conditions are

$$\sum \mathbf{F} = \mathbf{W} + \mathbf{N} + \mathbf{T} = 0$$

Substitute, and collect like terms

$$\sum \mathbf{F}_x = (600\sin\alpha - |\mathbf{T}_x|)\mathbf{i} = 0$$

$$\sum \mathbf{F}_y = (-600\cos\alpha + |\mathbf{N}_y|)\mathbf{j} = 0$$

Solve for the unknown tension when

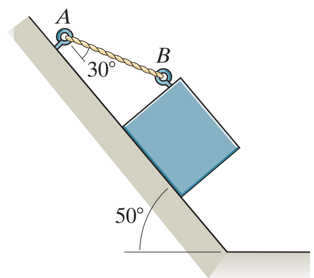
For $\alpha = 25^\circ$

$|\mathbf{T}_x| = 600\sin\alpha = 253.6$ lb.

For a tension of 400 lb, $(600\sin\alpha - 400) = 0$. Solve for the unknown angle

$$\sin\alpha = \frac{400}{600} = 0.667 \text{ or } \alpha = 41.84^\circ$$

Problem 3.15 The 80-lb box is held in place on the smooth inclined surface by the rope AB. Determine the tension in the rope and the normal force exerted on the box by the inclined surface.

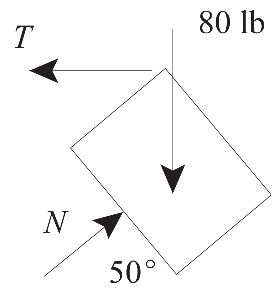


Solution: The equilibrium equations (in terms of a coordinate system with the x axis parallel to the inclined surface) are

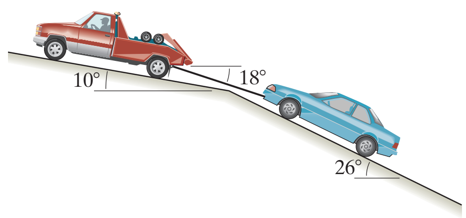
$$\sum F_x : (80 \text{ lb}) \sin 50^\circ - T \cos 50 = 0$$

$$\sum F_x : N - (80 \text{ lb}) \cos 50^\circ - T \sin 50 = 0$$

Solving: $\boxed{T = 95.34 \text{ lb}, N = 124 \text{ lb}}$



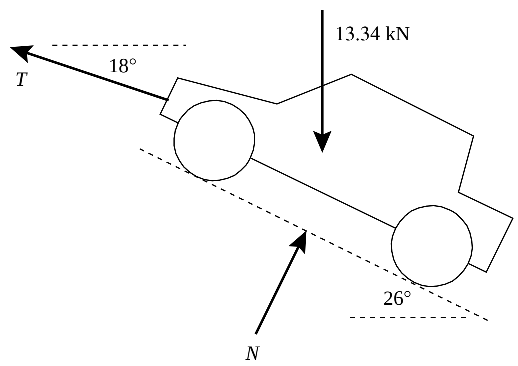
Problem 3.16 The 1360-kg car and the 2100-kg tow truck are stationary. The muddy surface on which the car's tires rest exerts negligible friction forces on them. What is the tension in the tow cable?



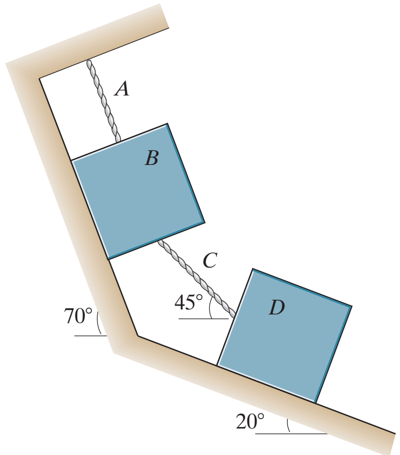
Solution: FBD of the car being towed

$$\sum F_{\nwarrow} : T \cos 8^\circ - 13.34 \text{ kN} \sin 26^\circ = 0$$

$$\boxed{T = 5.91 \text{ kN}}$$



Problem 3.17 Each box weighs 40 lb. The angles are measured relative to the horizontal. The surfaces are smooth. Determine the tension in the rope A and the normal force exerted on box B by the inclined surface.



Solution: The free-body diagrams are shown.
The equilibrium equations for box D are

$$\sum F_x : (40 \text{ lb}) \sin 20^\circ - T_C \cos 25^\circ = 0$$

$$\sum F_y : N_D - (40 \text{ lb}) \cos 20^\circ + T_C \sin 25^\circ = 0$$

The equilibrium equations for box B are

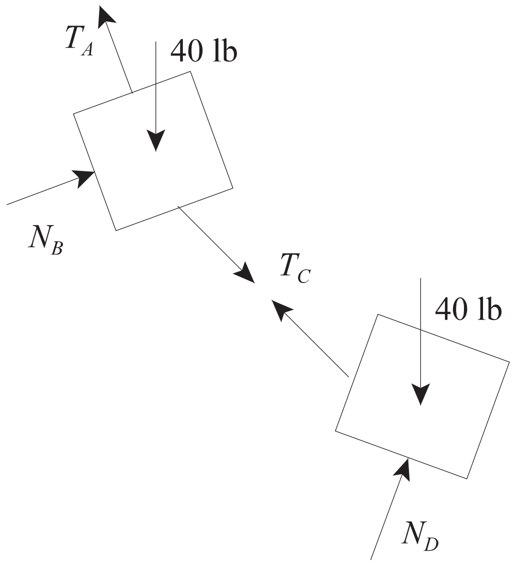
$$\sum F_x : (40 \text{ lb}) \sin 70^\circ + T_C \cos 25^\circ - T_A = 0$$

$$\sum F_y : N_B - (40 \text{ lb}) \cos 70^\circ + T_C \sin 25^\circ = 0$$

Solving these four equations yields:

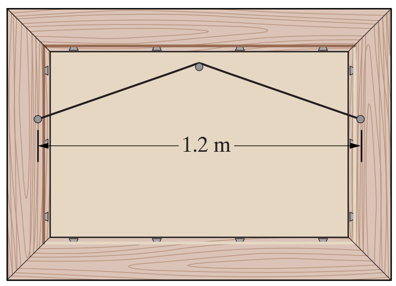
$$T_A = 51.2 \text{ lb}, T_C = 15.1 \text{ lb}, N_B = 7.30 \text{ lb}, N_D = 31.2 \text{ lb}$$

Thus $\boxed{T_A = 51.2 \text{ lb},\ N_B = 7.30 \text{ lb}}$



Problem 3.18 A 10-kg painting is hung with a wire supported by a nail. The length of the wire is 1.3 m.

(a) What is the tension in the wire?
(b) What is the magnitude of the force exerted on the nail by the wire?

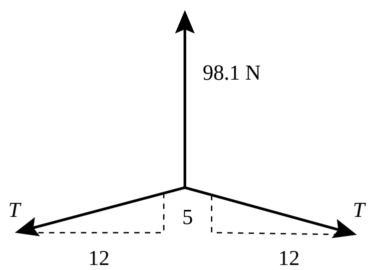


Solution:

(a) $\sum F_y : 98.1 \text{ N} - 2\frac{5}{13}T = 0$

$\boxed{T = 128 \text{ N}}$

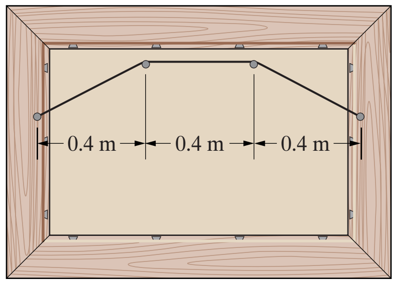
(b) $\boxed{\text{Force} = 98.1 \text{ N}}$



Problem 3.19 A 10-kg painting is hung with a wire supported by two nails. The length of the wire is 1.3 m.

(a) What is the tension in the wire?
(b) What is the magnitude of the force exerted on each nail by the wire? (Assume that the tension is the same in each part of the wire.)

Compare your answers to the answers to Problem 3.18.

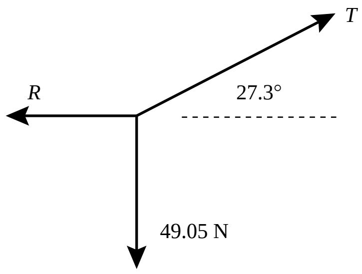


Solution:

(a) Examine the point on the left where the wire is attached to the picture. This point supports half of the weight

$$\sum F_y : T \sin 27.3^\circ - 49.05 \text{ N} = 0$$

$$\boxed{T = 107 \text{ N}}$$



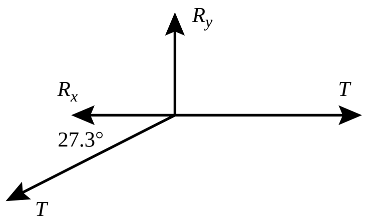
(b) Examine one of the nails

$$\sum F_x : -R_x - T \cos 27.3^\circ + T = 0$$

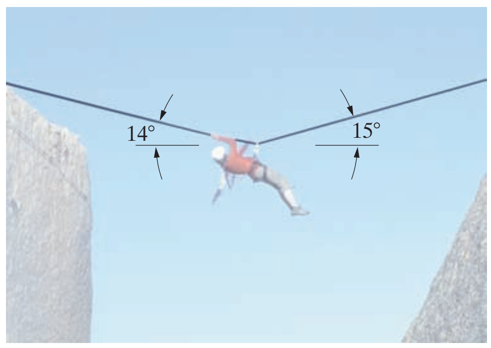
$$\sum F_y : R_y - T \sin 27.3^\circ = 0$$

$$R = \sqrt{{R_x}^2 + {R_y}^2}$$

$$\boxed{R = 50.5 \text{ N}}$$



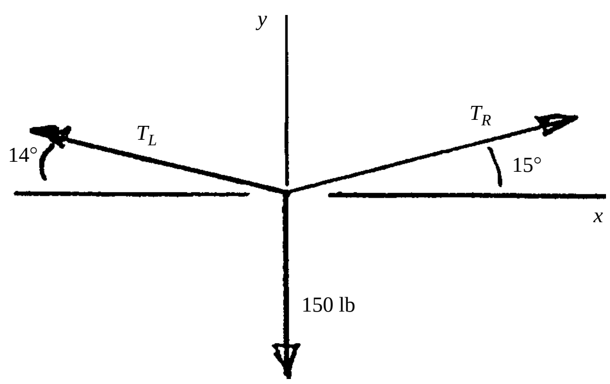
Problem 3.20 Assume that the 150-lb climber is in equilibrium. What are the tensions in the rope on the left and right sides?



Solution:

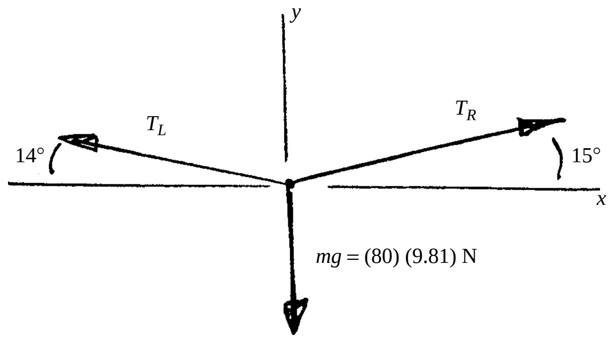
$$\begin{cases} \sum F_x = T_R \cos(15^\circ) - T_L \cos(14^\circ) = 0 \\ \sum F_y = T_R \sin(15^\circ) + T_L \sin(14^\circ) - 150 = 0 \end{cases}$$

Solving, we get $T_L = 299$ lb, $T_R = 300$ lb



Problem 3.21 If the mass of the climber shown in Problem 3.20 is 80 kg, what are the tensions in the rope on the left and right sides?

Solution:

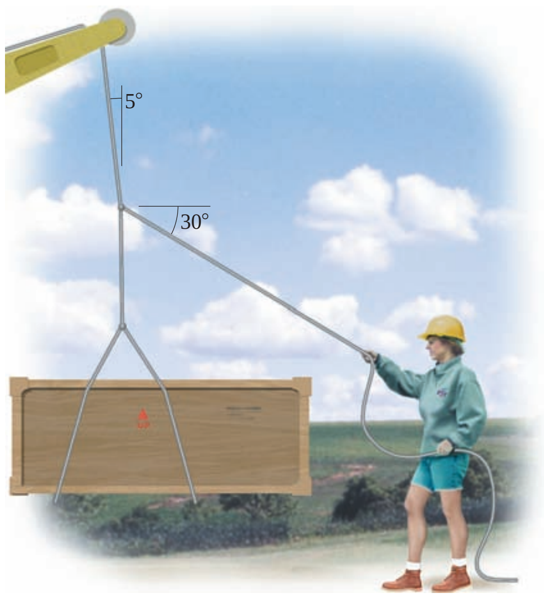


$$\begin{cases} \sum F_x = T_R\cos(15^\circ) - T_L\cos(14^\circ) = 0 \\ \sum F_y = T_R\sin(15^\circ) + T_L\sin(14^\circ) - mg = 0 \end{cases}$$

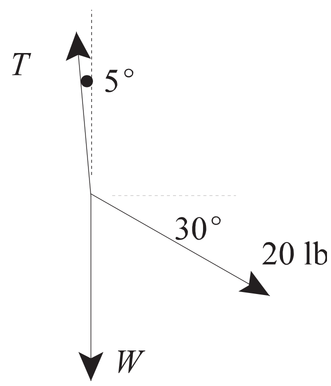
Solving, we get

$T_L = 1.56$ kN, $\quad T_R = 1.57$ kN

Problem 3.22 The construction worker exerts a 20-lb force on the rope to hold the crate in equilibrium in the position shown. What is the weight of the crate?



Solution: The free-body diagram is shown.
The equilibrium equations for the part of the rope system where the three ropes are joined are

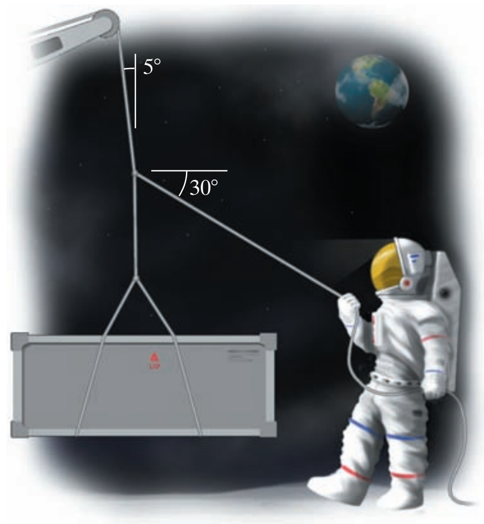


$$\sum F_x : (20\text{ lb})\cos 30^\circ - T\sin 5^\circ = 0$$

$$\sum F_y : -(20\text{ lb})\sin 30^\circ + T\cos 5^\circ - W = 0$$

Solving yields $\boxed{W = 188\text{ lb}}$

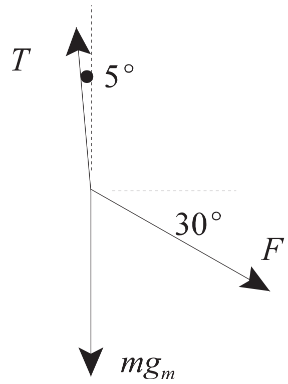
Problem 3.23 A construction worker on the moon, where the acceleration due to gravity is 1.62 m/s_2, holds the same crate described in Problem 3.22 in the position shown. What force must she exert on the cable to hold the crate ub equilibrium (a) in newtons; (b) in pounds?



Solution: The free-body diagram is shown.
From Problem 3.22 we know that the weight is $W = 188$ lb. Therefore its mass is

$$m = \frac{188 \text{ lb}}{32.2 \text{ ft/s}^2} = 5.84 \text{ slug}$$

$$m = 5.84 \text{ slug}\left(\frac{14.59 \text{ kg}}{\text{slug}}\right) = 85.2 \text{ kg}$$



The equilibrium equations for the part of the rope system where the three ropes are joined are

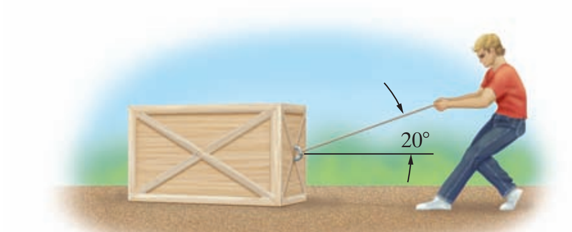
$$\sum F_x : F\cos 30^\circ - T\sin 5^\circ = 0$$

$$\sum F_y : -F\sin 30^\circ + T\cos 5^\circ - mg_m = 0$$

where $g_m = 1.62$ m/s^2.
Solving yields $F = 3.30$ lb $= 14.7$ N

$$\boxed{(a)\ F = 14.7 \text{ N},\ (b)\ F = 3.30 \text{ lb}}$$

Problem 3.24 The person wants to cause the 200-lb crate to start sliding toward the right. To achieve this, the *horizontal component* of the force exerted on the crate by the rope must equal 0.35 times the normal force exerted on the crate by the floor. In Fig.a, the person pulls on the rope in the direction shown. In Fig.b, the person attaches the rope to a support as shown and pulls upward on the rope. What is the magnitude of the force he must exert on the rope in each case?

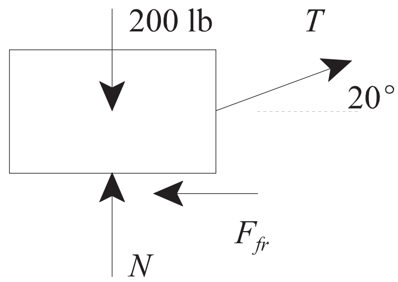


(a)

(b)

Solution: The friction force F_{fr} is given by

$$F_{fr} = 0.35N$$



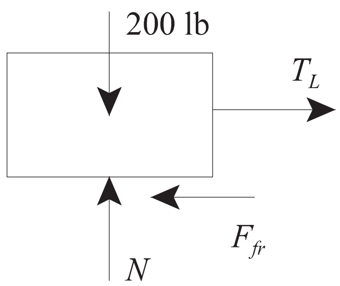
(a) For equilibrium we have

$$\sum F_x : T\cos 20^\circ - 0.35N = 0$$

$$\sum F_y : T\sin 20^\circ - 200\ \text{lb} + N = 0$$

Solving: $\boxed{T = 66.1\ \text{lb}}$

(b) The person exerts the force F. Using the free-body diagram of the crate and of the point on the rope where the person grabs the rope, we find



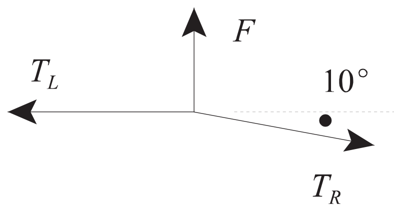
$$\sum F_x : T_L - 0.35N = 0$$

$$\sum F_y : N - 200\ \text{lb} = 0$$

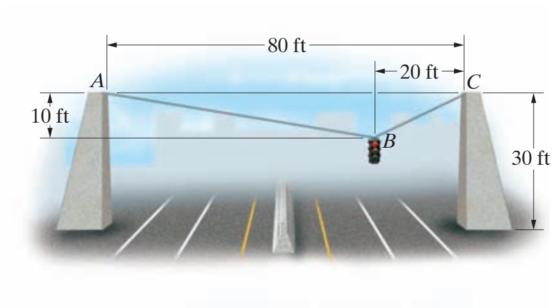
$$\sum F_x : -T_L + T_R\cos 10^\circ = 0$$

$$\sum F_y : F - T_R\sin 10^\circ = 0$$

Solving we find $\boxed{F = 12.3\ \text{lb}}$



Problem 3.25 A traffic engineer wants to suspend a 200-lb traffic light above the center of the two right lanes of a four-lane thoroughfare as shown. Determine the tensions in the cables AB and BC.

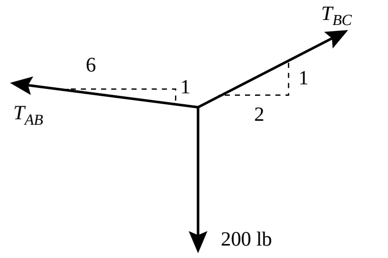


Solution:

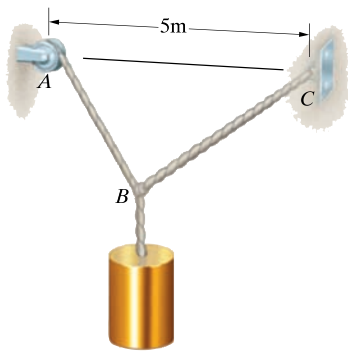
$$\sum F_x: -\frac{6}{\sqrt{37}}T_{AB} + \frac{2}{\sqrt{5}}T_{BC} = 0$$

$$\sum F_y: \frac{1}{\sqrt{37}}T_{AB} + \frac{1}{\sqrt{5}}T_{BC} - 200\text{ lb} = 0$$

Solving: $\boxed{T_{AB} = 304\text{ lb},\ T_{BC} = 335\text{ lb}}$



Problem 3.26 Cable AB is 3 m long and cable BC is 4 m long. The mass of the suspended object is 350 kg. Determine the tensions in cables AB and BC.

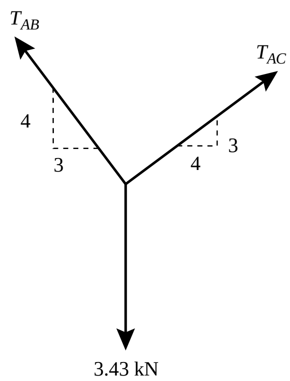


Solution:

$$\sum F_x: -\frac{3}{5}T_{AB} + \frac{4}{5}T_{BC} = 0$$

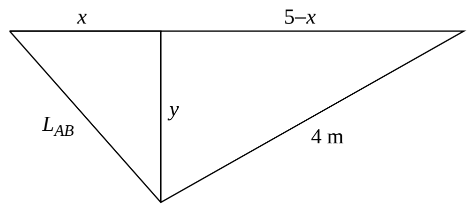
$$\sum F_y: \frac{4}{5}T_{AB} + \frac{3}{5}T_{BC} - 3.43\text{ kN} = 0$$

$\boxed{T_{AB} = 2.75\text{ kN},\ T_{BC} = 2.06\text{ kN}}$



Problem 3.27 In Problem 3.26, the length of cable AB is adjustable. If you don't want the tension in either cable AB or cable BC to exceed 3 kN, what is the minimum acceptable length of cable AB?

Solution: Consider the geometry:



We have the constraints

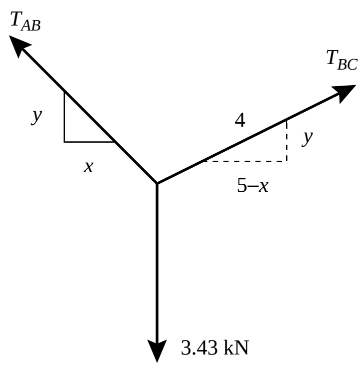
$$L_{AB}{}^2 = x^2 + y^2, \ \ (4 \text{ m})^2 = (5 \text{ m} - x)^2 + y^2$$

These constraint imply

$$y = \sqrt{(10 \text{ m})x - x^2 - 9 \text{ m}^2}$$

$$L = \sqrt{(10 \text{ m})x - 9 \text{ m}^2}$$

Now draw the FBD and write the equations in terms of x



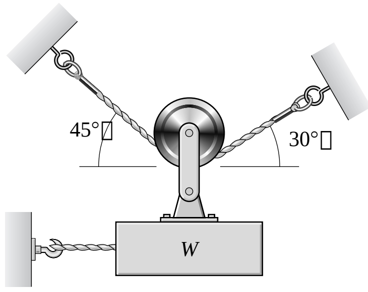
$$\sum F_x : -\frac{x}{\sqrt{10x - 9}} T_{AB} + \frac{5 - x}{4} T_{BC} = 0$$

$$\sum F_y : \frac{\sqrt{10x - x^2 - 9}}{\sqrt{10x - 9}} T_{AB} + \frac{\sqrt{10x - x^2 - 9}}{4} T_{BC} - 3.43 \text{ kN} = 0$$

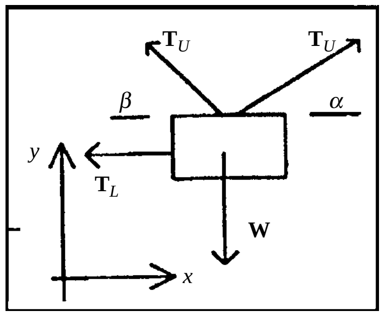
If we set $T_{AB} = 3$ kN and solve for x we find $x = 1.535$, $T_{BC} = 2.11 \text{ kN} < 3 \text{ kN}$

Using this value for x we find that $\boxed{L_{AB} = 2.52 \text{ m}}$

Problem 3.28 What are the tensions in the upper and lower cables? (Your answers will be in terms of W. Neglect the weight of the pulley.)



Solution: Isolate the weight. The frictionless pulley changes the direction but not the magnitude of the tension. The angle between the right hand upper cable and the x axis is α, hence



$$\mathbf{T}_{UR} = |\mathbf{T}_U|(\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha).$$

The angle between the positive x and the left hand upper pulley is $(180^\circ - \beta)$, hence

$$\mathbf{T}_{UL} = |\mathbf{T}_U|(\mathbf{i}\cos(180 - \beta) + \mathbf{j}\sin(180 - \beta))$$

$$= |\mathbf{T}_U|(-\mathbf{i}\cos\beta + \mathbf{j}\sin\beta).$$

The lower cable exerts a force: $\mathbf{T}_L = -|\mathbf{T}_L|\mathbf{i} + 0\mathbf{j}$

The weight: $\mathbf{W} = 0\mathbf{i} - |\mathbf{W}|\mathbf{j}$

The equilibrium conditions are

$$\sum \mathbf{F} = \mathbf{W} + \mathbf{T}_{UL} + \mathbf{T}_{UR} + \mathbf{T}_L = 0$$

Substitute and collect like terms,

$$\sum \mathbf{F}_x = (-|\mathbf{T}_U|\cos\beta + |\mathbf{T}_U|\cos\alpha - |\mathbf{T}_L|)\mathbf{i} = 0$$

$$\sum \mathbf{F}_y = (|\mathbf{T}_U|\sin\alpha + |\mathbf{T}_U|\sin\beta - |\mathbf{W}|)\mathbf{j} = 0.$$

Solve: $$|\mathbf{T}_U| = \left(\frac{|\mathbf{W}|}{(\sin\alpha + \sin\beta)}\right),$$

$$|\mathbf{T}_L| = |\mathbf{T}_U|(\cos\alpha - \cos\beta).$$

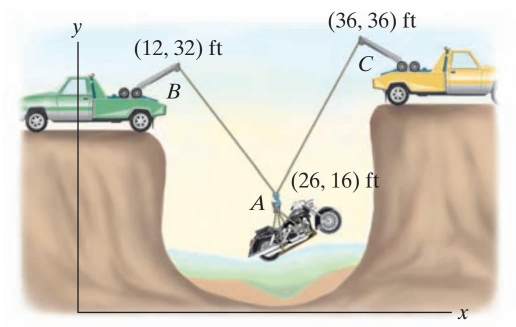
From which $$|\mathbf{T}_L| = |\mathbf{W}|\left(\frac{\cos\alpha - \cos\beta}{\sin\alpha + \sin\beta}\right).$$

For $\alpha = 30^\circ$ and $\beta = 45^\circ$

$|\mathbf{T}_U| = 0.828|\mathbf{W}|$,

$|\mathbf{T}_L| = 0.132|\mathbf{W}|$

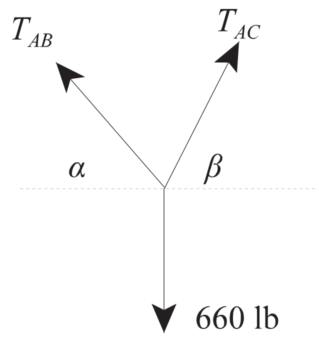
Problem 3.29 Two tow trucks lift a 660-lb motorcycle out of a ravine following an accident. If the motorcycle is in equilibrium in the position shown, what are the tensions in cables AB and AC?



Solution: The angles are

$$\alpha = \tan^{-1}\left(\frac{32-16}{26-12}\right) = 48.8^\circ$$

$$\beta = \tan^{-1}\left(\frac{36-16}{36-26}\right) = 63.4^\circ$$



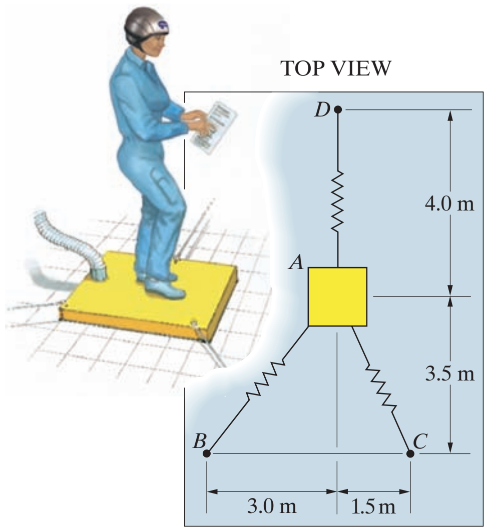
Now from equilibrium we have

$$\sum F_x : -T_{AB}\cos\alpha + T_{AC}\cos\beta = 0$$

$$\sum F_y : T_{AB}\sin\alpha + T_{AC}\sin\beta - 660 \text{ lb} = 0$$

Solving yields $\boxed{T_{AB} = 319 \text{ lb}, T_{AC} = 470 \text{ lb}}$

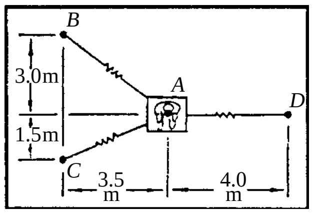
Problem 3.30 An astronaut candidate conducts experiments on an airbearing platform. While she carries out calibrations, the platform is held in place by the horizontal tethers AB, AC, and AD. The forces exerted by the tethers are the only horizontal forces acting on the platform. If the tension in tether AC is 2 N, what are the tensions in the other two tethers?

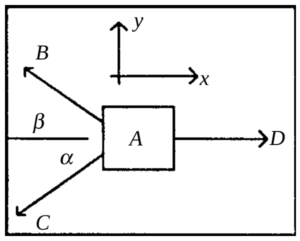


Solution: Isolate the platform. The angles α and β are

$$\tan\alpha = \left(\frac{1.5}{3.5}\right) = 0.429, \quad \alpha = 23.2^\circ.$$

Also, $$\tan\beta = \left(\frac{3.0}{3.5}\right) = 0.857, \quad \beta = 40.6^\circ.$$





The angle between the tether AB and the positive x axis is $(180^\circ - \beta)$, hence

$$\mathbf{T}_{AB} = |\mathbf{T}_{AB}|(\mathbf{i}\cos(180^\circ - \beta) + \mathbf{j}\sin(180^\circ - \beta))$$

$$\mathbf{T}_{AB} = |\mathbf{T}_{AB}|(-\mathbf{i}\cos\beta + \mathbf{j}\sin\beta).$$

The angle between the tether AC and the positive x axis is $(180^\circ + \alpha)$. The tension is

$$\mathbf{T}_{AC} = |\mathbf{T}_{AC}|(\mathbf{i}\cos(180^\circ + \alpha) + \mathbf{j}\sin(180^\circ + \alpha))$$

$$= |\mathbf{T}_{AC}|(-\mathbf{i}\cos\alpha - \mathbf{j}\sin\alpha).$$

The tether AD is aligned with the positive x axis, $\mathbf{T}_{AD} = |\mathbf{T}_{AD}|\mathbf{i} + 0\mathbf{j}$.

The equilibrium condition:

$$\sum \mathbf{F} = \mathbf{T}_{AD} + \mathbf{T}_{AB} + \mathbf{T}_{AC} = 0.$$

Substitute and collect like terms,

$$\sum \mathbf{F}_x = (-|\mathbf{T}_{AB}|\cos\beta - |\mathbf{T}_{AC}|\cos\alpha + |\mathbf{T}_{AD}|)\mathbf{i} = 0,$$

$$\sum \mathbf{F}_y = (|\mathbf{T}_{AB}|\sin\beta - |\mathbf{T}_{AC}|\sin\alpha)\mathbf{j} = 0.$$

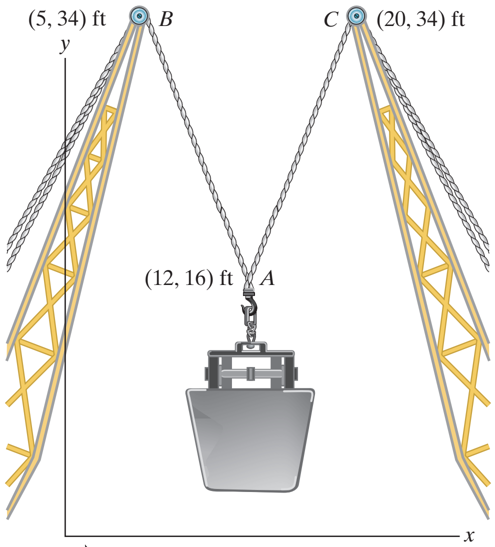
Solve: $$|\mathbf{T}_{AB}| = \left(\frac{\sin\alpha}{\sin\beta}\right)|\mathbf{T}_{AC}|,$$

$$|\mathbf{T}_{AD}| = \left(\frac{|\mathbf{T}_{AC}|\sin(\alpha+\beta)}{\sin\beta}\right).$$

For $|\mathbf{T}_{AC}| = 2$ N, $\alpha = 23.2^\circ$ and $\beta = 40.6^\circ$,

$$|\mathbf{T}_{AB}| = 1.21 \text{ N}, \quad |\mathbf{T}_{AD}| = 2.76 \text{ N}$$

Problem 3.31 The bucket contains concrete and weighs 5800 lb. What are the tensions in the cables AB and AC?



Solution: The angles are

$$\alpha = \tan^{-1}\left(\frac{34-16}{12-5}\right) = 68.7^\circ$$

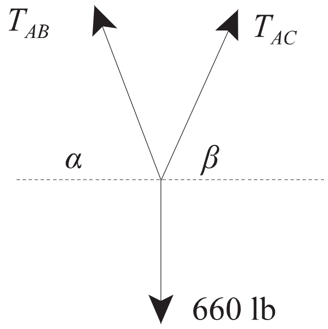
$$\beta = \tan^{-1}\left(\frac{34-16}{20-12}\right) = 66.0^\circ$$

Now from equilibrium we have

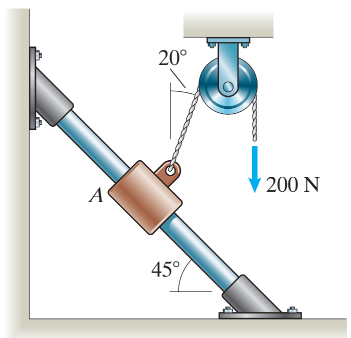
$$\sum F_x : -T_{AB}\cos\alpha + T_{AC}\cos\beta = 0$$

$$\sum F_y : T_{AB}\sin\alpha + T_{AC}\sin\beta - 660 \text{ lb} = 0$$

Solving yields $\boxed{T_{AB} = 319 \text{ lb},\ T_{AC} = 470 \text{ lb}}$



Problem 3.32 The slider A is in equilibrium and the bar is smooth. What is the mass of the slider?



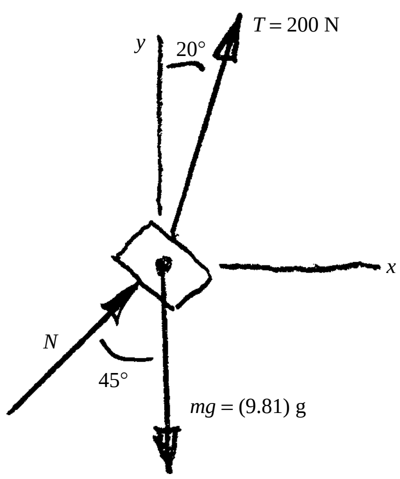
Solution: The pulley does not change the tension in the rope that passes over it. There is no friction between the slider and the bar.

Eqns. of Equilibrium:

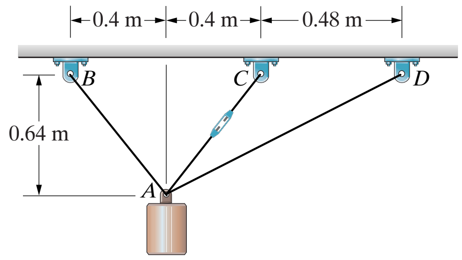
$$\begin{cases} \sum F_x = T\sin 20^\circ + N\cos 45^\circ = 0 \quad (T = 200 \text{ N}) \\ \sum F_y = N\sin 45^\circ + T\cos 20^\circ - mg = 0 \quad g = 9.81 \text{ m/s}^2 \end{cases}$$

Substituting for T and g, we have two eqns in two unknowns (N and m).

Solving, we get $N = -96.7$ N, $m = 12.2$ kg.



Problem 3.33 The 20-kg mass is suspended from three cables. Cable AC is equipped with a turnbuckle so that its tension can be adjusted and a strain gauge that allows its tension to be measured. If the tension in cable AC is 40 N, what are the tensions in cables AB and AD?



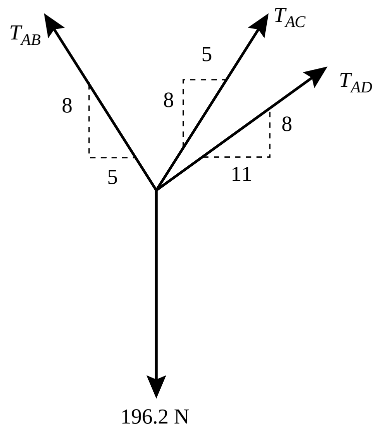
Solution:

$T_{AC} = 40$ N

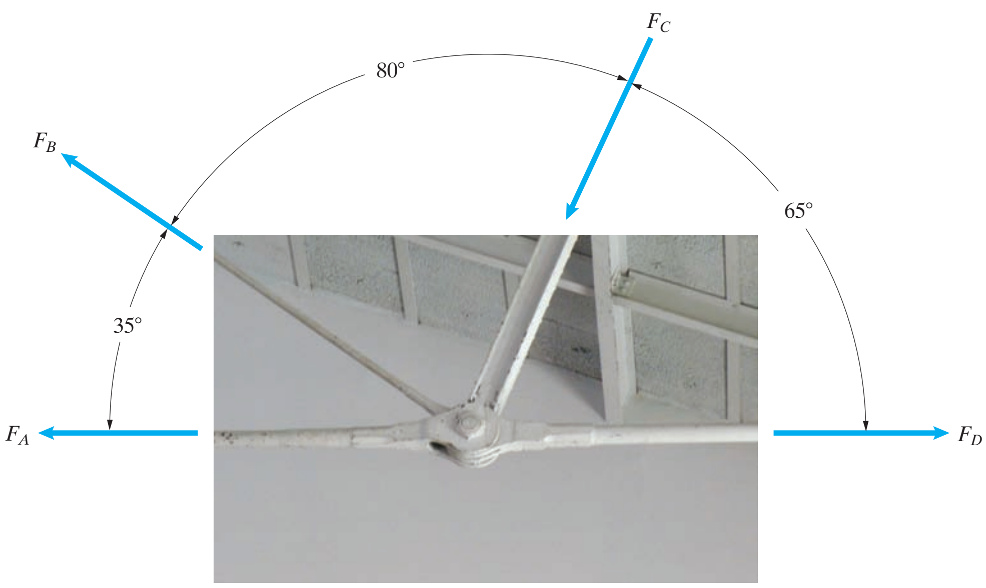
$$\sum F_x: -\frac{5}{\sqrt{89}}T_{AB} + \frac{5}{\sqrt{89}}T_{AC} + \frac{11}{\sqrt{185}}T_{AD} = 0$$

$$\sum F_y: \frac{8}{\sqrt{89}}T_{AB} + \frac{8}{\sqrt{89}}T_{AC} + \frac{8}{\sqrt{185}}T_{AD} - 196.2\text{ N} = 0$$

Solving: $\boxed{T_{AB} = 144.1\text{ N},\ T_{AD} = 68.2\text{ N}}$



Problem 3.34 The structural joint is in equilibrium. If $F_A = 1000$ lb and $F_D = 5000$ lb, what are F_B and F_C?



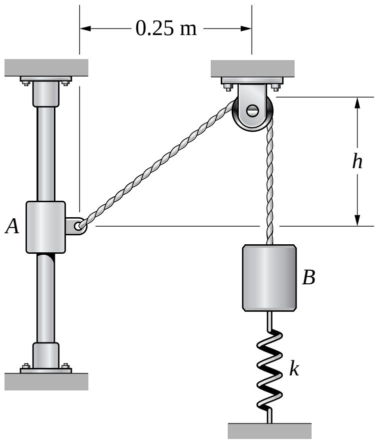
Solution: The equilibrium equations are

$$\sum F_x: F_D - F_C \cos 65^\circ - F_B \cos 35^\circ - F_A = 0$$

$$\sum F_y: -F_C \sin 65^\circ + F_B \sin 35^\circ = 0$$

Solving yields $\boxed{F_B = 3680\text{ lb},\ F_C = 2330\text{ lb}}$

Problem 3.35 The collar A slides on the smooth vertical bar. The masses $m_A = 20$ kg and $m_B = 10$ kg. When $h = 0.1$ m, the spring is unstretched. When the system is in equilibrium, $h = 0.3$ m. Determine the spring constant k.



Solution: The triangles formed by the rope segments and the horizontal line level with A can be used to determine the lengths L_u and L_s. The equations are

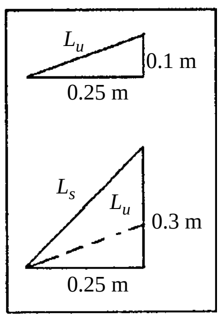
$L_u = \sqrt{(0.25)^2 + (0.1)^2}$ and $L_s = \sqrt{(0.25)^2 + (0.3)^2}$.

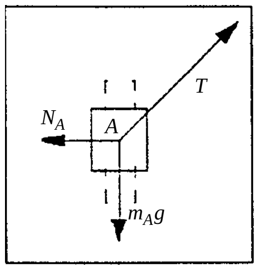
The stretch in the spring when in equilibrium is given by $\delta = L_s - L_u$. Carrying out the calculations, we get $L_u = 0.269$ m, $L_s = 0.391$ m, and $\delta = 0.121$ m. The angle, θ, between the rope at A and the horizontal when the system is in equilibrium is given by $\tan\theta = 0.3/0.25$, or $\theta = 50.2^\circ$. From the free body diagram for mass A, we get two equilibrium equations. They are

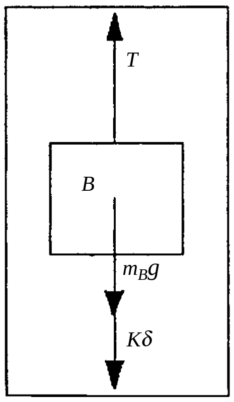
$$\sum F_x = -N_A + T\cos\theta = 0$$

and $$\sum F_y = T\sin\theta - m_A g = 0.$$

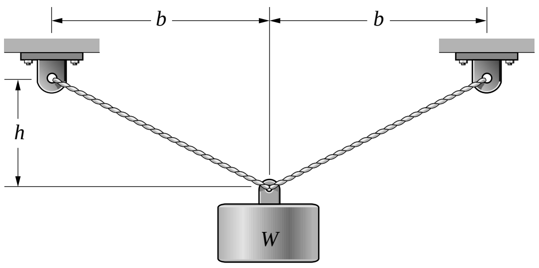
We have two equations in two unknowns and can solve. We get $N_A = 163.5$ N and $T = 255.4$ N. Now we go to the free body diagram for B, where the equation of equilibrium is $T - m_B g - k\delta = 0$. This equation has only one unknown. Solving, we get $\boxed{k = 1297 \text{ N/m}}$



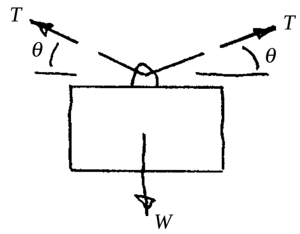




Problem 3.36* Suppose that you want to design a cable system to suspend an object of weight W from the ceiling. The two wires must be identical, and the dimension b is fixed. The ratio of the tension T in each wire to its cross-sectional area A must equal a specified value $T/A = \sigma$. The "cost" of your design is the total volume of material in the two wires, $V = 2A\sqrt{b^2 + h^2}$. Determine the value of h that minimizes the cost.



Solution: From the equation



$$\sum F_y = 2T\sin\theta - W = 0,$$

we obtain $T = \dfrac{W}{2\sin\theta} = \dfrac{W\sqrt{b^2+h^2}}{2h}$.

Since $T/A = \sigma$, $A = \dfrac{T}{\sigma} = \dfrac{W\sqrt{b^2+h^2}}{2\sigma h}$

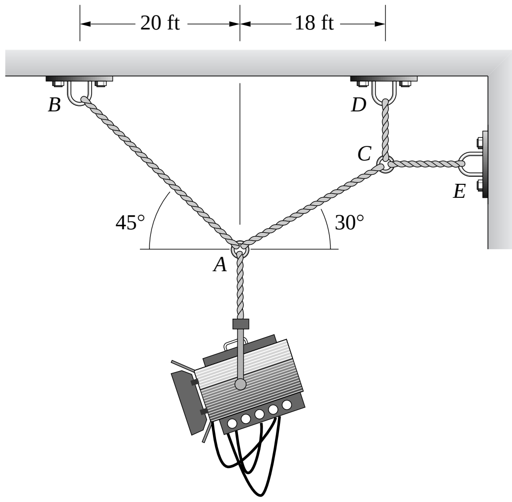
and the "cost" is $V = 2A\sqrt{b^2+h^2} = \dfrac{W(b^2+h^2)}{\sigma h}$.

To determine the value of h that minimizes V, we set

$$\frac{dV}{dh} = \frac{W}{\sigma}\left[-\frac{(b^2+h^2)}{h^2} + 2\right] = 0$$

and solve for h, obtaining $h = b$.

Problem 3.37 The system of cables suspends a 1000-lb bank of lights above a movie set. Determine the tensions in cables AB, CD, and CE.



Solution: Isolate juncture A, and solve the equilibrium equations. Repeat for the cable juncture C.

The angle between the cable AC and the positive x axis is α. The tension in AC is $\mathbf{T}_{AC} = |\mathbf{T}_{AC}|(\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha)$

The angle between the x axis and AB is $(180^\circ - \beta)$. The tension is

$$\mathbf{T}_{AB} = |\mathbf{T}_{AB}|(\mathbf{i}\cos(180 - \beta) + \mathbf{j}\sin(180 - \beta))$$

$$\mathbf{T}_{AB} = (-\mathbf{i}\cos\beta + \mathbf{j}\sin\beta).$$

The weight is $\mathbf{W} = 0\mathbf{i} - |\mathbf{W}|\mathbf{j}$.

The equilibrium conditions are

$$\sum \mathbf{F} = 0 = \mathbf{W} + \mathbf{T}_{AB} + \mathbf{T}_{AC} = 0.$$

Substitute and collect like terms,

$$\sum \mathbf{F}_x = (|\mathbf{T}_{AC}|\cos\alpha - |\mathbf{T}_{AB}|\cos\beta)\mathbf{i} = 0$$

$$\sum \mathbf{F}_y = (|\mathbf{T}_{AB}|\sin\beta + |\mathbf{T}_{AC}|\sin\alpha - |\mathbf{W}|)\mathbf{j} = 0.$$

Solving, we get

$$|\mathbf{T}_{AB}| = \left(\frac{\cos\alpha}{\cos\beta}\right)|\mathbf{T}_{AC}| \quad \text{and} \quad |\mathbf{T}_{AC}| = \left(\frac{|\mathbf{W}|\cos\beta}{\sin(\alpha+\beta)}\right),$$

$$|\mathbf{W}| = 1000 \text{ lb}, \text{ and } \alpha = 30^\circ, \ \beta = 45^\circ$$

$$|\mathbf{T}_{AC}| = (1000)\left(\frac{0.7071}{0.9659}\right) = 732.05 \text{ lb}$$

$$|\mathbf{T}_{AB}| = (732)\left(\frac{0.866}{0.7071}\right) = 896.5 \text{ lb}$$

Isolate juncture C. The angle between the positive x axis and the cable CA is $(180^\circ - \alpha)$. The tension is

$$\mathbf{T}_{CA} = |\mathbf{T}_{CA}|(\mathbf{i}\cos(180^\circ + \alpha) + \mathbf{j}\sin(180^\circ + \alpha)),$$

or $\mathbf{T}_{CA} = |\mathbf{T}_{CA}|(-\mathbf{i}\cos\alpha - \mathbf{j}\sin\alpha)$.

The tension in the cable CE is

$$\mathbf{T}_{CE} = \mathbf{i}|\mathbf{T}_{CE}| + 0\mathbf{j}.$$

The tension in the cable CD is $\mathbf{T}_{CD} = 0\mathbf{i} + \mathbf{j}|\mathbf{T}_{CD}|$.

The equilibrium conditions are

$$\sum \mathbf{F} = 0 = \mathbf{T}_{CA} + \mathbf{T}_{CE} + \mathbf{T}_{CD} = 0$$

Substitute t and collect like terms,

$$\sum \mathbf{F}_x = (|\mathbf{T}_{CE}| - |\mathbf{T}_{CA}|\cos\alpha)\mathbf{i} = 0,$$

$$\sum \mathbf{F}_y = (|\mathbf{T}_{CD}| - |\mathbf{T}_{CA}|\sin\alpha)\mathbf{j} = 0.$$

Solve: $|\mathbf{T}_{CE}| = |\mathbf{T}_{CA}|\cos\alpha$,

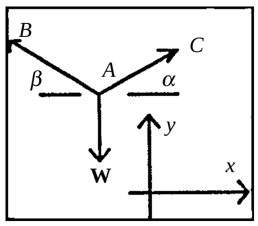
$$|\mathbf{T}_{CD}| = |\mathbf{T}_{CA}|\sin\alpha;$$

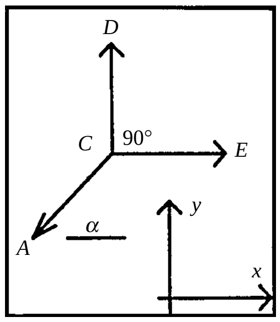
for $|\mathbf{T}_{CA}| = 732$ lb and $\alpha = 30^\circ$,

$$|\mathbf{T}_{AB}| = 896.6 \text{ lb},$$

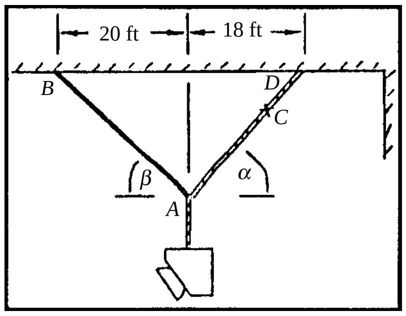
$$|\mathbf{T}_{CE}| = 634 \text{ lb},$$

$$|\mathbf{T}_{CD}| = 366 \text{ lb}$$





Problem 3.38 Consider the 1000-lb bank of lights in Problem 3.37. A technician changes the position of the lights by removing the cable CE. What is the tension in cable AB after the change?



Solution: The original configuration in Problem 3.35 is used to solve for the dimensions and the angles. Isolate the juncture A, and solve the equilibrium conditions.

The lengths are calculated as follows: The vertical interior distance in the triangle is 20 ft, since the angle is 45 deg. and the base and altitude of a 45 deg triangle are equal. The length AB is given by

$$\overline{AB} = \frac{20 \text{ ft}}{\cos 45^\circ} = 28.284 \text{ ft}.$$

The length AC is given by

$$\overline{AC} = \frac{18 \text{ ft}}{\cos 30^\circ} = 20.785 \text{ ft}.$$

The altitude of the triangle for which AC is the hypotenuse is $18 \tan 30^\circ = 10.392$ ft. The distance CD is given by $20 - 10.392 = 9.608$ ft.

The distance AD is given by

$$AD = AC + CD = 20.784 + 9.608 = 30.392$$

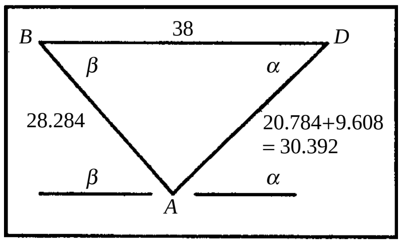
The new angles are given by the cosine law

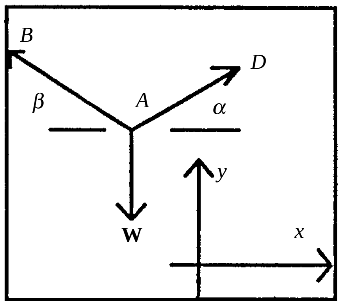
$$AB^2 = 38^2 + AD^2 - 2(38)(AD)\cos\alpha.$$

Reduce and solve:

$$\cos\alpha = \left(\frac{38^2 + (30.392)^2 - (28.284)^2}{2(38)(30.392)}\right) = 0.6787, \;\; \alpha = 47.23^\circ.$$

$$\cos\beta = \left(\frac{(28.284)^2 + (38)^2 - (30.392)^2}{2(28.284)(38)}\right) = 0.6142, \;\; \beta = 52.1^\circ.$$





Isolate the juncture A. The angle between the cable AD and the positive x axis is α. The tension is:

$$\mathbf{T}_{AD} = |\mathbf{T}_{AD}|(\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha).$$

The angle between x and the cable AB is $(180^\circ - \beta)$. The tension is

$$\mathbf{T}_{AB} = |\mathbf{T}_{AB}|(-\mathbf{i}\cos\beta + \mathbf{j}\sin\beta).$$

The weight is $\mathbf{W} = 0\mathbf{i} - |\mathbf{W}|\mathbf{j}$

The equilibrium conditions are

$$\sum \mathbf{F} = 0 = \mathbf{W} + \mathbf{T}_{AB} + \mathbf{T}_{AD} = 0.$$

Substitute and collect like terms,

$$\sum \mathbf{F}_x = (|\mathbf{T}_{AD}|\cos\alpha - |\mathbf{T}_{AB}|\cos\beta)\mathbf{i} = 0,$$

$$\sum \mathbf{F}_y = (|\mathbf{T}_{AB}|\sin\beta + |\mathbf{T}_{AD}|\sin\alpha - |\mathbf{W}|)\mathbf{j} = 0.$$

Solve: $|\mathbf{T}_{AB}| = \left(\dfrac{\cos\alpha}{\cos\beta}\right)|\mathbf{T}_{AD}|,$

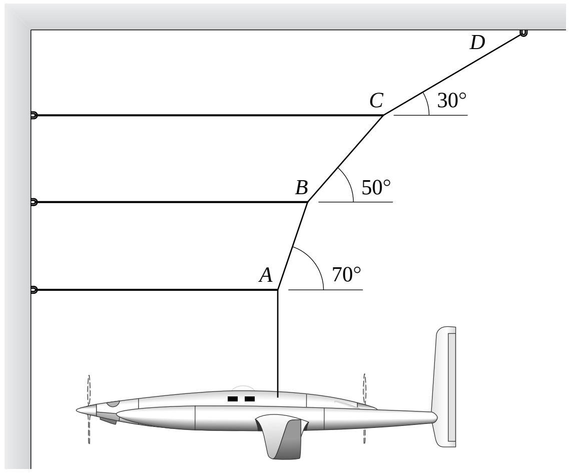
and $|\mathbf{T}_{AD}| = \left(\dfrac{|\mathbf{W}|\cos\beta}{\sin(\alpha+\beta)}\right).$

For $|\mathbf{W}| = 1000$ lb, and $\alpha = 51.2^\circ$, $\beta = 47.2^\circ$

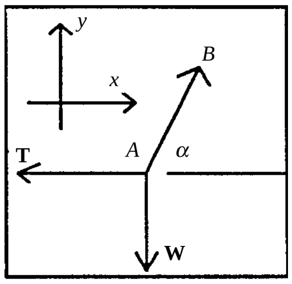
$$|\mathbf{T}_{AD}| = (1000)\left(\frac{0.6142}{0.989}\right) = 621.03 \text{ lb},$$

$$|\mathbf{T}_{AB}| = (622.3)\left(\frac{0.6787}{0.6142}\right) = 687.9 \text{ lb}$$

Problem 3.39 While working on another exhibit, a curator at the Smithsonian Institution pulls the suspended *Voyager* aircraft to one side by attaching three horizontal cables as shown. The mass of the aircraft is 1250 kg. Determine the tensions in the cable segments AB, BC, and CD.



Solution: Isolate each cable juncture, beginning with A and solve the equilibrium equations at each juncture. The angle between the cable AB and the positive x axis is $\alpha = 70°$; the tension in cable AB is $\mathbf{T}_{AB} = |\mathbf{T}_{AB}|(\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha)$. The weight is $\mathbf{W} = 0\mathbf{i} - |\mathbf{W}|\mathbf{j}$. The tension in cable AT is $\mathbf{T} = -|\mathbf{T}|\mathbf{i} + 0\mathbf{j}$. The equilibrium conditions are



$$\sum \mathbf{F} = \mathbf{W} + \mathbf{T} + \mathbf{T}_{AB} = 0.$$

Substitute and collect like terms

$$\sum \mathbf{F}_x (|\mathbf{T}_{AB}|\cos\alpha - |\mathbf{T}|)\mathbf{i} = 0,$$

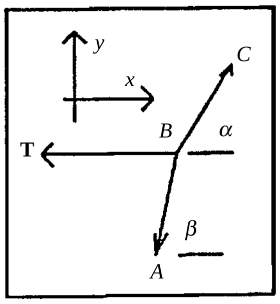
$$\sum \mathbf{F}_y = (|\mathbf{T}_{AB}|\sin\alpha - |\mathbf{W}|)\mathbf{j} = 0.$$

Solve: the tension in cable AB is $|\mathbf{T}_{AB}| = \left(\dfrac{|\mathbf{W}|}{\sin\alpha}\right)$.

For $|\mathbf{W}| = (1250 \text{ kg})\left(9.81 \ \dfrac{\text{m}}{\text{s}^2}\right) = 12262.5 \text{ N}$ and $\alpha = 70°$

$$|\mathbf{T}_{AB}| = \left(\frac{12262.5}{0.94}\right) = 13049.5 \text{ N}$$

Isolate juncture B. The angles are $\alpha = 50°$, $\beta = 70°$, and the tension cable BC is $\mathbf{T}_{BC} = |\mathbf{T}_{BC}|(\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha)$. The angle between the cable BA and the positive x axis is $(180 + \beta)$; the tension is



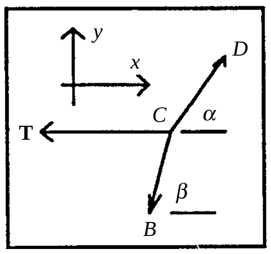
$$\mathbf{T}_{BA} = |\mathbf{T}_{BA}|(\mathbf{i}\cos(180 + \beta) + \mathbf{j}\sin(180 + \beta))$$

$$= |\mathbf{T}_{BA}|(-\mathbf{i}\cos\beta - \mathbf{j}\sin\beta)$$

The tension in the left horizontal cable is $\mathbf{T} = -|\mathbf{T}|\mathbf{i} + 0\mathbf{j}$. The equilibrium conditions are

$$\sum \mathbf{F} = \mathbf{T}_{BA} + \mathbf{T}_{BC} + \mathbf{T} = 0.$$

Substitute and collect like terms



$$\sum \mathbf{F}_x = (|\mathbf{T}_{BC}|\cos\alpha - |\mathbf{T}_{BA}|\cos\beta - |\mathbf{T}|)\mathbf{i} = 0$$

$$\sum \mathbf{F}_y = (|\mathbf{T}_{BC}|\sin\alpha - |\mathbf{T}_{BA}|\sin\beta)\mathbf{j} = 0.$$

Solve: $|\mathbf{T}_{BC}| = \left(\dfrac{\sin\beta}{\sin\alpha}\right)|\mathbf{T}_{BA}|$.

For $|\mathbf{T}_{BA}| = 13049.5$ N, and $\alpha = 50°$, $\beta = 70°$,

$$|\mathbf{T}_{BC}| = (13049.5)\left(\frac{0.9397}{0.7660}\right) = 16007.6 \text{ N}$$

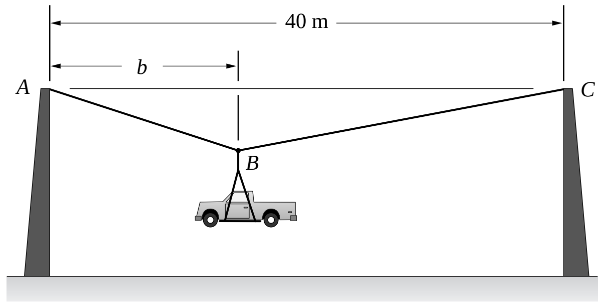
Isolate the cable juncture C. The angles are $\alpha = 30°$, $\beta = 50°$. By symmetry with the cable juncture B above, the tension in cable CD is

$$|\mathbf{T}_{CD}| = \left(\frac{\sin\beta}{\sin\alpha}\right)|\mathbf{T}_{CB}|.$$

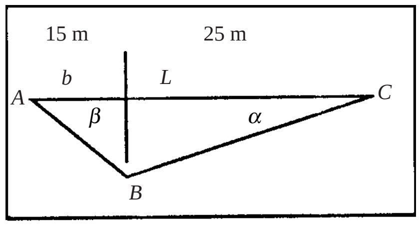
Substitute: $|\mathbf{T}_{CD}| = (16007.6)\left(\dfrac{0.7660}{0.5}\right) = 24525.0 \text{ N}.$

This completes the problem solution.

Problem 3.40 A truck dealer wants to suspend a 4000-kg truck as shown for advertising. The distance $b = 15$ m, and the sum of the lengths of the cables AB and BC is 42 m. Points A and C are at the same height. What are the tensions in the cables?



Solution: Determine the dimensions and angles of the cables. Isolate the cable juncture B, and solve the equilibrium conditions. The dimensions of the triangles formed by the cables:



$b = 15$ m, $L = 25$ m, $AB + BC = S = 42$ m.

Subdivide into two right triangles with a common side of unknown length. Let the unknown length of this common side be d, then by the Pythagorean Theorem $b^2 + d^2 = AB^2$, $L^2 + d^2 = BC^2$.

Subtract the first equation from the second to eliminate the unknown d, $L^2 - b^2 = BC^2 - AB^2$.

Note that $BC^2 - AB^2 = (BC - AB)(BC + AB)$.

Substitute and reduce to the pair of simultaneous equations in the unknowns

$$BC - AB = \left(\frac{L^2 - b^2}{S}\right), \quad BC + AB = S$$

Solve: $$BC = \left(\frac{1}{2}\right)\left(\frac{L^2 - b^2}{S} + S\right)$$

$$= \left(\frac{1}{2}\right)\left(\frac{25^2 - 15^2}{42} + 42\right) = 25.762 \text{ m}$$

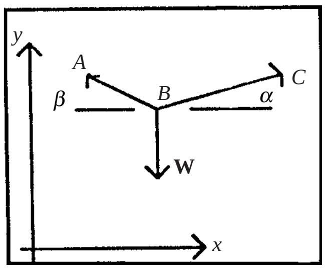
and $AB = S - BC = 42 - 25.762 = 16.238$ m.

The interior angles are found from the cosine law:

$$\cos\alpha = \left(\frac{(L+b)^2 + BC^2 - AB^2}{2(L+b)(BC)}\right) = 0.9704 \quad \alpha = 13.97^\circ$$

$$\cos\beta = \left(\frac{(L+b)^2 + AB^2 - BC^2}{2(L+b)(AB)}\right) = 0.9238 \quad \beta = 22.52^\circ$$

Isolate cable juncture B. The angle between BC and the positive x axis is α; the tension is



$$\mathbf{T}_{BC} = |\mathbf{T}_{BC}|(\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha)$$

The angle between BA and the positive x axis is $(180^\circ - \beta)$; the tension is

$$\mathbf{T}_{BA} = |\mathbf{T}_{BA}|(\mathbf{i}\cos(180 - \beta) + \mathbf{j}\sin(180 - \beta))$$

$$= |\mathbf{T}_{BA}|(-\mathbf{i}\cos\beta + \mathbf{j}\sin\beta).$$

The weight is $\mathbf{W} = 0\mathbf{i} - |\mathbf{W}|\mathbf{j}$.

The equilibrium conditions are

$$\sum \mathbf{F} = \mathbf{W} + \mathbf{T}_{BA} + \mathbf{T}_{BC} = 0.$$

Substitute and collect like terms

$$\sum \mathbf{F}_x = (|\mathbf{T}_{BC}|\cos\alpha - |\mathbf{T}_{BA}|\cos\beta)\mathbf{i} = 0,$$

$$\sum \mathbf{F}_y = (|\mathbf{T}_{BC}|\sin\alpha + |\mathbf{T}_{BA}|\sin\beta - |\mathbf{W}|)\mathbf{j} = 0$$

Solve: $$|\mathbf{T}_{BC}| = \left(\frac{\cos\beta}{\cos\alpha}\right)|\mathbf{T}_{BA}|,$$

and $$|\mathbf{T}_{BA}| = \left(\frac{|\mathbf{W}|\cos\alpha}{\sin(\alpha + \beta)}\right).$$

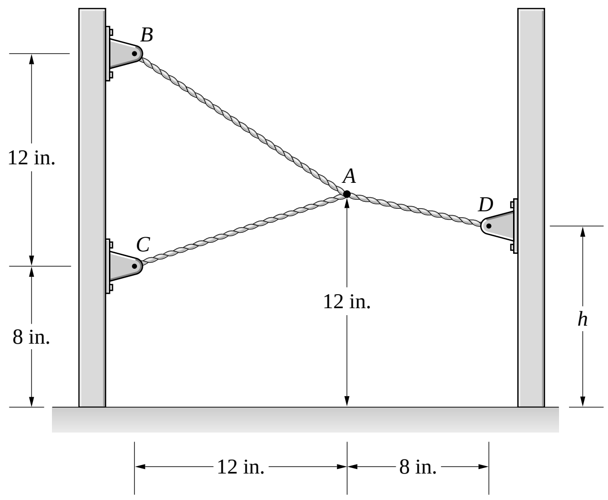
For $|\mathbf{W}| = (4000)(9.81) = 39240$ N,

and $\alpha = 13.97^\circ$, $\beta = 22.52^\circ$,

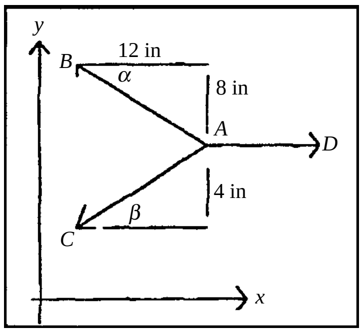
$|\mathbf{T}_{BA}| = 64033 = 64$ kN,

$|\mathbf{T}_{BC}| = 60953 = 61$ kN

Problem 3.41 The distance $h = 12$ in, and the tension in cable AD is 200 lb. What are the tensions in cables AB and AC?



Solution: Isolated the cable juncture. From the sketch, the angles are found from



$$\tan\alpha = \left(\frac{8}{12}\right) = 0.667 \quad \alpha = 33.7^\circ$$

$$\tan\beta = \left(\frac{4}{12}\right) = 0.333 \quad \beta = 18.4^\circ$$

The angle between the cable AB and the positive x axis is $(180^\circ - \alpha)$, the tension in AB is:

$$\mathbf{T}_{AB} = |\mathbf{T}_{AB}|(\mathbf{i}\cos(180 - \alpha) + \mathbf{j}\sin(180 - \alpha))$$

$$\mathbf{T}_{AB} = |\mathbf{T}_{AB}|(-\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha).$$

The angle between AC and the positive x axis is $(180 + \beta)$. The tension is

$$\mathbf{T}_{AC} = |\mathbf{T}_{AC}|(\mathbf{i}\cos(180 + \beta) + \mathbf{j}\sin(180 + \beta))$$

$$\mathbf{T}_{AC} = |\mathbf{T}_{AC}|(-\mathbf{i}\cos\beta - \mathbf{j}\sin\beta).$$

The tension in the cable AD is

$$\mathbf{T}_{AD} = |\mathbf{T}_{AD}|\mathbf{i} + 0\mathbf{j}.$$

The equilibrium conditions are

$$\sum \mathbf{F} = \mathbf{T}_{AC} + \mathbf{T}_{AB} + \mathbf{T}_{AD} = 0.$$

Substitute and collect like terms,

$$\sum \mathbf{F}_x = (-|\mathbf{T}_{AB}|\cos\alpha - |\mathbf{T}_{AC}|\cos\beta + |\mathbf{T}_{AD}|)\mathbf{i} = 0$$

$$\sum \mathbf{F}_y = (|\mathbf{T}_{AB}|\sin\alpha - |\mathbf{T}_{AC}|\sin\beta)\mathbf{j} = 0.$$

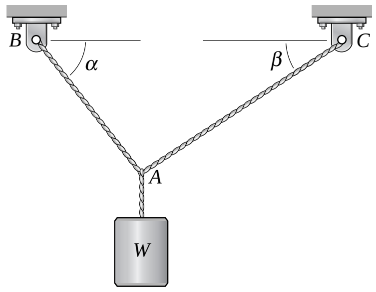
Solve: $|\mathbf{T}_{AB}| = \left(\frac{\sin\beta}{\sin\alpha}\right)|\mathbf{T}_{AC}|$,

and $|\mathbf{T}_{AC}| = \left(\frac{\sin\alpha}{\sin(\alpha + \beta)}\right)|\mathbf{T}_{AD}|$.

For $|\mathbf{T}_{AD}| = 200$ lb, $\alpha = 33.7^\circ$, $\beta = 18.4^\circ$

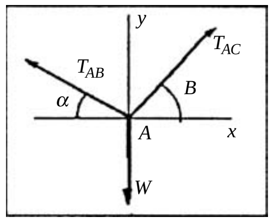
$|\mathbf{T}_{AC}| = 140.6$ lb, $|\mathbf{T}_{AB}| = 80.1$ lb

Problem 3.42 You are designing a cable system to support a suspended object of weight W. Because your design requires points A and B to be placed as shown, you have no control over the angle α, but you can choose the angle β by placing point C wherever you wish. Show that to minimize the tensions in cables AB and BC, you must choose $\beta = \alpha$ if the angle $\alpha \geq 45^\circ$.



Strategy: Draw a diagram of the sum of the forces exerted by the three cables at A.

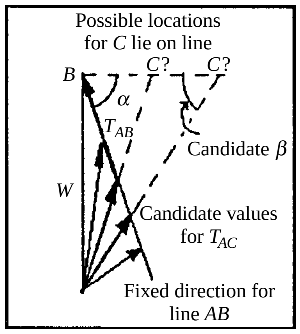
Solution: Draw the free body diagram of the knot at point A. Then draw the force triangle involving the three forces. Remember that α is fixed and the force W has both fixed magnitude and direction. From the force triangle, we see that the force T_{AC} can be smaller than T_{AB} for a large range of values for β. By inspection, we see that the minimum simultaneous values for T_{AC} and T_{AB} occur when the two forces are equal. This occurs when $\alpha = \beta$. Note: this does not happen when $\alpha < 45^\circ$.



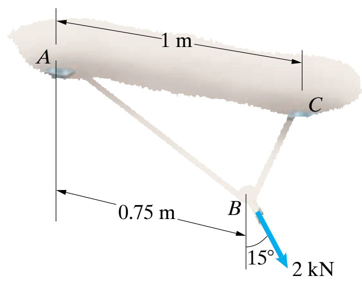
In this case, we solved the problem without writing the equations of equilibrium. For reference, these equations are:

$$\sum F_x = -T_{AB}\cos\alpha + T_{AC}\cos\beta = 0$$

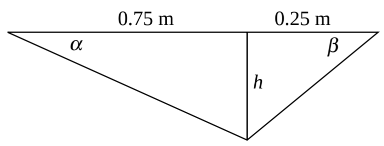
and $\sum F_y = T_{AB}\sin\alpha + T_{AC}\sin\beta - W = 0.$



Problem 3.43* The length of the cable ABC is 1.4 m. The 2-kN force is applied to a small pulley. The system is stationary. What is the tension in the cable?



Solution: Examine the geometry

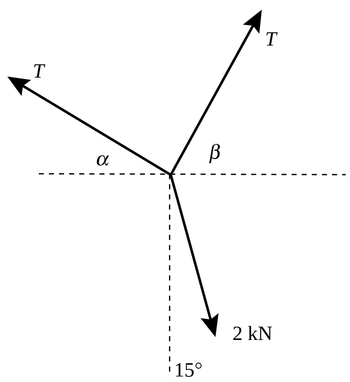


$$\sqrt{h^2 + (0.75\text{ m})^2} + \sqrt{h^2 + (0.25\text{ m})^2} = 1.4\text{ m}$$

$$\tan\alpha = \frac{h}{0.75\text{ m}},\quad \tan\beta = \frac{h}{0.25\text{ m}}$$

$$\Rightarrow \quad h = 0.458\text{ m},\ \alpha = 31.39^\circ,\ \beta = 61.35^\circ$$

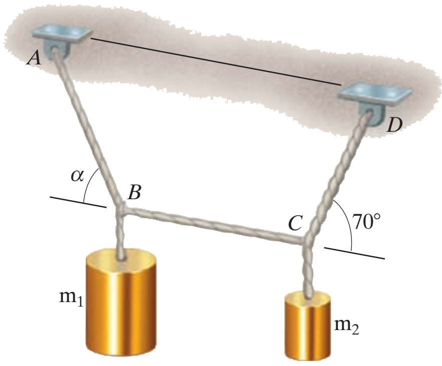
Now draw a FBD and solve for the tension. We can use either of the equilibrium equations

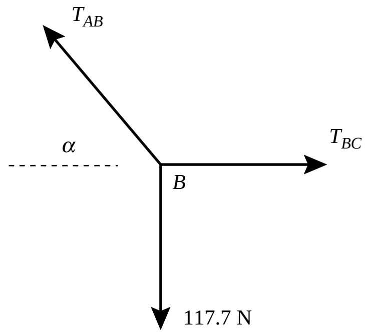


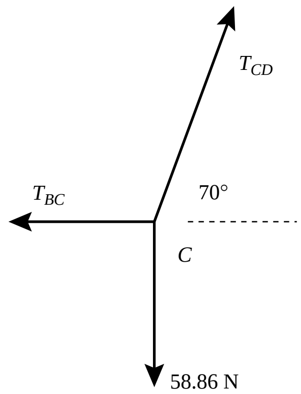
$$\sum F_x : -T\cos\alpha + T\cos\beta + (2\text{ kN})\sin 15^\circ = 0$$

$$\boxed{T = 1.38\text{ kN}}$$

Problem 3.44 The masses $m_1 = 12$ kg and $m_2 = 6$ kg are suspended by the cable system shown. The cable BC is horizontal. Determine the angle α and the tensions in the cables AB, BC, and CD.







Solution: We have 4 unknowns and 4 equations

$$\sum F_{Bx} : -T_{AB}\cos\alpha + T_{BC} = 0$$

$$\sum F_{By} : T_{AB}\sin\alpha - 117.7\text{ N} = 0$$

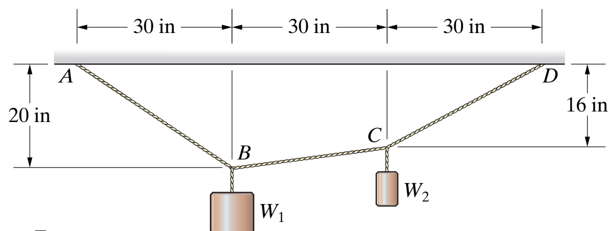
$$\sum F_{Cx} : -T_{BC} + T_{CD}\cos 70^\circ = 0$$

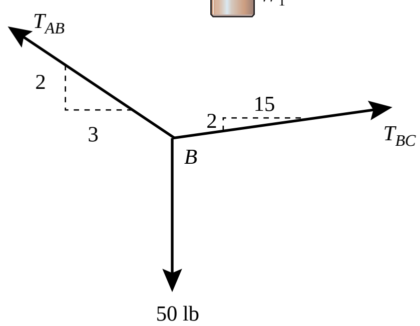
$$\sum F_{Cy} : T_{CD}\sin 70^\circ - 58.86\text{ N} = 0$$

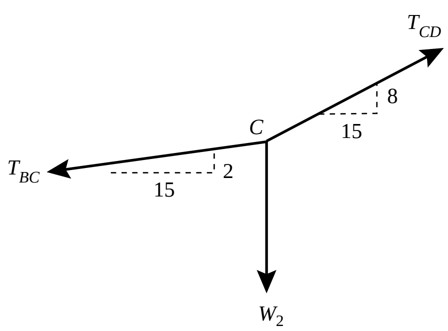
Solving we find

$$\boxed{\alpha = 79.7^\circ,\ T_{AB} = 119.7\text{ N},\ T_{BC} = 21.4\text{ N},\ T_{CD} = 62.6\text{ N}}$$

Problem 3.45 The weights $W_1 = 50$ lb and W_2 are suspended by the cable system shown. Determine the weight W_2 and the tensions in the cables AB, BC, and CD.







Solution: We have 4 unknowns and 4 equilibrium equations to use

$$\sum F_{Bx} : -\frac{3}{\sqrt{13}}T_{AB} + \frac{15}{\sqrt{229}}T_{BC} = 0$$

$$\sum F_{By} : \frac{2}{\sqrt{13}}T_{AB} + \frac{2}{\sqrt{229}}T_{BC} - 50\text{ lb} = 0$$

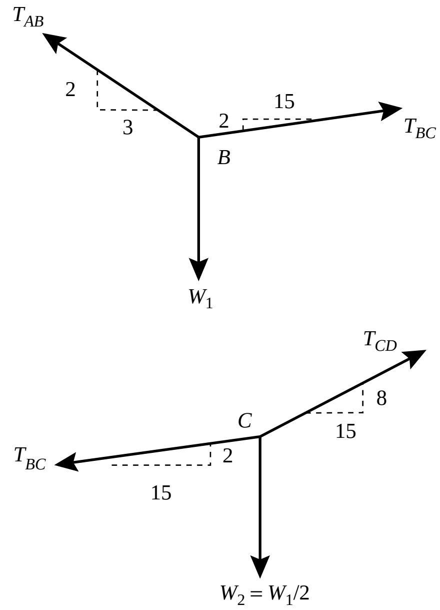
$$\sum F_{Cx} : -\frac{15}{\sqrt{229}}T_{BC} + \frac{15}{17}T_{CD} = 0$$

$$\sum F_{Cy} : -\frac{2}{\sqrt{229}}T_{BC} + \frac{8}{17}T_{CD} - W_2 = 0$$

$$\Rightarrow \boxed{\begin{array}{l} W_2 = 25\text{ lb},\ T_{AB} = 75.1\text{ lb} \\ T_{BC} = 63.1\text{ lb},\ T_{CD} = 70.8\text{ lb} \end{array}}$$

Problem 3.46 In the system shown in Problem 3.45, assume that $W_2 = W_1/2$. If you don't want the tension anywhere in the supporting cable to exceed 200 lb, what is the largest acceptable value of W_1?

Solution:



$$\sum F_{Bx}: -\frac{3}{\sqrt{13}}T_{AB} + \frac{15}{\sqrt{229}}T_{BC} = 0$$

$$\sum F_{By}: \frac{2}{\sqrt{13}}T_{AB} + \frac{2}{\sqrt{229}}T_{BC} - W_1 = 0$$

$$\sum F_{Cx}: -\frac{15}{\sqrt{229}}T_{BC} + \frac{15}{17}T_{CD} = 0$$

$$\sum F_{Cy}: -\frac{2}{\sqrt{229}}T_{BC} + \frac{8}{17}T_{CD} - \frac{W_1}{2} = 0$$

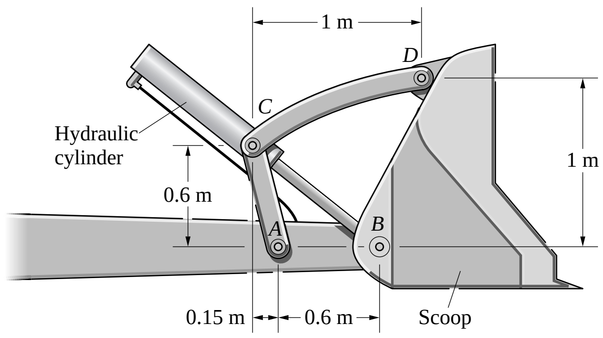
$T_{AB} = 1.502W_1$, $T_{BC} = 1.262W_1$, $T_{CD} = 1.417W_1$

AB is the critical cable

$$\boxed{200 \text{ lb} = 1.502W_1 \Rightarrow W_1 = 133.2\ lb}$$

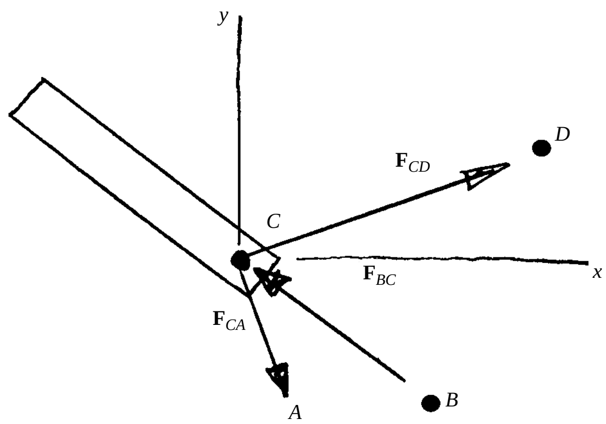
Problem 3.47 The hydraulic cylinder is subjected to three forces. An 8-kN force is exerted on the cylinder at B that is parallel to the cylinder and points from B toward C. The link AC exerts a force at C that is parallel to the line from A to C. The link CD exerts a force at C that is parallel to the line from C to D.

(a) Draw the free-body diagram of the cylinder. (The cylinder's weight is negligible).
(b) Determine the magnitudes of the forces exerted by the links AC and CD.



Solution: From the figure, if C is at the origin, then points A, B, and D are located at

$A(0.15, -0.6)$
$B(0.75, -0.6)$
$D(1.00, 0.4)$



and forces $\mathbf{F}_{CA}$, $\mathbf{F}_{BC}$, and $\mathbf{F}_{CD}$ are parallel to CA, BC, and CD, respectively.

We need to write unit vectors in the three force directions and express the forces in terms of magnitudes and unit vectors. The unit vectors are given by

$$\mathbf{e}_{CA} = \frac{\mathbf{r}_{CA}}{|\mathbf{r}_{CA}|} = 0.243\mathbf{i} - 0.970\mathbf{j}$$

$$\mathbf{e}_{CB} = \frac{\mathbf{r}_{CB}}{|\mathbf{r}_{CB}|} = 0.781\mathbf{i} - 0.625\mathbf{j}$$

$$\mathbf{e}_{CD} = \frac{\mathbf{r}_{CD}}{|\mathbf{r}_{CD}|} = 0.928\mathbf{i} + 0.371\mathbf{j}$$

Now we write the forces in terms of magnitudes and unit vectors. We can write $\mathbf{F}_{BC}$ as $\mathbf{F}_{CB} = -8\mathbf{e}_{CB}$ kN or as $\mathbf{F}_{CB} = 8(-\mathbf{e}_{CB})$ kN (because we were told it was directed from B toward C and had a magnitude of 8 kN. Either way, we must end up with

$$\mathbf{F}_{CB} = -6.25\mathbf{i} + 5.00\mathbf{j} \text{ kN}$$

Similarly,

$$\mathbf{F}_{CA} = 0.243F_{CA}\mathbf{i} - 0.970F_{CA}\mathbf{j}$$

$$\mathbf{F}_{CD} = 0.928F_{CD}\mathbf{i} + 0.371F_{CD}\mathbf{j}$$

For equilibrium, $\mathbf{F}_{CA} + \mathbf{F}_{CB} + \mathbf{F}_{CD} = 0$

In component form, this gives

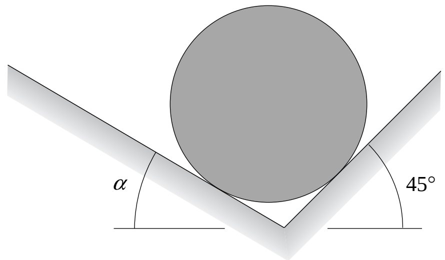
$$\begin{cases} \sum F_x = 0.243F_{CA} + 0.928F_{CD} - 6.25 \text{ (kN)} = 0 \\ \sum F_y = -0.970F_{CA} + 0.371F_{CD} + 5.00 \text{ (kN)} = 0 \end{cases}$$

Solving, we get

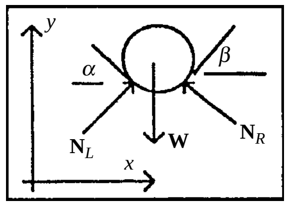
$$F_{CA} = 7.02 \text{ kN}, \quad F_{CD} = 4.89 \text{ kN}$$

Problem 3.48 The 50-lb cylinder rests on two smooth surfaces.

(a) Draw the free-body diagram of the cylinder.
(b) If $\alpha = 30^\circ$, what are the magnitudes of the forces exerted on the cylinder by the left and right surfaces?



Solution: Isolate the cylinder. (a) The free body diagram of the isolated cylinder is shown. (b) The forces acting are the weight and the normal forces exerted by the surfaces. The angle between the normal force on the right and the x axis is $(90 + \beta)$. The normal force is



$$\mathbf{N}_R = |\mathbf{N}_R|(\mathbf{i}\cos(90 + \beta) + \mathbf{j}\sin(90 + \beta))$$

$$\mathbf{N}_R = |\mathbf{N}_R|(-\mathbf{i}\sin\beta + \mathbf{j}\cos\beta).$$

The angle between the positive x axis and the left hand force is normal $(90 - \alpha)$; the normal force is $\mathbf{N}_L = |\mathbf{N}_L|(\mathbf{i}\sin\alpha + \mathbf{j}\cos\alpha)$. The weight is $\mathbf{W} = 0\mathbf{i} - |\mathbf{W}|\mathbf{j}$. The equilibrium conditions are

$$\sum \mathbf{F} = \mathbf{W} + \mathbf{N}_R + \mathbf{N}_L = 0.$$

Substitute and collect like terms,

$$\sum \mathbf{F}_x = (-|\mathbf{N}_R|\sin\beta + |\mathbf{N}_L|\sin\alpha)\mathbf{i} = 0,$$

$$\sum \mathbf{F}_y = (|\mathbf{N}_R|\cos\beta + |\mathbf{N}_L|\cos\alpha - |\mathbf{W}|)\mathbf{j} = 0.$$

Solve: $|\mathbf{N}_R| = \left(\dfrac{\sin\alpha}{\sin\beta}\right)|N_L|,$

and $|\mathbf{N}_L| = \left(\dfrac{|\mathbf{W}|\sin\beta}{\sin(\alpha + \beta)}\right).$

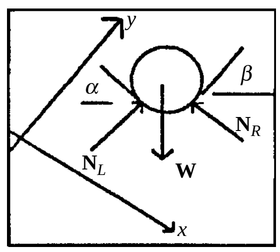
For $|\mathbf{W}| = 50$ lb, and $\alpha = 30^\circ$, $\beta = 45^\circ$, the normal forces are

$|\mathbf{N}_L| = 36.6$ lb, $|\mathbf{N}_R| = 25.9$ lb

Problem 3.49 For the 50-lb cylinder in Problem 3.48, obtain an equation for the force exerted on the cylinder by the left surface in terms of the angle α in two ways: (a) using a coordinate system with the y axis vertical, (b) using a coordinate system with the y axis parallel to the right surface.

Solution: The solution for Part (a) is given in Problem 3.48 (see free body diagram).

$$|\mathbf{N}_R| = \left(\frac{\sin\alpha}{\sin\beta}\right)|N_L| \quad |\mathbf{N}_L| = \left(\frac{|\mathbf{W}|\sin\beta}{\sin(\alpha + \beta)}\right).$$



Part (b): The isolated cylinder with the coordinate system is shown. The angle between the right hand normal force and the positive x axis is 180°. The normal force: $\mathbf{N}_R = -|\mathbf{N}_R|\mathbf{i} + 0\mathbf{j}$. The angle between the left hand normal force and the positive x is $180 - (\alpha + \beta)$. The normal force is $\mathbf{N}_L = |\mathbf{N}_L|(-\mathbf{i}\cos(\alpha + \beta) + \mathbf{j}\sin(\alpha + \beta))$.

The angle between the weight vector and the positive x axis is $-\beta$. The weight vector is $\mathbf{W} = |\mathbf{W}|(\mathbf{i}\cos\beta - \mathbf{j}\sin\beta)$.
The equilibrium conditions are

$$\sum \mathbf{F} = \mathbf{W} + \mathbf{N}_R + \mathbf{N}_L = 0.$$

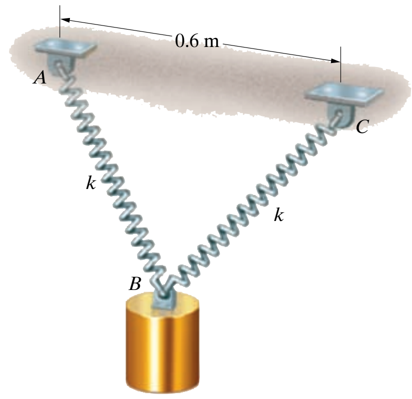
Substitute and collect like terms,

$$\sum \mathbf{F}_x = (-|\mathbf{N}_R| - |\mathbf{N}_L|\cos(\alpha + \beta) + |\mathbf{W}|\cos\beta)\mathbf{i} = 0,$$

$$\sum \mathbf{F}_y = (|\mathbf{N}_L|\sin(\alpha + \beta) - |\mathbf{W}|\sin\beta)\mathbf{j} = 0.$$

Solve: $|\mathbf{N}_L| = \left(\dfrac{|\mathbf{W}|\sin\beta}{\sin(\alpha + \beta)}\right)$

Problem 3.50 The two springs are identical, with unstretched length 0.4 m. When the 50-kg mass is suspended at B, the length of each spring increases to 0.6 m. What is the spring constant k?

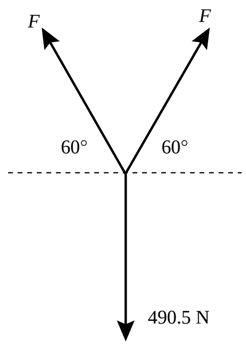


Solution:

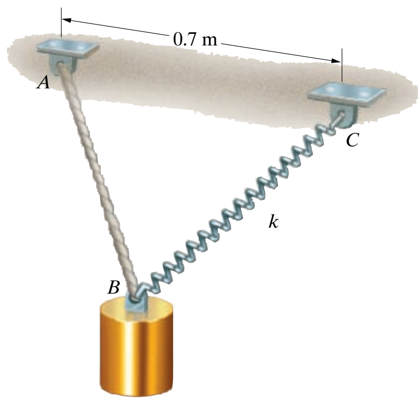
$F = k(0.6\text{ m} - 0.4\text{ m})$

$\sum F_y : 2F \sin 60^\circ - 490.5\text{ N} = 0$

$\boxed{k = 1416\text{ N/m}}$



Problem 3.51 The cable AB is 0.5 m in length. The *unstretched* length of the spring is 0.4 m. When the 50-kg mass is suspended at B, the length of the spring increases to 0.45 m. What is the spring constant k?



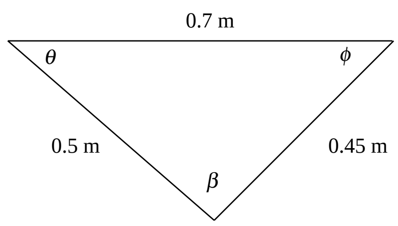
Solution: The Geometry

Law of Cosines and Law of Sines

$0.7^2 = 0.5^2 + 0.45^2 - 2(0.5)(0.45)\cos\beta$

$\dfrac{\sin\theta}{0.45\text{ m}} = \dfrac{\sin\phi}{0.5\text{ m}} = \dfrac{\sin\beta}{0.7\text{ m}}$

$\beta = 94.8^\circ,\ \ \theta = 39.8^\circ\ \phi = 45.4^\circ$



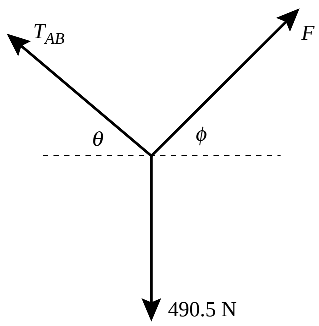
Now do the statics

$F = k(0.45\text{ m} - 0.4\text{ m})$

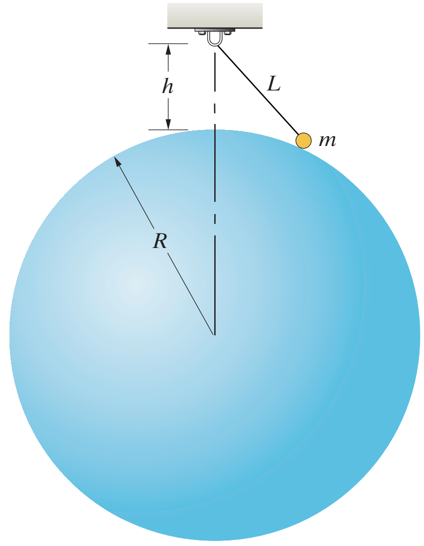
$\sum F_x : -T_{AB}\cos\theta + F\cos\phi = 0$

$\sum F_y : T_{AB}\sin\theta + F\sin\phi - 490.5\text{ N} = 0$

Solving: $\boxed{k = 7560\text{ N/m}}$



Problem 3.52 The small sphere of mass m is attached to a string of length L and rests on the smooth surface of a fixed sphere of radius R. The center of the sphere is directly below the point where the string is attached. Obtain an equation for the tension in the string in terms of m, L, h, and R.



Solution: From the geometry we have

$$\cos\theta = \frac{R+h-y}{L}, \ \sin\theta = \frac{x}{L}$$

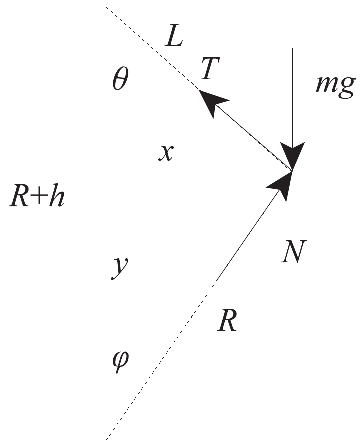
$$\cos\phi = \frac{y}{R}, \ \sin\phi = \frac{x}{R}$$

Thus the equilibrium equations can be written

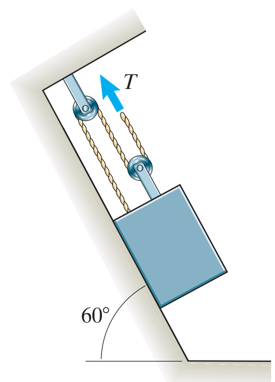
$$\sum F_x : -\frac{x}{L}T + \frac{x}{R}N = 0$$

$$\sum F_y : \frac{R+h-y}{L}T + \frac{y}{R}N - mg = 0$$

Solving, we find $\boxed{T = \dfrac{mgL}{R+h}}$



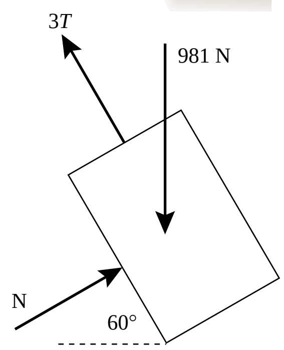
Problem 3.53 The inclined surface is smooth. Determine the force T that must be exerted on the cable to hold the 100-kg crate in equilibrium and compare your answer to the answer of Problem 3.11.



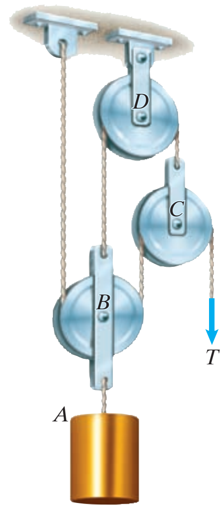
Solution:

$$\sum F_{\nwarrow} : 3\,\mathrm{T} - 981\ \mathrm{N}\sin 60^\circ = 0$$

$$\boxed{T = 283\ \mathrm{N}}$$



Problem 3.54 In Example 3.3, suppose that the mass of the suspended object is m_A and the masses of the pulleys are $m_B = 0.3m_A$, $m_C = 0.2m_A$, and $m_D = 0.2m_A$. Show that the force T necessary for the system to be in equilibrium is $0.275m_A g$.



Solution: From the free-body diagram of pulley C

$$T_D - 2T - m_C g = 0 \Rightarrow T_D = 2T + m_C g$$

Then from the free-body diagram of pulley B

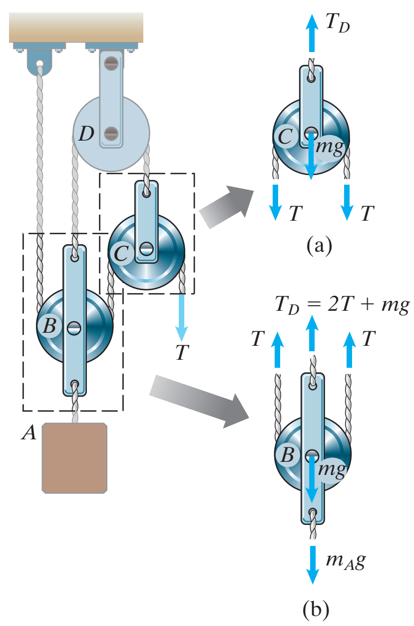
$$T + T + 2T + m_C g - m_B g - m_A g = 0$$

Thus

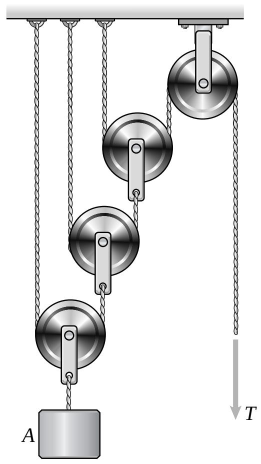
$$T = \frac{1}{4}(m_A + m_B - m_C)g$$

$$T = \frac{1}{4}(m_A + 0.3m_A - 0.2m_A)g = 0.275m_A g$$

$$\boxed{T = 0.275m_A g}$$



Problem 3.55 The mass of each pulley of the system is m and the mass of the suspended object A is m_A. Determine the force T necessary for the system to be in equilibrium.



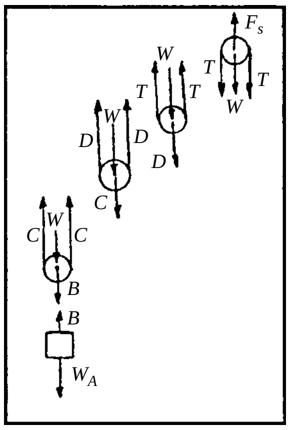
Solution: Draw free body diagrams of each pulley and the object A. Each pulley and the object A must be in equilibrium. The weights of the pulleys and object A are $W = mg$ and $W_A = m_A g$. The equilibrium equations for the weight A, the lower pulley, second pulley, third pulley, and the top pulley are, respectively, $B - W_A = 0$, $2C - B - W = 0$, $2D - C - W = 0$, $2T - D - W = 0$, and $F_S - 2T - W = 0$. Begin with the first equation and solve for B, substitute for B in the second equation and solve for C, substitute for C in the third equation and solve for D, and substitute for D in the fourth equation and solve for T, to get T in terms of W and W_A. The result is

$$B = W_A, \quad C = \frac{W_A}{2} + \frac{W}{2},$$

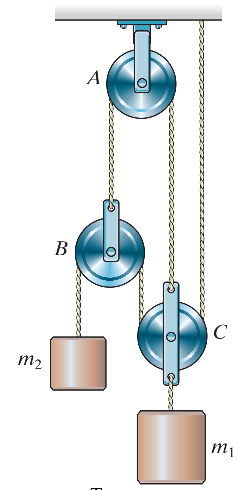
$$D = \frac{W_A}{4} + \frac{3W}{4}, \text{ and } T = \frac{W_A}{8} + \frac{7W}{8},$$

or in terms of the masses,

$$T = \frac{g}{8}(m_A + 7m).$$



Problem 3.56 The suspended mass $m_1 = 50$ kg. Neglecting the masses of the pulleys, determine the value of the mass m_2 necessary for the system to be in equilibrium.

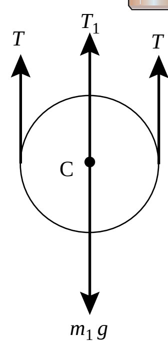


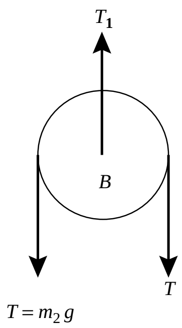
Solution:

$\sum F_C : T_1 + 2m_2 g - m_1 g = 0$

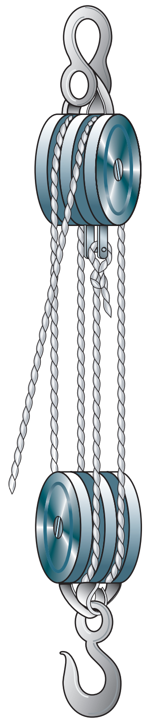
$\sum F_B : T_1 - 2m_2 g = 0$

$$\boxed{m_2 = \frac{m_1}{4} = 12.5 \text{ kg}}$$





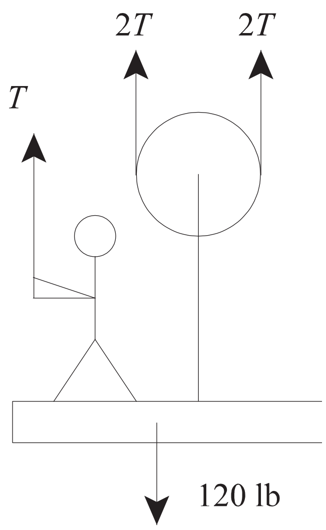
Problem 3.57 The boy is lifting himself using the block and tackle shown. If the weight of the block and tackle is negligible, and the combined weight of the boy and the beam he is sitting on is 120 lb, what force does he have to exert on the rope to raise himself at a constant rate? (Neglect the deviation of the ropes from the vertical.)



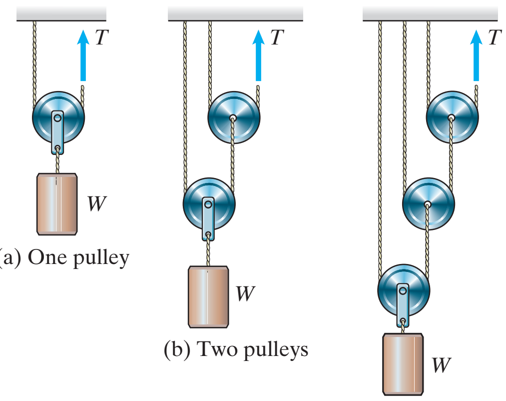
Solution: A free-body diagram can be obtained by cutting the four ropes between the two pulleys of the block and tackle *and the rope the boy is holding*. The tension has the same value T in all five of these ropes. So the upward force on the free-body diagram is $5T$ and the downward force is the 120-lb weight. Therefore the force the boy must exert is

$T = (120 \text{ lb})/5 = 24 \text{ lb}$

$\boxed{T = 24 \text{ lb}}$



Problem 3.58 Pulley systems containing one, two, and three pulleys are shown. Neglecting the weights of the pulleys, determine the force T required to support the weight W in each case.



(a) One pulley

(b) Two pulleys

(c) Three pulleys

Solution:

(a) $\boxed{\sum F_y : 2T - W = 0 \Rightarrow T = \frac{W}{2}}$

(b) $\sum F_{\text{upper}} : 2T - T_1 = 0$

$\sum F_{\text{lower}} : 2T_1 - W = 0$

$\boxed{T = \frac{W}{4}}$

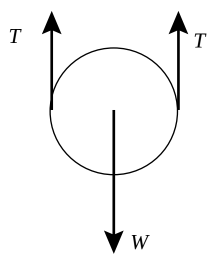
(c) $\sum F_{\text{upper}} : 2T - T_1 = 0$

$\sum F_{\text{middle}} : 2T_1 - T_2 = 0$

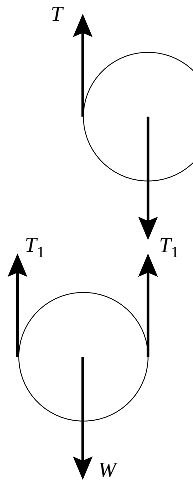
$\sum F_{\text{lower}} : 2T_2 - W = 0$

$\boxed{T = \frac{W}{8}}$

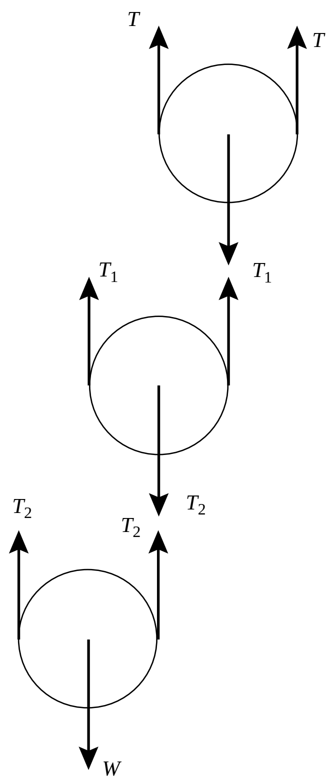
(a) For one pulleys



(b) For two pulleys



(c) For three pulleys



Problem 3.59 Problem 3.58 shows pulley systems containing one, two, and three pulleys. The number of pulleys in the type of system shown could obviously be extended to an arbitrary number N.

(a) Neglecting the weights of the pulleys, determine the force T required to support the weight W as a function of the number of pulleys N in the system.
(b) Using the result of part (a), determine the force T required to support the weight W for a system with 10 pulleys.

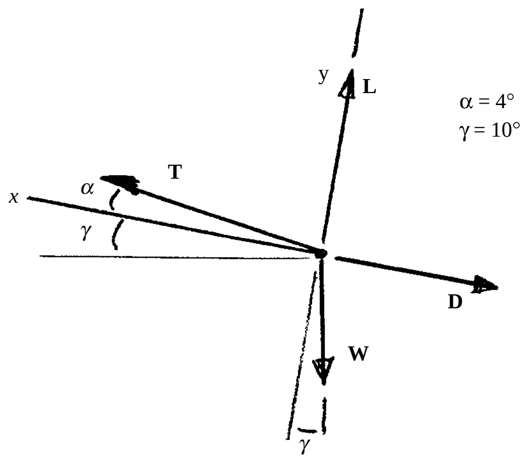
Solution: By extrapolation of the previous problem

(a) $\boxed{T = \dfrac{W}{2^N}}$

(b) $\boxed{T = \dfrac{W}{1024}}$

Problem 3.60 A 14,000-kg airplane is in steady flight in the vertical plane. The flight path angle is $\gamma = 10^\circ$, the angle of attack is $\alpha = 4^\circ$, and the thrust force exerted by the engine is $T = 60$ kN. What are the magnitudes of the lift and drag forces acting on the airplane? (See Example 3.4).

Solution: Let us draw a more detailed free body diagram to see the angles involved more clearly. Then we will write the equations of equilibrium and solve them.



$W = mg = (14{,}000)(9.81)$ N

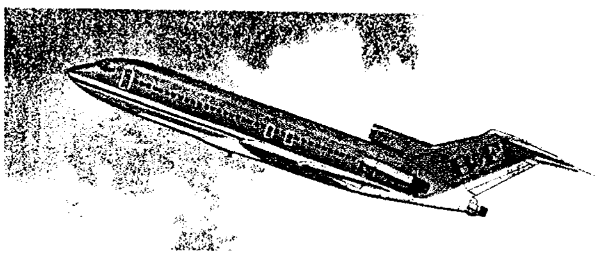
The equilibrium equations are

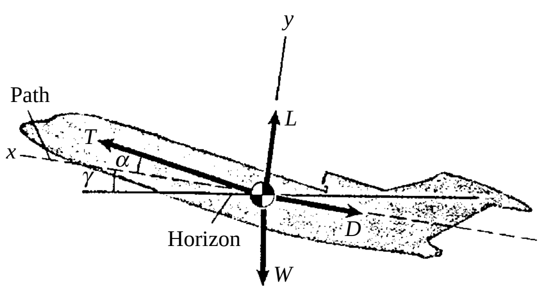
$$\begin{cases} \sum F_x = T\cos\alpha - D - W\sin\gamma = 0 \\ \sum F_y = T\sin\alpha + L - W\cos\gamma = 0 \end{cases}$$

$T = 60$ kN $= 60000$ N

Solving, we get

$D = 36.0$ kN, $L = 131.1$ kN





Problem 3.61 An airplane is in steady flight, the angle of attack $\alpha = 0$, the thrust-to-drag ratio $T/D = 2$, and the lift-to-drag ratio $L/D = 4$. What is the flight path angle γ? (See Example 3.4).

Solution: Use the same strategy as in Problem 3.52. The angle between the thrust vector and the positive x axis is α,

$\mathbf{T} = |\mathbf{T}|(\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha)$

The lift vector: $\mathbf{L} = 0\mathbf{i} + |\mathbf{L}|\mathbf{j}$

The drag: $\mathbf{D} = -|\mathbf{D}|\mathbf{i} + 0\mathbf{j}$. The angle between the weight vector and the positive x axis is $(270 - \gamma)$;

$\mathbf{W} = |\mathbf{W}|(-\mathbf{i}\sin\gamma - \mathbf{j}\cos\gamma)$.

The equilibrium conditions are

$\sum \mathbf{F} = \mathbf{T} + \mathbf{L} + \mathbf{D} + \mathbf{W} = 0$.

Substitute and collect like terms

$$\sum \mathbf{F}_x = (|\mathbf{T}|\cos\alpha - |\mathbf{D}| - |\mathbf{W}|\sin\gamma)\mathbf{i} = 0,$$

and $\sum \mathbf{F}_y = (|\mathbf{T}|\sin\alpha + |\mathbf{L}| - |\mathbf{W}|\cos\gamma)\mathbf{j} = 0$

Solve the equations for the terms in γ:

$$|\mathbf{W}|\sin\gamma = |\mathbf{T}|\cos\alpha - |\mathbf{D}|,$$

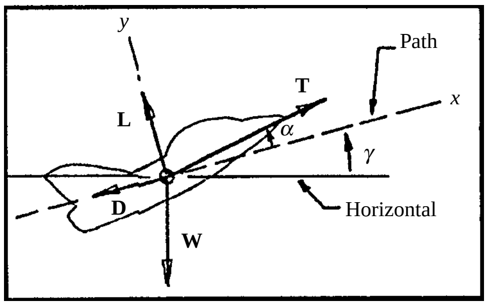
and $|\mathbf{W}|\cos\gamma = |\mathbf{T}|\sin\alpha + |\mathbf{L}|$.

Take the ratio of the two equations

$$\tan\gamma = \left(\frac{|\mathbf{T}|\cos\alpha - |\mathbf{D}|}{|\mathbf{T}|\sin\alpha + |\mathbf{L}|}\right).$$

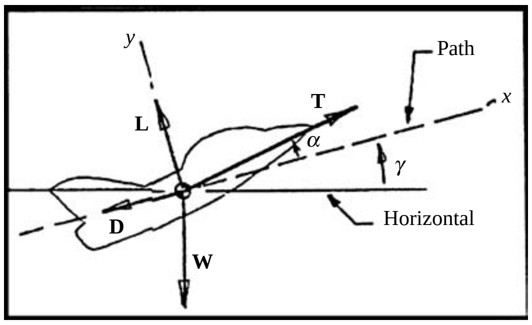
Divide top and bottom on the right by $|\mathbf{D}|$.

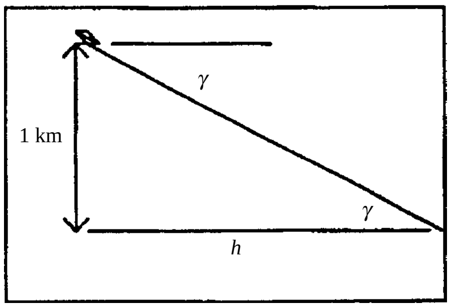
For $\alpha = 0$, $\dfrac{|\mathbf{T}|}{|\mathbf{D}|} = 2$, $\dfrac{|\mathbf{L}|}{|\mathbf{D}|} = 4$, $\tan\gamma = \left(\dfrac{2-1}{4}\right) = \dfrac{1}{4}$ or $\gamma = 14^\circ$



Problem 3.62 An airplane glides in steady flight ($T = 0$), and its lift-to-drag ratio is $L/D = 4$.

(a) What is the flight path angle γ?
(b) If the airplane glides from an altitude of 1000 m to zero altitude, what horizontal distance does it travel?
(See Example 3.4.)





Solution: See Example 3.4. The angle between the thrust vector and the positive $\boldsymbol{x}$ axis is α:

$\mathbf{T} = |\mathbf{T}|(\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha)$.

The lift vector: $\mathbf{L} = 0\mathbf{i} + |\mathbf{L}|\mathbf{j}$.

The drag: $\mathbf{D} = -|\mathbf{D}|\mathbf{i} + 0\mathbf{j}$. The angle between the weight vector and the positive $\mathbf{x}$ axis is $(270 - \gamma)$:

$\mathbf{W} = |\mathbf{W}|(-\mathbf{i}\sin\gamma - \mathbf{j}\cos\gamma)$.

The equilibrium conditions are

$\sum \mathbf{F} = \mathbf{T} + \mathbf{L} + \mathbf{D} + \mathbf{W} = 0$.

Substitute and collect like terms:

$\sum \mathbf{F}_x = (|\mathbf{T}|\cos\alpha - |\mathbf{D}| - |\mathbf{W}|\sin\gamma)\mathbf{i} = 0$

$\sum \mathbf{F}_y = (|\mathbf{T}|\sin\alpha + |\mathbf{L}| - |\mathbf{W}|\cos\gamma)\mathbf{j} = 0$

Solve the equations for the terms in γ,

$|\mathbf{W}|\sin\gamma = |\mathbf{T}|\cos\alpha - |\mathbf{D}|$,

and $|\mathbf{W}|\cos\gamma = |\mathbf{T}|\sin\alpha + |\mathbf{L}|$

Part (a): Take the ratio of the two equilibrium equations:

$$\tan\gamma = \left(\frac{|\mathbf{T}|\cos\alpha - |\mathbf{D}|}{|\mathbf{T}|\sin\alpha + |\mathbf{L}|}\right).$$

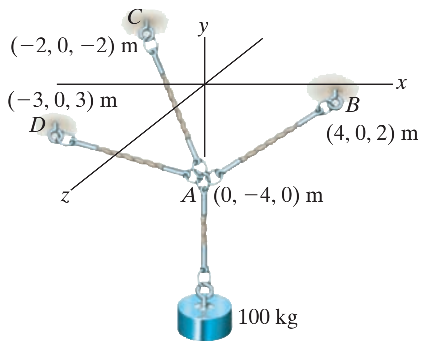
Divide top and bottom on the right by $|\mathbf{D}|$.

For $\alpha = 0$, $|\mathbf{T}| = 0$, $\dfrac{|\mathbf{L}|}{|\mathbf{D}|} = 4$, $\quad \tan\gamma = \left(\dfrac{-1}{4}\right)$ $\gamma = -14^\circ$

Part (b): The flight path angle is a negative angle measured from the horizontal, hence from the equality of opposite interior angles the angle γ is also the positive elevation angle of the airplane measured at the point of landing.

$$\tan\gamma = \frac{1}{h}, \quad h = \frac{1}{\tan\gamma} = \frac{1}{\left(\frac{1}{4}\right)} = 4 \text{ km}$$

Problem 3.63 In Active Example 3.5, suppose that the attachment point B is moved to the point (5,0,0) m. What are the tensions in cables AB, AC, and AD?



Solution: The position vector from point A to point B can be used to write the force $\mathbf{T}_{AB}$.

$$\mathbf{r}_{AB} = (5\mathbf{i} + 4\mathbf{j})\ \text{m}$$

$$\mathbf{T}_{AB} = T_{AB}\frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = T_{AB}(0.781\mathbf{i} + 0.625\mathbf{j})$$

Using the other forces from Active Example 3.5, we have

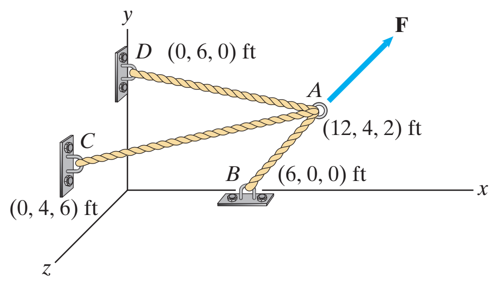
$$\sum F_x : 0.781T_{AB} - 0.408T_{AC} - 0.514T_{AD} = 0$$

$$\sum F_y : 0.625T_{AB} + 0.816T_{AC} + 0.686T_{AD} - 981\ \text{N} = 0$$

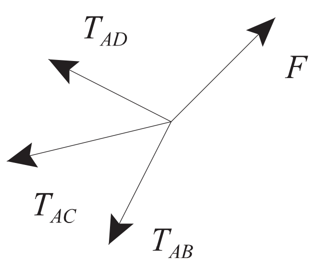
$$\sum F_z : -0.408T_{AC} + 0.514T_{AD} = 0$$

Solving yields $\boxed{T_{AB} = 509\ \text{N},\ T_{AC} = 487\ \text{N},\ T_{AD} = 386\ \text{N}}$

Problem 3.64 The force $\mathbf{F} = 800\mathbf{i} + 200\mathbf{j}$ (lb) acts at point A where the cables AB, AC, and AD are joined. What are the tensions in the three cables?



Solution: We first write the position vectors



$$\mathbf{r}_{AB} = (-6\mathbf{i} - 4\mathbf{j} - 2\mathbf{k})\ \text{ft}$$

$$\mathbf{r}_{AC} = (-12\mathbf{i} + 6\mathbf{k})\ \text{ft}$$

$$\mathbf{r}_{AD} = (-12\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})\ \text{ft}$$

Now we can use these vectors to define the force vectors

$$\mathbf{T}_{AB} = T_{AB}\frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = T_{AB}(-0.802\mathbf{i} - 0.535\mathbf{j} - 0.267\mathbf{k})$$

$$\mathbf{T}_{AC} = T_{AC}\frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = T_{AC}(-0.949\mathbf{i} + 0.316\mathbf{k})$$

$$\mathbf{T}_{AD} = T_{AD}\frac{\mathbf{r}_{AD}}{|\mathbf{r}_{AD}|} = T_{AD}(-0.973\mathbf{i} + 0.162\mathbf{j} - 0.162\mathbf{k})$$

The equilibrium equations are then

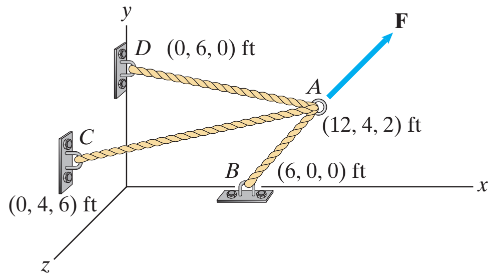
$$\sum F_x : -0.802T_{AB} - 0.949T_{AC} - 0.973T_{AD} + 800\ \text{lb} = 0$$

$$\sum F_y : -0.535T_{AB} + 0.162T_{AD} + 200\ \text{lb} = 0$$

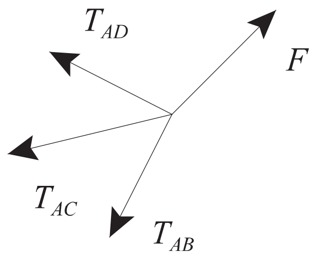
$$\sum F_z : -0.267T_{AB} + 0.316T_{AC} - 0.162T_{AD} = 0$$

Solving, we find $\boxed{T_{AB} = 405\ \text{lb},\ T_{AC} = 395\ \text{lb},\ T_{AD} = 103\ \text{lb}}$

Problem 3.65 Suppose that you want to apply a 1000-lb force **F** at point A in a direction such that the resulting tensions in cables AB, AC, and AD are equal. Determine the components of **F**.



Solution: We first write the position vectors



$$\mathbf{r}_{AB} = (-6\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}) \text{ ft}$$

$$\mathbf{r}_{AC} = (-12\mathbf{i} + 6\mathbf{k}) \text{ ft}$$

$$\mathbf{r}_{AD} = (-12\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) \text{ ft}$$

Now we can use these vectors to define the force vectors

$$\mathbf{T}_{AB} = T\frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = T(-0.802\mathbf{i} - 0.535\mathbf{j} - 0.267\mathbf{k})$$

$$\mathbf{T}_{AC} = T\frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = T(-0.949\mathbf{i} + 0.316\mathbf{k})$$

$$\mathbf{T}_{AD} = T\frac{\mathbf{r}_{AD}}{|\mathbf{r}_{AD}|} = T(-0.973\mathbf{i} + 0.162\mathbf{j} - 0.162\mathbf{k})$$

The force **F** can be written $\mathbf{F} = (F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k})$
The equilibrium equations are then

$$\sum F_x : -0.802T - 0.949T - 0.973T + F_x = 0 \Rightarrow F_x = 2.72T$$

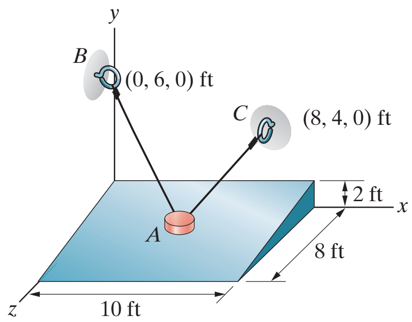
$$\sum F_y : -0.535T + 0.162T + F_y = 0 \Rightarrow F_y = 0.732T$$

$$\sum F_z : -0.267T + 0.316T - 0.162T + F_z = 0 \Rightarrow F_z = 0.113T$$

We also have the constraint equation $\sqrt{F_x{}^2 + F_y{}^2 + F_z{}^2} = 1000$ lb
$\Rightarrow T = 363$ lb

Solving, we find $\boxed{F_x = 990 \text{ lb}, F_y = 135 \text{ lb}, F_z = 41.2 \text{ lb}}$

Problem 3.66 The 10-lb metal disk A is supported by the smooth inclined surface and the strings AB and AC. The disk is located at coordinates (5,1,4) ft. What are the tensions in the strings?



Solution: The position vectors are

$\mathbf{r}_{AB} = (-5\mathbf{i} + 5\mathbf{j} - 4\mathbf{k})$ ft

$\mathbf{r}_{AC} = (3\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$ ft

The angle α between the inclined surface the horizontal is

$$\alpha = \tan^{-1}(2/8) = 14.0^\circ$$

We identify the following force:

$$\mathbf{T}_{AB} = T_{AB}\frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = T_{AB}(-0.615\mathbf{i} + 0.615\mathbf{j} - 0.492\mathbf{k})$$

$$\mathbf{T}_{AC} = T_{AC}\frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = T_{AC}(0.514\mathbf{i} + 0.514\mathbf{j} - 0.686\mathbf{k})$$

$$\mathbf{N} = N(\cos\alpha\mathbf{j} + \sin\alpha\mathbf{k}) = N(0.970\mathbf{j} + 0.243\mathbf{k})$$

$$\mathbf{W} = -(10\text{ lb})\mathbf{j}$$

The equilibrium equations are then

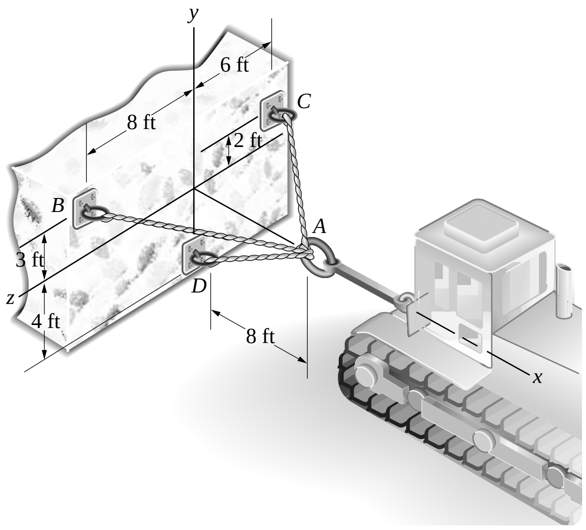
$$\sum F_x : -0.615T_{AB} + 0.514T_{AC} = 0$$

$$\sum F_y : 0.615T_{AB} + 0.514T_{AC} + 0.970N - 10\text{ lb} = 0$$

$$\sum F_z : -0.492T_{AB} - 0.686T_{AC} + 0.243N = 0$$

Solving, we find $N = 8.35$ lb $\boxed{T_{AB} = 1.54\text{ lb}, T_{AC} = 1.85\text{ lb}}$

Problem 3.67 The bulldozer exerts a force $\mathbf{F} = 2\mathbf{i}$ (kip) at A. What are the tensions in cables AB, AC, and AD?



Solution: Isolate the cable juncture. Express the tensions in terms of unit vectors. Solve the equilibrium equations. The coordinates of points A, B, C, D are:

$A(8, 0, 0)$, $B(0, 3, 8)$, $C(0, 2, -6)$, $D(0, -4, 0)$.

The radius vectors for these points are

$$\mathbf{r}_A = 8\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}, \quad \mathbf{r}_B = 0\mathbf{i} + 3\mathbf{j} + 8\mathbf{k},$$

$$\mathbf{r}_C = 0\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}, \quad \mathbf{r}_D = 0\mathbf{i} + 4\mathbf{j} + 0\mathbf{k}.$$

By definition, the unit vector parallel to the tension in cable AB is

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|}.$$

Carrying out the operations for each of the cables, the results are:

$$\mathbf{e}_{AB} = -0.6835\mathbf{i} + 0.2563\mathbf{j} + 0.6835\mathbf{k},$$

$$\mathbf{e}_{AC} = -0.7845\mathbf{i} + 0.1961\mathbf{j} - 0.5883\mathbf{k},$$

$$\mathbf{e}_{AD} = -0.8944\mathbf{i} - 0.4472\mathbf{j} + 0\mathbf{k}.$$

The tensions in the cables are expressed in terms of the unit vectors,

$$\mathbf{T}_{AB} = |\mathbf{T}_{AB}|\mathbf{e}_{AB}, \quad \mathbf{T}_{AC} = |\mathbf{T}_{AC}|\mathbf{e}_{AC}, \quad \mathbf{T}_{AD} = |\mathbf{T}_{AD}|\mathbf{e}_{AD}.$$

The external force acting on the juncture is $\mathbf{F} = 2000\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$. The equilibrium conditions are

$$\sum \mathbf{F} = 0 = \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{F} = 0.$$

Substitute the vectors into the equilibrium conditions:

$$\sum \mathbf{F}_x = (-0.6835|\mathbf{T}_{AB}| - 0.7845|\mathbf{T}_{AC}| - 0.8944|\mathbf{T}_{AD}| + 2000)\mathbf{i} = 0$$

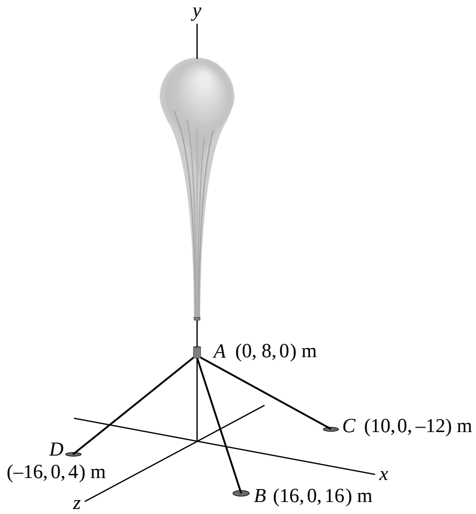
$$\sum \mathbf{F}_y = (0.2563|\mathbf{T}_{AB}| + 0.1961|\mathbf{T}_{AC}| - 0.4472|\mathbf{T}_{AD}|)\mathbf{j} = 0$$

$$\sum \mathbf{F}_z = (0.6835|\mathbf{T}_{AB}| - 0.5883|\mathbf{T}_{AC}| + 0|\mathbf{T}_{AD}|)\mathbf{k} = 0$$

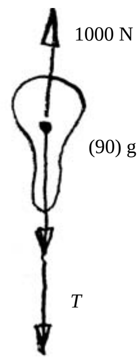
The commercial program **TK Solver Plus** was used to solve these equations. The results are

$$\boxed{|\mathbf{T}_{AB}| = 780.31 \text{ lb}}, \quad \boxed{|\mathbf{T}_{AC}| = 906.49 \text{ lb}}, \quad \boxed{|\mathbf{T}_{AD}| = 844.74 \text{ lb}}.$$

Problem 3.68 Prior to its launch, a balloon carrying a set of experiments to high altitude is held in place by groups of student volunteers holding the tethers at B, C, and D. The mass of the balloon, experiments package, and the gas it contains is 90 kg, and the buoyancy force on the balloon is 1000 N. The supervising professor conservatively estimates that each student can exert at least a 40-N tension on the tether for the necessary length of time. Based on this estimate, what minimum numbers of students are needed at B, C, and D?



Solution:



$$\sum F_y = 1000 - (90)(9.81) - T = 0$$

$$T = 117.1 \text{ N}$$

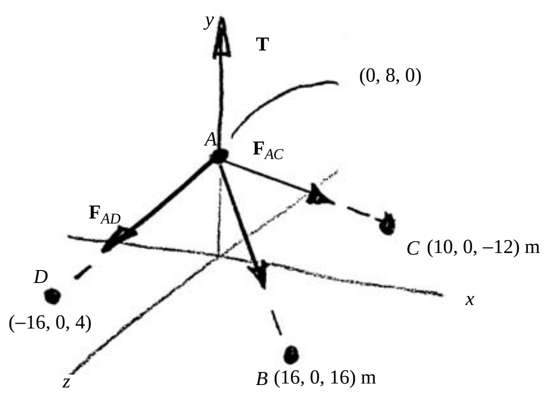
$A(0, 8, 0)$

$B(16, 0, 16)$

$C(10, 0, -12)$

$D(-16, 0, 4)$

We need to write unit vectors $\mathbf{e}_{AB}$, $\mathbf{e}_{AC}$, and $\mathbf{e}_{AD}$.



$$\mathbf{e}_{AB} = 0.667\mathbf{i} - 0.333\mathbf{j} + 0.667\mathbf{k}$$

$$\mathbf{e}_{AC} = 0.570\mathbf{i} - 0.456\mathbf{j} - 0.684\mathbf{k}$$

$$\mathbf{e}_{AD} = -0.873\mathbf{i} - 0.436\mathbf{j} + 0.218\mathbf{k}$$

We now write the forces in terms of magnitudes and unit vectors

$$\begin{cases} \mathbf{F}_{AB} = 0.667F_{AB}\mathbf{i} - 0.333F_{AB}\mathbf{j} + 0.667F_{AB}\mathbf{k} \\ \mathbf{F}_{AC} = 0.570F_{AC}\mathbf{i} - 0.456F_{AC}\mathbf{j} - 0.684F_{AC}\mathbf{k} \\ \mathbf{F}_{AD} = -0.873F_{AD}\mathbf{i} - 0.436F_{AC}\mathbf{j} + 0.218F_{AC}\mathbf{k} \\ \quad \mathbf{T} = 117.1\mathbf{j} \text{ (N)} \end{cases}$$

The equations of equilibrium are

$$\sum F_x = 0.667F_{AB} + 0.570F_{AC} - 0.873F_{AD} = 0$$

$$\sum F_y = -0.333F_{AB} - 0.456F_{AC} - 0.436F_{AC} + 117.1 = 0$$

$$\sum F_z = 0.667F_{AB} - 0.684F_{AC} + 0.218F_{AC} = 0$$

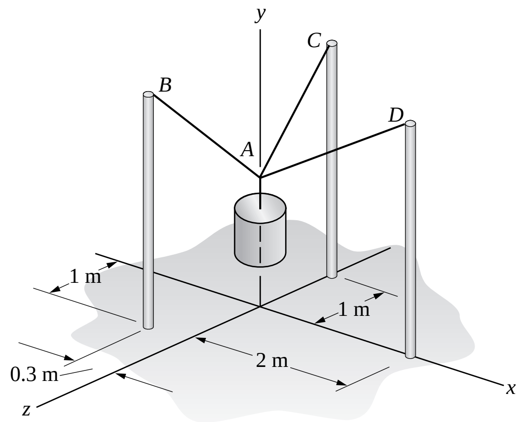
Solving, we get

$F_{AB} = 64.8 \text{ N} \sim 2$ students

$F_{AC} = 99.8 \text{ N} \sim 3$ students

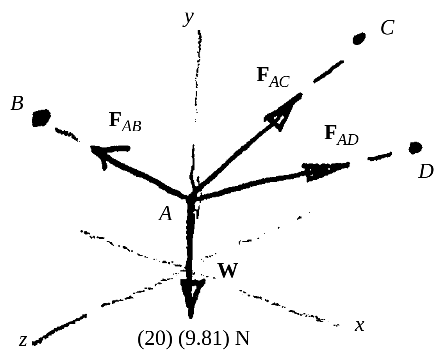
$F_{AD} = 114.6 \text{ N} \sim 3$ students

Problem 3.69 The 20-kg mass is suspended by cables attached to three vertical 2-m posts. Point A is at (0, 1.2, 0) m. Determine the tensions in cables AB, AC, and AD.



Solution: Points A, B, C, and D are located at

$A(0, 1.2, 0)$, $B(-0.3, 2, 1)$,
$C(0, 2, -1)$, $D(2, 2, 0)$



Write the unit vectors $\mathbf{e}_{AB}$, $\mathbf{e}_{AC}$, $\mathbf{e}_{AD}$

$$\mathbf{e}_{AB} = -0.228\mathbf{i} + 0.608\mathbf{j} + 0.760\mathbf{k}$$

$$\mathbf{e}_{AC} = 0\mathbf{i} + 0.625\mathbf{j} - 0.781\mathbf{k}$$

$$\mathbf{e}_{AD} = 0.928\mathbf{i} + 0.371\mathbf{j} + 0\mathbf{k}$$

The forces are

$$\mathbf{F}_{AB} = -0.228F_{AB}\mathbf{i} + 0.608F_{AB}\mathbf{j} + 0.760F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = 0F_{AC}\mathbf{i} + 0.625F_{AC}\mathbf{j} - 0.781F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AD} = 0.928F_{AD}\mathbf{i} + 0.371F_{AD}\mathbf{j} + 0\mathbf{k}$$

$$\mathbf{W} = -(20)(9.81)\mathbf{j}$$

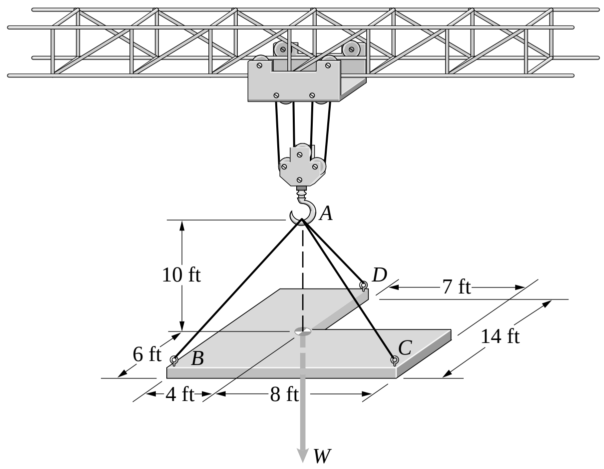
The equations of equilibrium are

$$\begin{cases} \sum F_x = -0.228F_{AB} + 0 + 0.928F_{AD} = 0 \\ \sum F_y = 0.608F_{AB} + 0.625F_{AC} + 0.371F_{AD} - 20(9.81) = 0 \\ \sum F_z = 0.760F_{AB} - 0.781F_{AC} + 0 = 0 \end{cases}$$

We have 3 eqns in 3 unknowns solving, we get

$$\boxed{\begin{array}{l} F_{AB} = 150.0\ \text{N} \\ F_{AC} = 146.1\ \text{N} \\ F_{AD} = 36.9\ \text{N} \end{array}}$$

Problem 3.70 The weight of the horizontal wall section is $W = 20{,}000$ lb. Determine the tensions in the cables AB, AC, and AD.



Solution: Set the coordinate origin at A with axes as shown. The upward force, T, at point A will be equal to the weight, W, since the cable at A supports the entire wall. The upward force at A is $\mathbf{T} = W\mathbf{k}$. From the figure, the coordinates of the points in feet are

$A(4, 6, 10)$, $B(0, 0, 0)$, $C(12, 0, 0)$, and $D(4, 14, 0)$.

The three unit vectors are of the form

$$\mathbf{e}_{AI} = \frac{(x_I - x_A)\mathbf{i} + (y_I - y_A)\mathbf{j} + (z_I - z_A)\mathbf{k}}{\sqrt{(x_I - x_A)^2 + (y_I - y_A)^2 + (z_I - z_A)^2}},$$

where I takes on the values B, C, and D. The denominators of the unit vectors are the distances AB, AC, and AD, respectively. Substitution of the coordinates of the points yields the following unit vectors:

$$\mathbf{e}_{AB} = -0.324\mathbf{i} - 0.487\mathbf{j} - 0.811\mathbf{k},$$

$$\mathbf{e}_{AC} = 0.566\mathbf{i} - 0.424\mathbf{j} - 0.707\mathbf{k},$$

and $\mathbf{e}_{AD} = 0\mathbf{i} + 0.625\mathbf{j} - 0.781\mathbf{k}$.

The forces are

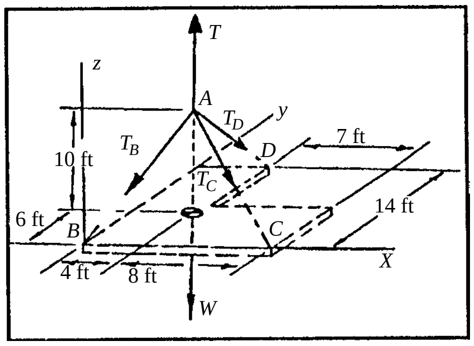
$\mathbf{T}_{AB} = T_{AB}\mathbf{e}_{AB}$, $\mathbf{T}_{AC} = T_{AC}\mathbf{e}_{AC}$, and $\mathbf{T}_{AD} = T_{AD}\mathbf{e}_{AD}$.

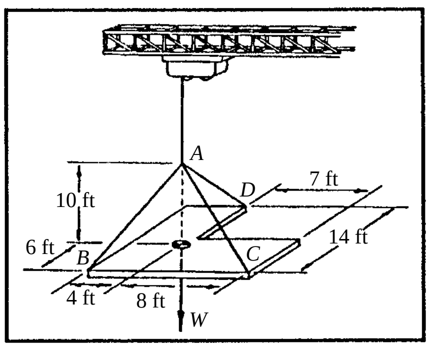
The equilibrium equation for the knot at point A is

$$\mathbf{T} + \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} = 0.$$

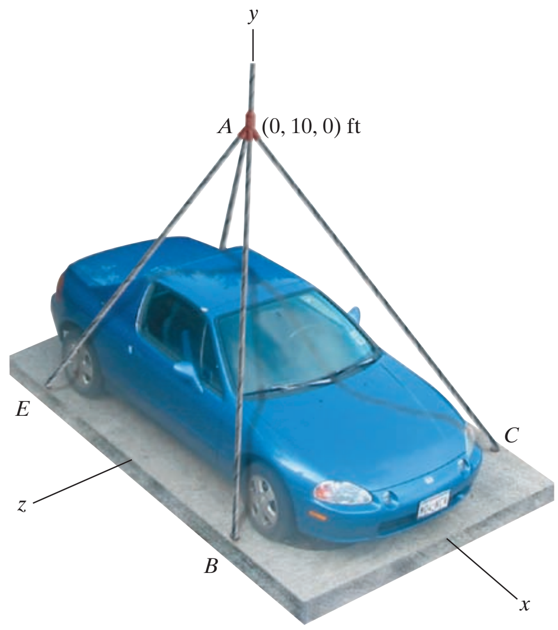
From the vector equilibrium equation, write the scalar equilibrium equations in the x, y, and z directions. We get three linear equations in three unknowns. Solving these equations simultaneously, we get

$$\boxed{T_{AB} = 9393 \text{ lb}, \ T_{AC} = 5387 \text{ lb}, \text{ and } \ T_{AD} = 10{,}977 \text{ lb}}$$

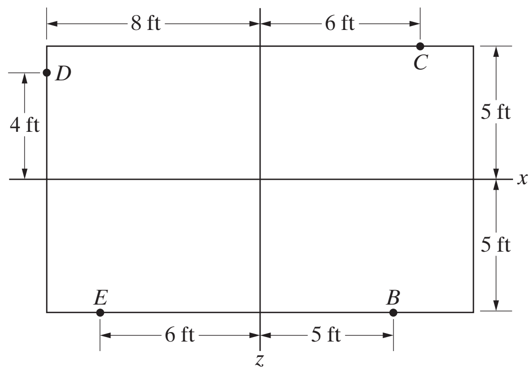




Problem 3.71 The car in Fig. a and the pallet supporting it weigh 3000 lb. They are supported by four cables AB, AC, AD, and AE. The locations of the attachment points on the pallet are shown in Fig. b. The tensions in cables AB and AE are equal. Determine the tensions in the cables.



(a)



(b)

Solution: Isolate the knot at A. Let $\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{T}_{AD}$ and $\mathbf{T}_{AE}$ be the forces exerted by the tensions in the cables. The force exerted by the vertical cable is (3000 lb)$\mathbf{j}$. We first find the position vectors and then express all of the forces as vectors.

$$\mathbf{r}_{AB} = (5\mathbf{i} - 10\mathbf{j} + 5\mathbf{k}) \text{ ft}$$

$$\mathbf{r}_{AC} = (6\mathbf{i} - 10\mathbf{j} - 5\mathbf{k}) \text{ ft}$$

$$\mathbf{r}_{AD} = (-8\mathbf{i} - 10\mathbf{j} - 4\mathbf{k}) \text{ ft}$$

$$\mathbf{r}_{AE} = (-6\mathbf{i} - 10\mathbf{j} + 5\mathbf{k}) \text{ ft}$$

$$\mathbf{T}_{AB} = T_{AB}\frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = T_{AB}(0.408\mathbf{i} - 0.816\mathbf{j} + 0.408\mathbf{k})$$

$$\mathbf{T}_{AC} = T_{AC}\frac{\mathbf{r}_{AC}}{|\mathbf{r}_{AC}|} = T_{AC}(0.473\mathbf{i} - 0.788\mathbf{j} - 0.394\mathbf{k})$$

$$\mathbf{T}_{AD} = T_{AD}\frac{\mathbf{r}_{AD}}{|\mathbf{r}_{AD}|} = T_{AD}(-0.596\mathbf{i} - 0.745\mathbf{j} - 0.298\mathbf{k})$$

$$\mathbf{T}_{AE} = T_{AE}\frac{\mathbf{r}_{AE}}{|\mathbf{r}_{AE}|} = T_{AE}(-0.473\mathbf{i} - 0.788\mathbf{j} + 0.394\mathbf{k})$$

The equilibrium equations are

$$\sum F_x : 0.408T_{AB} + 0.473T_{AC} - 0.596T_{AD} - 0.473T_{AE} = 0$$

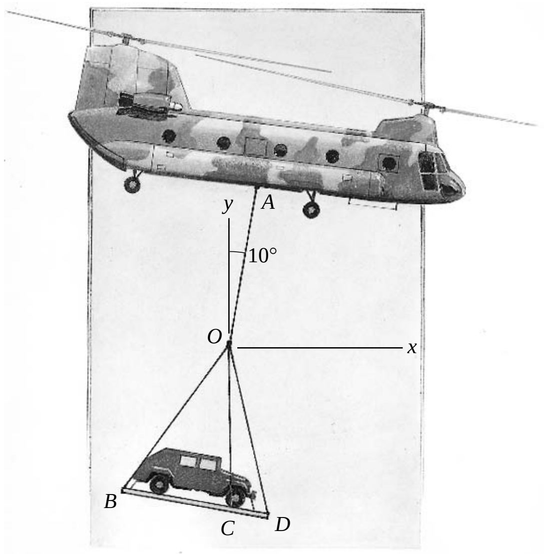
$$\sum F_y : -0.816T_{AB} - 0.788T_{AC} - 0.745T_{AD} - 0.788T_{AE} + 3000 \text{ lb} = 0$$

$$\sum F_z : 0.408T_{AB} - 0.394T_{AC} - 0.298T_{AD} + 0.394T_{AE} = 0$$

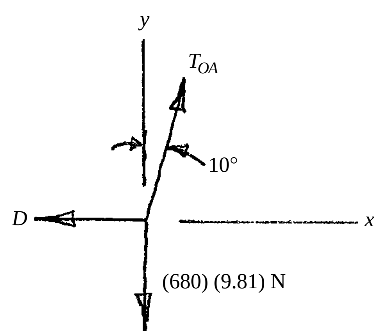
Solving, we find

$$\boxed{T_{AB} = 896 \text{ lb}, T_{AC} = 1186 \text{ lb}, T_{AD} = 843 \text{ lb}, T_{AE} = 896 \text{ lb}}$$

Problem 3.72 The 680-kg load suspended from the helicopter is in equilibrium. The aerodynamic drag force on the load is horizontal. The y axis is vertical, and cable OA lies in the x-y plane. Determine the magnitude of the drag force and the tension in cable OA.



Solution:



$$\sum F_x = T_{OA}\sin 10^\circ - D = 0,$$

$$\sum F_y = T_{OA}\cos 10^\circ - (680)(9.81) = 0.$$

Solving, we obtain $D = 1176$ N, $T_{OA} = 6774$ N.

Problem 3.73 In Problem 3.72, the coordinates of the three cable attachment points B, C, and D are $(-3.3, -4.5, 0)$ m, $(1.1, -5.3, 1)$ m, and $(1.6, -5.4, -1)$ m, respectively. What are the tensions in cables OB, OC, and OD?

Solution: The position vectors from O to pts B, C, and D are

$$\mathbf{r}_{OB} = -3.3\mathbf{i} - 4.5\mathbf{j} \text{ (m)},$$

$$\mathbf{r}_{OC} = 1.1\mathbf{i} - 5.3\mathbf{j} + \mathbf{k} \text{ (m)},$$

$$\mathbf{r}_{OD} = 1.6\mathbf{i} - 5.4\mathbf{j} - \mathbf{k} \text{ (m)}.$$

Dividing by the magnitudes, we obtain the unit vectors

$$\mathbf{e}_{OB} = -0.591\mathbf{i} - 0.806\mathbf{j},$$

$$\mathbf{e}_{OC} = 0.200\mathbf{i} - 0.963\mathbf{j} + 0.182\mathbf{k},$$

$$\mathbf{e}_{OD} = 0.280\mathbf{i} - 0.944\mathbf{j} - 0.175\mathbf{k}.$$

Using these unit vectors, we obtain the equilibrium equations

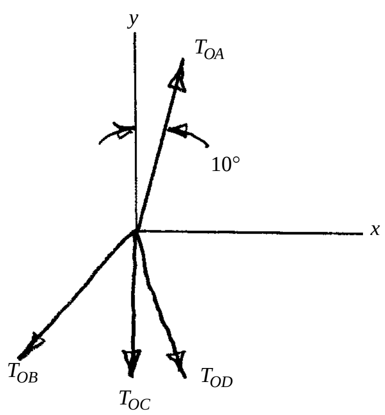
$$\sum F_x = T_{OA}\sin 10^\circ - 0.591T_{OB} + 0.200T_{OC} + 0.280T_{OD} = 0,$$

$$\sum F_y = T_{OA}\cos 10^\circ - 0.806T_{OB} - 0.963T_{OC} - 0.944T_{OD} = 0,$$

$$\sum F_z = 0.182T_{OC} - 0.175T_{OD} = 0.$$

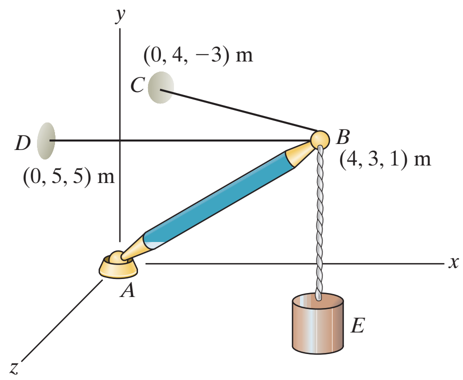
From the solution of Problem 3.72, $T_{OA} = 6774$ N. Solving these equations, we obtain

$$T_{OB} = 3.60 \text{ kN}, \quad T_{OC} = 1.94 \text{ kN}, \quad T_{OD} = 2.02 \text{ kN}.$$



Problem 3.74 If the mass of the bar AB is negligible compared to the mass of the suspended object E, the bar exerts a force on the "ball" at B that points from A toward B. The mass of the object E is 200 kg. The y-axis points upward. Determine the tensions in the cables BC and CD.

Strategy: Draw a free-body diagram of the ball at B. (The weight of the ball is negligible.)



Solution:

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{-4\mathbf{i} - 3\mathbf{j} - \mathbf{k}}{\sqrt{26}}\right), \ \mathbf{T}_{BC} = T_{BC}\left(\frac{-4\mathbf{i} + \mathbf{j} - 4\mathbf{k}}{\sqrt{33}}\right),$$

The forces

$$\mathbf{T}_{BD} = T_{BD}\left(\frac{-4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}}{6}\right), \ \mathbf{W} = -(200 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j}$$

The equilibrium equations

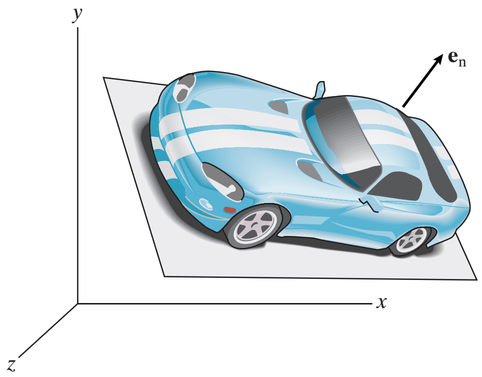
$$\sum F_x : -\frac{4}{\sqrt{26}}F_{AB} - \frac{4}{\sqrt{33}}T_{BC} - \frac{4}{6}T_{BD} = 0$$

$$\sum F_y : -\frac{3}{\sqrt{26}}F_{AB} + \frac{1}{\sqrt{33}}T_{BC} + \frac{2}{6}T_{BD} - 1962 \text{ N} = 0$$

$$\sum F_z : -\frac{1}{\sqrt{26}}F_{AB} - \frac{4}{\sqrt{33}}T_{BC} + \frac{4}{6}T_{BD} = 0$$

$$\Rightarrow \boxed{\begin{matrix} T_{BC} = 1610 \text{ N} \\ T_{BD} = 1009 \text{ N} \end{matrix}}$$

Problem 3.75* The 3400-lb car is at rest on the plane surface. The unit vector $\mathbf{e}_n = 0.456\mathbf{i} + 0.570\mathbf{j} + 0.684\mathbf{k}$ is perpendicular to the surface. Determine the magnitudes of the total normal force $\mathbf{N}$ and the total friction force $\mathbf{f}$ exerted on the surface by the car's wheels.



Solution: The forces on the car are its weight, the normal force, and the friction force.

The normal force is in the direction of the unit vector, so it can be written

$$\mathbf{N} = N\mathbf{e}_n = N(0.456\mathbf{i} + 0.570\mathbf{j} + 0.684\mathbf{k})$$

The equilibrium equation is

$$N\mathbf{e}_n + \mathbf{f} - (3400 \text{ lb})\mathbf{j} = 0$$

The friction force $\mathbf{f}$ is perpendicular to $\mathbf{N}$, so we can eliminate the friction force from the equilibrium equation by taking the dot product of the equation with $\mathbf{e}_n$.

$$(N\mathbf{e}_n + \mathbf{f} - (3400 \text{ lb})\mathbf{j}) \cdot \mathbf{e}_n = N - (3400 \text{ lb})(\mathbf{j} \cdot \mathbf{e}_n) = 0$$

$$N = (3400 \text{ lb})(0.57) = 1940 \text{ lb}$$

Now we can solve for the friction force $\mathbf{f}$.

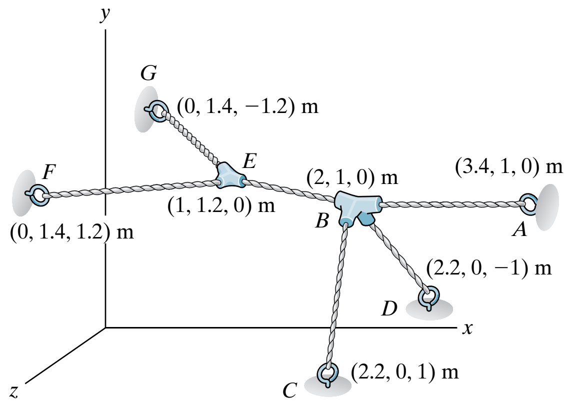
$$\mathbf{f} = (3400 \text{ lb})\mathbf{j} - N\mathbf{e}_n = (3400 \text{ lb})\mathbf{j} - (1940 \text{ lb})(0.456\mathbf{i} + 0.570\mathbf{j} + 0.684\mathbf{k})$$

$$\mathbf{f} = (-884\mathbf{i} + 2300\mathbf{j} - 1330\mathbf{k}) \text{ lb}$$

$$|\mathbf{f}| = \sqrt{(-884 \text{ lb})^2 + (2300 \text{ lb})^2 + (-1330 \text{ lb})^2} = 2790 \text{ lb}$$

$$\boxed{|\mathbf{N}| = 1940 \text{ lb}, |\mathbf{f}| = 2790 \text{ lb}}$$

Problem 3.76 The system shown anchors a stanchion of a cable-suspended roof. If the tension in cable AB is 900 kN, what are the tensions in cables EF and EG?



Solution: Using the coordinates for the points we find

$$\mathbf{r}_{BA} = [(3.4-2)\mathbf{i} + (1-1)\mathbf{j} + (0-0)\mathbf{k}]\text{ m}$$

$$\mathbf{r}_{\text{BA}} = (1.4\text{ m})\mathbf{i}\ \mathbf{e}_{BA} = \frac{\mathbf{r}_{BA}}{|\mathbf{r}_{BA}|} = \mathbf{i}$$

Using the same procedure we find the other unit vectors that we need.

$$\mathbf{e}_{BC} = 0.140\mathbf{i} - 0.700\mathbf{j} + 0.700\mathbf{k}$$

$$\mathbf{e}_{BD} = 0.140\mathbf{i} - 0.700\mathbf{j} + 0.700\mathbf{k}$$

$$\mathbf{e}_{BE} = -0.981\mathbf{i} + 0.196\mathbf{j}$$

$$\mathbf{e}_{EG} = -0.635\mathbf{i} + 0.127\mathbf{j} - 0.762\mathbf{k}$$

$$\mathbf{e}_{EF} = -0.635\mathbf{i} + 0.127\mathbf{j} + 0.762\mathbf{k}$$

We can now write the equilibrium equations for the connections at B and E.

$$(900\text{ kN})\mathbf{e}_{BA} + T_{BC}\mathbf{e}_{BC} + T_{BD}\mathbf{e}_{BD} + T_{BE}\mathbf{e}_{BE} = 0,\ T_{BE}(-\mathbf{e}_{BE}) + T_{EF}\mathbf{e}_{EF} + T_{EG}\mathbf{e}_{EG}0$$

Breaking these equations into components, we have the following six equations to solve for five unknows (one of the equations is redundant).

$$(900\text{ kN}) + T_{BC}(0.140) + T_{BD}(0.140) + T_{BE}(-0.981) = 0$$

$$T_{BC}(-0.700) + T_{BD}(-0.700) + T_{BE}(0.196) = 0$$

$$T_{BC}(0.700) + T_{BD}(-0.700) = 0$$

$$T_{BE}(0.981) + T_{EG}(-0.635) + T_{EF}(-0.635) = 0$$

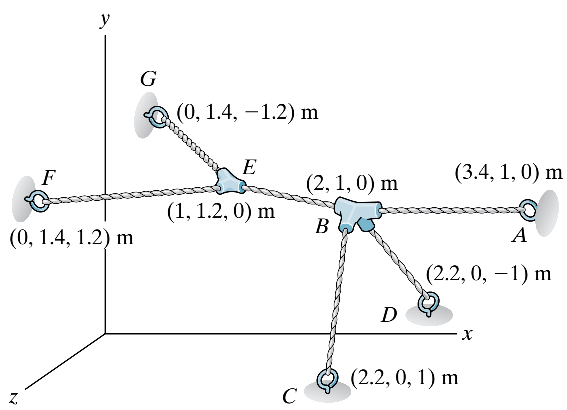
$$T_{BE}(-0.196) + T_{EG}(0.127) + T_{EF}(0.127) = 0$$

$$T_{EG}(-0.726) + T_{EF}(0.726) = 0$$

Solving, we find $T_{BC} = T_{BD} = 134$ kN, $T_{BE} = 956$ kN

$$\boxed{T_{EF} = T_{EG} = 738\text{ kN}}$$

Problem 3.77* The cables of the system will each safely support a tension of 1500 kN. Based on this criterion, what is the largest safe value of the tension in cable AB?

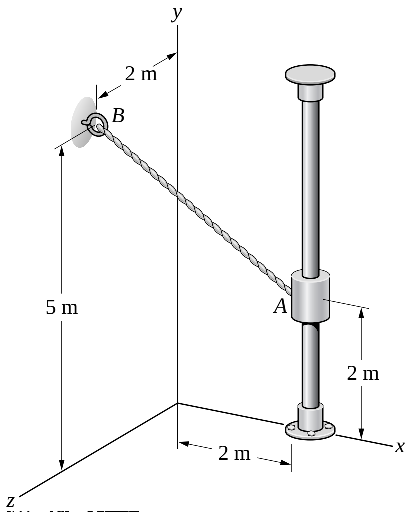


Solution: From Problem 3.76 we know that if the tension in AB is 900 kN, then the largest force in the system occurs in cable BE and that tension is 956 kN. To solve this problem, we can just scale the results from Problem 3.76

$$T_{AB} = \left(\frac{1500\text{ kN}}{956\text{ kN}}\right)(900\text{ kN}) \quad \boxed{T_{AB} = 1410\text{ kN}}$$

Problem 3.78 The 200-kg slider at A is held in place on the smooth vertical bar by the cable AB.

(a) Determine the tension in the cable.
(b) Determine the force exerted on the slider by the bar.

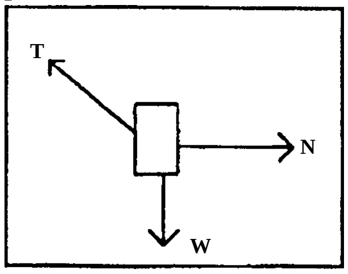


Solution: The coordinates of the points A, B are $A(2, 2, 0)$, $B(0, 5, 2)$. The vector positions

$$\mathbf{r}_A = 2\mathbf{i} + 2\mathbf{j} + 0\mathbf{k}, \quad \mathbf{r}_B = 0\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$$

The equilibrium conditions are:

$$\sum \mathbf{F} = \mathbf{T} + \mathbf{N} + \mathbf{W} = 0.$$



Eliminate the slider bar normal force as follows: The bar is parallel to the y axis, hence the unit vector parallel to the bar is $\mathbf{e}_B = 0\mathbf{i} + 1\mathbf{j} + 0\mathbf{k}$. The dot product of the unit vector and the normal force vanishes: $\mathbf{e}_B \cdot \mathbf{N} = 0$. Take the dot product of $\mathbf{e}_B$ with the equilibrium conditions: $\mathbf{e}_B \cdot \mathbf{N} = 0$.

$$\sum \mathbf{e}_B \cdot \mathbf{F} = \mathbf{e}_B \cdot \mathbf{T} + \mathbf{e}_B \cdot \mathbf{W} = 0.$$

The weight is

$$\mathbf{e}_B \cdot \mathbf{W} = 1\mathbf{j} \cdot (-\mathbf{j}|\mathbf{W}|) = -|\mathbf{W}| = -(200)(9.81) = -1962 \text{ N}.$$

The unit vector parallel to the cable is by definition,

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|}.$$

Substitute the vectors and carry out the operation:

$$\mathbf{e}_{AB} = -0.4851\mathbf{i} + 0.7278\mathbf{j} + 0.4851\mathbf{k}.$$

(a) The tension in the cable is $\mathbf{T} = |\mathbf{T}|\mathbf{e}_{AB}$. Substitute into the modified equilibrium condition

$$\sum \mathbf{e}_B \mathbf{F} = (0.7276|\mathbf{T}| - 1962) = 0.$$

Solve: $|\mathbf{T}| = 2696.5$ N from which the tension vector is

$$\mathbf{T} = |\mathbf{T}|\mathbf{e}_{AB} = -1308\mathbf{i} + 1962\mathbf{j} + 1308\mathbf{k}.$$

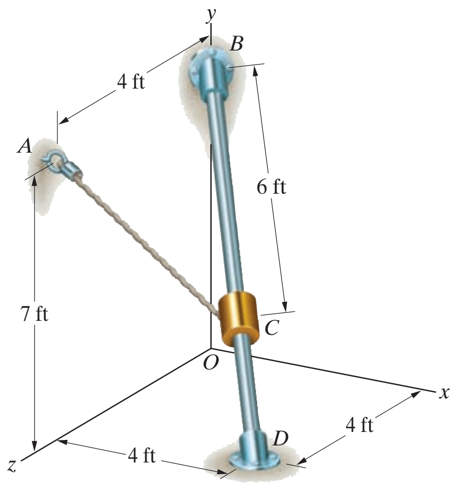
(b) The equilibrium conditions are

$$\sum \mathbf{F} = 0 = \mathbf{T} + \mathbf{N} + \mathbf{W} = -1308\mathbf{i} + 1308\mathbf{k} + \mathbf{N} = 0.$$

Solve for the normal force: $\mathbf{N} = 1308\mathbf{i} - 1308\mathbf{k}$. The magnitude is $|\mathbf{N}| = 1850$ N.

Note: For this specific configuration, the problem can be solved without eliminating the slider bar normal force, since it does not appear in the y-component of the equilibrium equation (the slider bar is parallel to the y-axis). However, in the general case, the slider bar will not be parallel to an axis, and the unknown normal force will be projected onto all components of the equilibrium equations (see Problem 3.79 below). In this general situation, it will be necessary to eliminate the slider bar normal force by some procedure equivalent to that used above. *End Note.*

Problem 3.79 In Example 3.6, suppose that the cable AC is replaced by a longer one so that the distance from point B to the slider C increases from 6 ft to 8 ft. Determine the tension in the cable.



Solution: The vector from B to C is now

$$\mathbf{r}_{BC} = (8 \text{ ft})\,\mathbf{e}_{BD}$$

$$\mathbf{r}_{BC} = (8 \text{ ft})\left(\frac{4}{9}\mathbf{i} - \frac{7}{9}\mathbf{j} + \frac{4}{9}\mathbf{k}\right)$$

$$\mathbf{r}_{BC} = (3.56\mathbf{i} - 6.22\mathbf{j} + 3.56\mathbf{k}) \text{ ft}$$

We can now find the unit vector form C to A.

$$\mathbf{r}_{CA} = \mathbf{r}_{OA} - (\mathbf{r}_{OB} + \mathbf{r}_{BC}) = [(7\mathbf{j} + 4\mathbf{k})$$

$$- \{(7\mathbf{j}) + (3.56\mathbf{i} - 6.22\mathbf{j} + 3.56\mathbf{k})\}] \text{ ft}$$

$$\mathbf{r}_{CA} = (-3.56\mathbf{i} + 6.22\mathbf{j} + 0.444\mathbf{k}) \text{ ft}$$

$$\mathbf{e}_{CA} = \frac{\mathbf{r}_{CA}}{|\mathbf{r}_{CA}|} = (-0.495\mathbf{i} + 0.867\mathbf{j} + 0.0619\mathbf{k})$$

Using $\mathbf{N}$ to stand for the normal force between the bar and the slider, we can write the equilibrium equation:

$$T\mathbf{e}_{CA} + \mathbf{N} - (100 \text{ lb})\mathbf{j} = 0$$

We can use the dot product to eliminate $\mathbf{N}$ from the equation

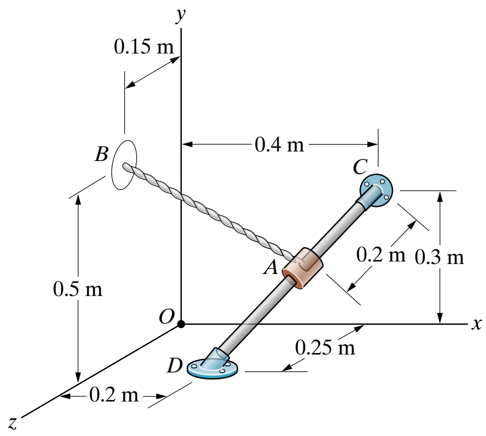
$$[T\mathbf{e}_{CA} + \mathbf{N} - (100 \text{ lb})\mathbf{j}] \cdot \mathbf{e}_{BD} = T(\mathbf{e}_{CA} \cdot \mathbf{e}_{BD}) - (100 \text{ lb})(\mathbf{j} \cdot \mathbf{e}_{BD}) = 0$$

$$T\left(\left[\frac{4}{9}\right][-0.495] + \left[-\frac{7}{9}\right][0.867] + \left[\frac{4}{9}\right][0.0619]\right) - (100 \text{ lb})(-0.778) = 0$$

$$T(-0.867) + (77.8 \text{ lb}) = 0 \Rightarrow T = 89.8 \text{ lb}$$

$$\boxed{T = 89.8 \text{ lb}}$$

Problem 3.80 The cable AB keeps the 8-kg collar A in place on the smooth bar CD. The y axis points upward. What is the tension in the cable?



Solution: We develop the following position vectors and unit vectors

$$\mathbf{r}_{CD} = (-0.2\mathbf{i} - 0.3\mathbf{j} + 0.25\mathbf{k})\ \text{m}$$

$$\mathbf{e}_{CD} = \frac{\mathbf{r}_{CD}}{|\mathbf{r}_{CD}|} = (-0.456\mathbf{i} - 0.684\mathbf{j} + 0.570\mathbf{k})$$

$$\mathbf{r}_{CA} = (0.2\ \text{m})\mathbf{e}_{CD} = (-0.091\mathbf{i} - 0.137\mathbf{j} + 0.114\mathbf{k})\ \text{m}$$

$$\mathbf{r}_{AB} = \mathbf{r}_{OB} - (\mathbf{r}_{OC} + \mathbf{r}_{CA})$$

$$\mathbf{r}_{AB} = [(0.5\mathbf{j} + 0.15\mathbf{k}) - (\{0.4\mathbf{i} + 0.3\mathbf{j}\} + \{-0.091\mathbf{i} - 0.137\mathbf{j} + 0.114\mathbf{k}\})]\ \text{m}$$

$$\mathbf{r}_{AB} = (-0.309\mathbf{i} + 0.337\mathbf{j} + 0.036\mathbf{k})\ \text{m}$$

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|} = (-0.674\mathbf{i} + 0.735\mathbf{j} + 0.079\mathbf{k})$$

We can now write the equilibrium equation for the slider using $\mathbf{N}$ to stand for the normal force between the slider and the bar CD.

$$T\mathbf{e}_{AB} + \mathbf{N} - (8\ \text{kg})(9.81\ \text{m/s}^2)\mathbf{j} = 0$$

To eliminate the normal force $\mathbf{N}$ we take a dot product with $\mathbf{e}_{CD}$.

$$[T\mathbf{e}_{AB} + \mathbf{N} - (8\ \text{kg})(9.81\ \text{m/s}^2)\mathbf{j}] \cdot \mathbf{e}_{CD} = 0$$

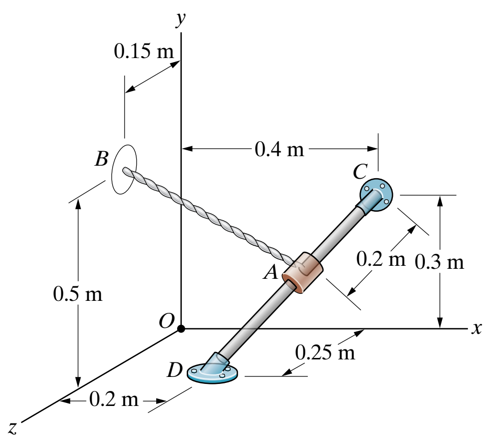
$$T(\mathbf{e}_{AB} \cdot \mathbf{e}_{CD}) - (78.5\ \text{N})(\mathbf{j} \cdot \mathbf{e}_{CD}) = 0$$

$$T([-0.674][-0.456] + [0.735][-0.684] + [0.079][0.570]) - (78.5\ \text{N})(-0.684) = 0$$

$$T(-0.150) + 53.6\ \text{N} = 0$$

$$\boxed{T = 357\ \text{N}}$$

Problem 3.81 Determine the magnitude of the normal force exerted on the collar A by the smooth bar.



Solution: From Problem 3.81 we have

$$\mathbf{e}_{AB} = (-0.674\mathbf{i} + 0.735\mathbf{j} + 0.079\mathbf{k})$$

$$T = 357\ \text{N}$$

The equilibrium equation is

$$T\mathbf{e}_{AB} + \mathbf{N} - (78.5\ \text{N})\mathbf{j} = 0$$

We can now solve for the normal force $\mathbf{N}$.

$$\mathbf{N} = (78.5\ \text{N})\mathbf{j} - (357\ \text{N})(-0.674\mathbf{i} + 0.735\mathbf{j} + 0.079\mathbf{k})$$

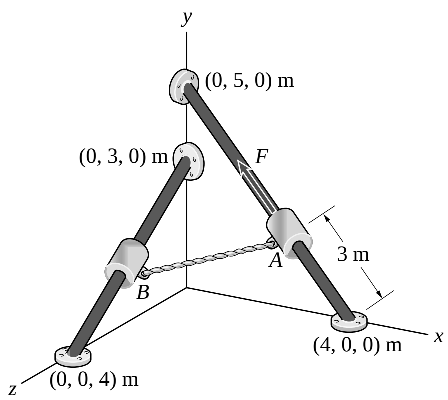
$$\mathbf{N} = (240\mathbf{i} - 184\mathbf{j} - 28.1\mathbf{k})\ \text{N}$$

The magnitude of $\mathbf{N}$ is

$$|\mathbf{N}| = \sqrt{(240\ \text{N})^2 + (-184\ \text{N})^2 + (-28.1\ \text{N})^2}$$

$$\boxed{|\mathbf{N}| = 304\ \text{N}}$$

Problem 3.82* The 10-kg collar A and 20-kg collar B are held in place on the smooth bars by the 3-m cable from A to B and the force F acting on A. The force F is parallel to the bar. Determine F.



Solution: The geometry is the first part of the Problem. To ease our work, let us name the points C, D, E, and G as shown in the figure. The unit vectors from C to D and from E to G are essential to the location of points A and B. The diagram shown contains two free bodies plus the pertinent geometry. The unit vectors from C to D and from E to G are given by

$$\mathbf{e}_{CD} = \mathbf{er}_{CDx}\mathbf{i} + \mathbf{e}_{CDy}\mathbf{j} + \mathbf{e}_{CDz}\mathbf{k},$$

and $\mathbf{e}_{EG} = \mathbf{er}_{EGx}\mathbf{i} + \mathbf{e}_{EGy}\mathbf{j} + \mathbf{e}_{EGz}\mathbf{k}$.

Using the coordinates of points C, D, E, and G from the picture, the unit vectors are

$$\mathbf{e}_{CD} = -0.625\mathbf{i} + 0.781\mathbf{j} + 0\mathbf{k},$$

and $\mathbf{e}_{EG} = 0\mathbf{i} + 0.6\mathbf{j} + 0.8\mathbf{k}$.

The location of point A is given by

$$x_A = x_C + CA\mathbf{e}_{CDx}, \quad y_A = y_C + CA\mathbf{e}_{CDy},$$

and $z_A = z_C + CA\mathbf{e}_{CDz}$,

where $CA = 3$ m. From these equations, we find that the location of point A is given by A (2.13, 2.34, 0) m. Once we know the location of point A, we can proceed to find the location of point B. We have two ways to determine the location of B. First, B is 3 m from point A along the line AB (which we do not know). Also, B lies on the line EG. The equations for the location of point B based on line AB are:

$$x_B = x_A + AB\mathbf{e}_{ABx}, \quad y_B = y_A + AB\mathbf{e}_{ABy},$$

and $z_B = z_A + AB\mathbf{e}_{ABz}$.

The equations based on line EG are:

$$x_B = x_E + EB\mathbf{e}_{EGx}, \quad y_B = y_E + EB\mathbf{e}_{EGy},$$

and $z_B = z_E + EB\mathbf{e}_{EGz}$.

We have six new equations in the three coordinates of B and the distance EB. Some of the information in the equations is redundant. However, we can solve for EB (and the coordinates of B). We get that the length EB is 2.56 m and that point B is located at (0, 1.53, 1.96) m. We next write equilibrium equations for bodies A and B. From the free body diagram for A, we get

$$N_{Ax} + T_{AB}\mathbf{e}_{ABx} + F\mathbf{e}_{CDx} = 0,$$

$$N_{Ay} + T_{AB}\mathbf{e}_{ABy} + F\mathbf{e}_{CDy} - m_A g = 0,$$

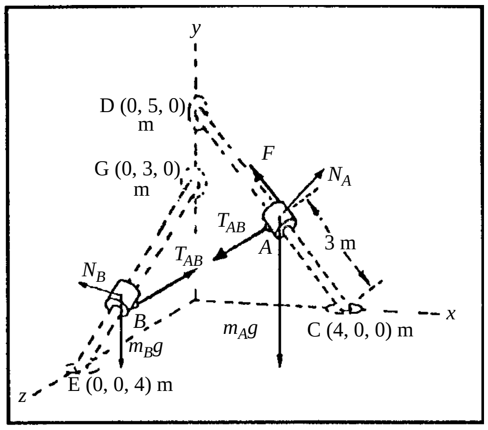
and $N_{Az} + T_{AB}\mathbf{e}_{ABz} + F\mathbf{e}_{CDz} = 0$.

From the free body diagram for B, we get

$$N_{Bx} - T_{AB}\mathbf{e}_{ABx} = 0,$$

$$N_{by} - T_{AB}\mathbf{e}_{ABy} - m_B g = 0,$$

and $N_{Bz} - T_{AB}\mathbf{e}_{ABz} = 0$.



We now have two fewer equation than unknowns. Fortunately, there are two conditions we have not yet invoked. The bars at A and B are smooth. This means that the normal force on each bar can have no component along that bar. This can be expressed by using the dot product of the normal force and the unit vector along the bar. The two conditions are

$$\mathbf{N}_A \cdot \mathbf{e}_{CD} = N_{Ax}\mathbf{e}_{CDx} + N_{Ay}\mathbf{e}_{CDy} + N_{Az}\mathbf{e}_{CDz} = 0$$

for slider A and

$$\mathbf{N}_B \cdot \mathbf{e}_{EG} = N_{Bx}\mathbf{e}_{EGx} + N_{By}\mathbf{e}_{EGy} + N_{Bz}\mathbf{e}_{EGz} = 0.$$

Solving the eight equations in the eight unknowns, we obtain

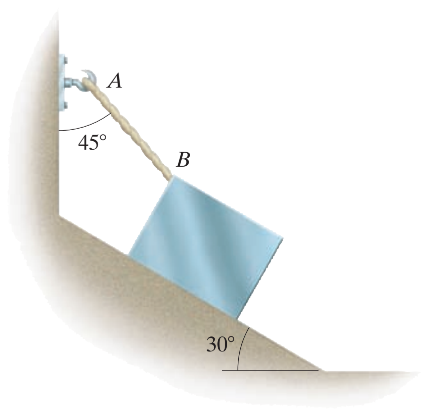
$\boxed{F = 36.6 \text{ N}}$.

Other values obtained in the solution are $EB = 2.56$ m,

$N_{Ax} = 145$ N, $\quad N_{Ay} = 116$ N, $\quad N_{Az} = -112$ N,

$N_{Bx} = -122$ N, $\quad N_{By} = 150$ N, $\quad$ and $\quad N_{Bz} = 112$ N.

Problem 3.83 The 100-lb crate is held in place on the smooth surface by the rope AB. Determine the tension in the rope and the magnitude of the normal force exerted on the crate by the surface.



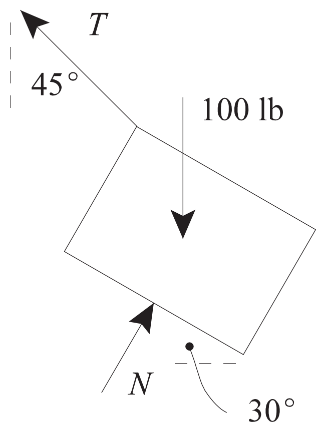
Solution: The free-body diagram is sketched. The equilibrium equations are

$$\sum F_x \nwarrow : T\cos(45^\circ - 30^\circ) - (100\ \text{lb})\sin 30^\circ = 0$$

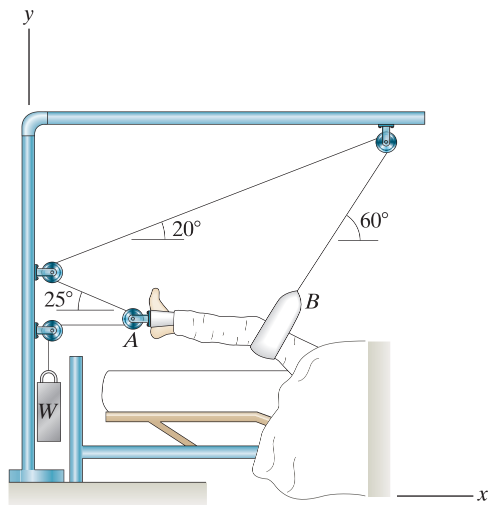
$$\sum F \nearrow : T\sin(45^\circ - 30^\circ) - (100\ \text{lb})\cos 30^\circ + N = 0$$

Solving, we find

$$\boxed{T = 51.8\ \text{lb}, N = 73.2\ \text{lb}}$$



Problem 3.84 The system shown is called Russell's traction. If the sum of the downward forces exerted at A and B by the patient's leg is 32.2 lb, what is the weight W?



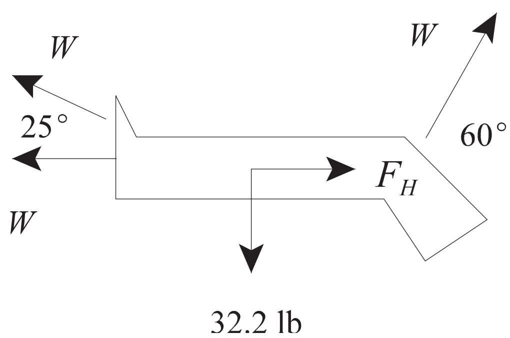
Solution: The force in the cable is W everywhere. The free-body diagram of the leg is shown. The downward force is given, but the horizontal force F_H is unknown.
The equilibrium equation in the vertical direction is

$$\sum F_y : W\sin 25^\circ + W\sin 60^\circ - 32.2\ \text{lb} = 0$$

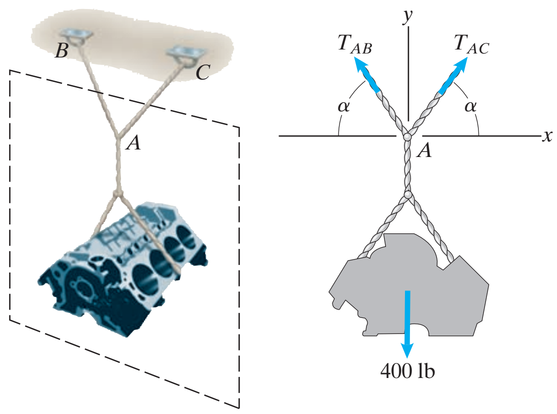
Thus

$$W = \frac{32.2\ \text{lb}}{\sin 25^\circ + \sin 60^\circ}$$

$$\boxed{W = 25.0\ \text{lb}}$$



Problem 3.85 The 400-lb engine block is suspended by the cables AB and AC. If you don't want either T_{AB} or T_{AC} to exceed 400 lb, what is the smallest acceptable value of the angle α?



Solution: The equilibrium equations are

$$\sum F_x : -T_{AB}\cos\alpha + T_{AC}\cos\alpha = 0$$

$$\sum F_y : T_{AB}\sin\alpha + T_{AC}\sin\alpha - (400\text{ lb}) = 0$$

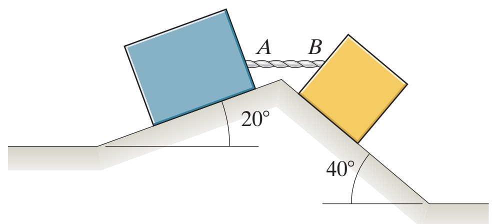
Solving, we find

$$T_{AB} = T_{AC} = \frac{400\text{ lb}}{2\sin\alpha}$$

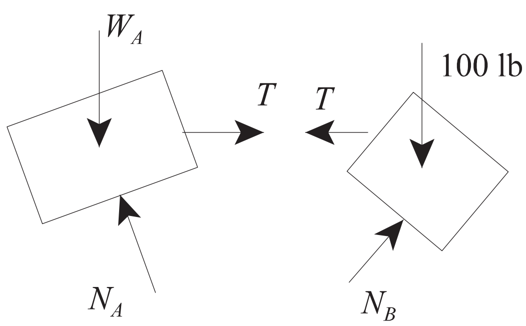
If we limit the tensions to 400 lb, we have

$$400\text{ lb} = \frac{400\text{ lb}}{2\sin\alpha} \Rightarrow \sin\alpha = \frac{1}{2} \Rightarrow \boxed{\alpha = 30^\circ}$$

Problem 3.86 The cable AB is horizontal, and the box on the right weighs 100 lb. The surface are smooth.
(a) What is the tension in the cable?
(b) What is the weight of the box on the left?



Solution: We have the following equilibrium equations



$$\sum F_{yB} : N_B\cos 40^\circ - 100\text{ lb} = 0$$

$$\sum F_{xB} : N_B\sin 40^\circ - T = 0$$

$$\sum F_{xA} : T - N_A\sin 20^\circ = 0$$

$$\sum F_{yA} : N_A\cos 20^\circ - W_A = 0$$

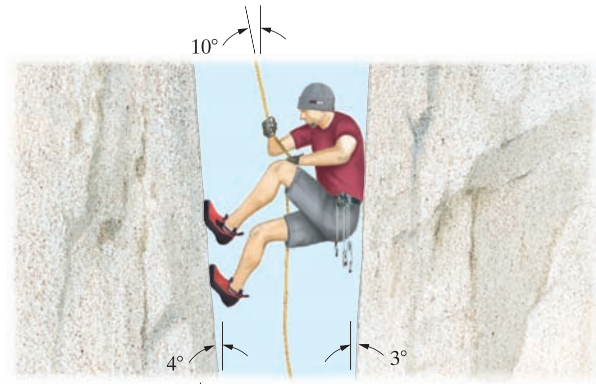
Solving these equations sequentially, we find

$N_B = 131$ lb, $T = 83.9$ lb

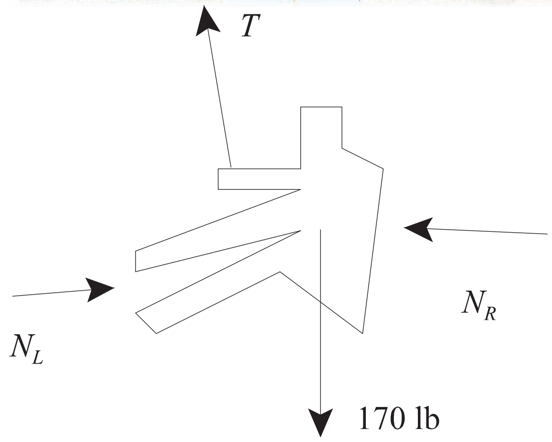
$N_A = 245$ lb, $W_A = 230.5$ lb

Thus we have $\boxed{T = 83.9\text{ lb},\ W_A = 230.5\text{ lb}}$

Problem 3.87 Assume that the forces exerted on the 170-lb climber by the slanted walls of the "chimney" are perpendicular to the walls. If he is in equilibrium and is exerting a 160-lb force on the rope, what are the magnitudes of the forces exerted on him by the left and right walls?



Solution: The forces in the free-body diagram are in the directions shown on the figure. The equilibrium equations are:



$$\sum F_x : -T \sin 10^\circ + N_L \cos 4^\circ - N_R \cos 3^\circ = 0$$

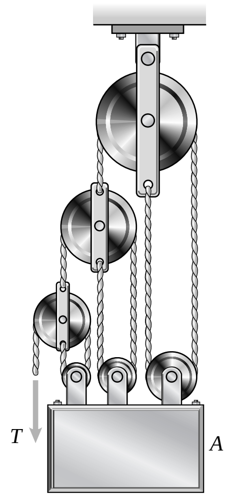
$$\sum F_y : T \cos 10^\circ - (170 \text{ lb}) + N_L \sin 40^\circ + N_R \sin 3^\circ = 0$$

where $T = 160$ lb. Solving we find

$$N_L = 114 \text{ lb}, N_R = 85.8 \text{ lb}$$

Left Wall: 114 lb
Right Wall: 85.8 lb

Problem 3.88 The mass of the suspended object A is m_A and the masses of the pulleys are negligible. Determine the force T necessary for the system to be in equilibrium.



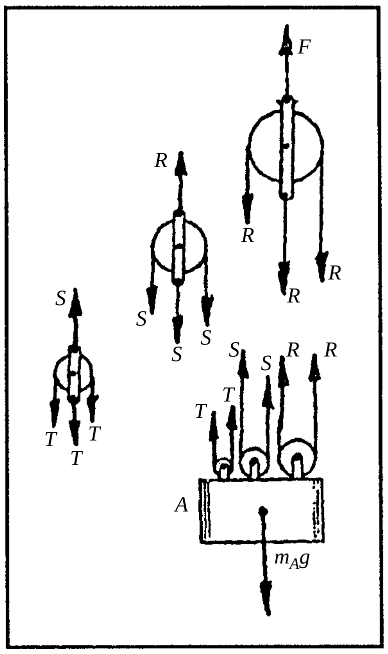
Solution: Break the system into four free body diagrams as shown. Carefully label the forces to ensure that the tension in any single cord is uniform. The equations of equilibrium for the four objects, starting with the leftmost pulley and moving clockwise, are:

$$S - 3T = 0, \quad R - 3S = 0, \quad F - 3R = 0,$$

and $2T + 2S + 2R - m_A g = 0$.

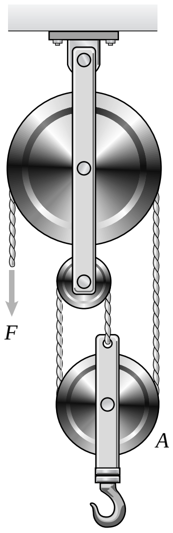
We want to eliminate S, R, and F from our result and find T in terms of m_A and g. From the first two equations, we get $S = 3T$, and $R = 3S = 9T$. Substituting these into the last equilibrium equation results in $2T + 2(3T) + 2(9T) = m_A g$.

Solving, we get $\boxed{T = m_A g/26}$.

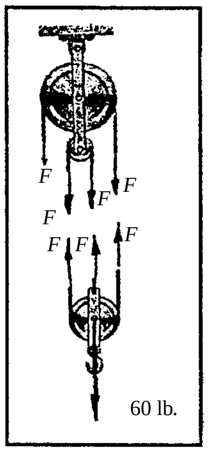


Note: We did not have to solve for F to find the appropriate value of T. The final equation would give us the value of F in terms of m_A and g. We would get $F = 27m_A g/26$. If we then drew a free body diagram of the entire assembly, the equation of equilibrium would be $F - T - m_A g = 0$. Substituting in the known values for T and F, we see that this equation is also satisfied. Checking the equilibrium solution by using the "extra" free body diagram is often a good procedure.

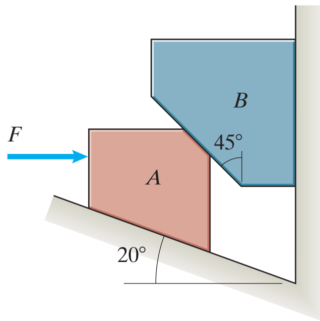
Problem 3.89 The assembly A, including the pulley, weighs 60 lb. What force F is necessary for the system to be in equilibrium?



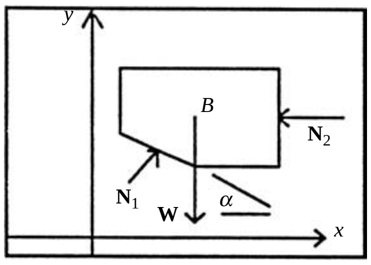
Solution: From the free body diagram of the assembly A, we have $3F - 60 = 0$, or $\boxed{F = 20\text{ lb}}$



Problem 3.90 The mass of block A is 42 kg, and the mass of block B is 50 kg. The surfaces are smooth. If the blocks are in equilibrium, what is the force F?



Solution: Isolate the top block. Solve the equilibrium equations. The weight is. The angle between the normal force $\mathbf{N}_1$ and the positive x axis is. The normal force is. The force $\mathbf{N}_2$ is. The equilibrium conditions are

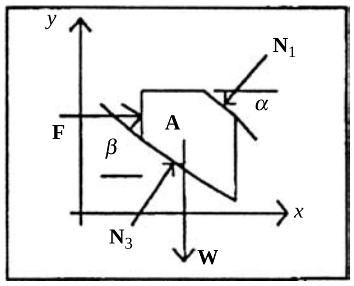


$$\sum \mathbf{F} = \mathbf{N}_1 + \mathbf{N}_2 + \mathbf{W} = 0$$

from which $\sum \mathbf{F}_x = (0.7071|\mathbf{N}_1| - |\mathbf{N}_2|)\mathbf{i} = 0$

$$\sum \mathbf{F}_y = (0.7071|\mathbf{N}_1| - 490.5)\mathbf{j} = 0.$$

Solve: $\mathbf{N}_1 = 693.7$ N, $|\mathbf{N}_2| = 490.5$ N



Isolate the bottom block. The weight is

$\mathbf{W} = 0\mathbf{i} - |W|\mathbf{j} = 0\mathbf{i} - (42)(9.81)\mathbf{j} = 0\mathbf{i} - 412.02\mathbf{j}$ (N).

The angle between the normal force $\mathbf{N}_1$ and the positive x axis is $(270^\circ - 45^\circ) = 225^\circ$.

The normal force:

$\mathbf{N}_1 = |\mathbf{N}_1|(\mathbf{i}\cos 225^\circ + \mathbf{j}\sin 225^\circ) = |\mathbf{N}_1|(-0.7071\mathbf{i} - 0.7071\mathbf{j})$.

The angle between the normal force $\mathbf{N}_3$ and the positive x-axis is $(90^\circ - 20^\circ) = 70^\circ$.

The normal force is

$\mathbf{N}_1 = |\mathbf{N}_3|(\mathbf{i}\cos 70^\circ + \mathbf{j}\sin 70^\circ) = |\mathbf{N}_3|(0.3420\mathbf{i} + 0.9397\mathbf{j})$.

The force is $\ldots \mathbf{F} = |\mathbf{F}|\mathbf{i} + 0\mathbf{j}$. The equilibrium conditions are

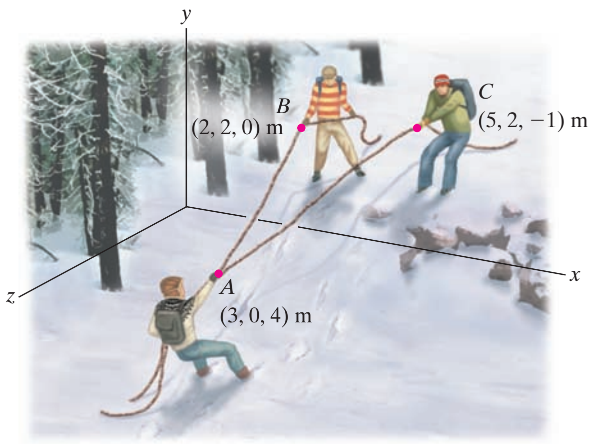
$$\sum \mathbf{F} = \mathbf{W} + \mathbf{N}_1 + \mathbf{N}_3 + \mathbf{F} = 0,$$

from which: $\sum \mathbf{F}_x = (-0.7071|\mathbf{N}_1| + 0.3420|\mathbf{N}_3| + |\mathbf{F}|)\mathbf{i} = 0$

$$\sum \mathbf{F}_y = (-0.7071|\mathbf{N}_1| + 0.9397|\mathbf{N}_3| - 412)\mathbf{j} = 0$$

For $|\mathbf{N}_1| = 693.7$ N from above: $\boxed{|\mathbf{F}| = 162 \text{ N}}$

Problem 3.91 The climber A is being helped up an icy slope by two friends. His mass is 80 kg, and the direction cosines of the force exerted on him by the slope are $\cos\theta_x = -0.286$, $\cos\theta_y = 0.429$, $\cos\theta_z = 0.857$. The y axis is vertical. If the climber is in equilibrium in the position shown, what are the tensions in the ropes AB and AC and the magnitude of the force exerted on him by the slope?



Solution: Get the unit vectors parallel to the ropes using the coordinates of the end points. Express the tensions in terms of these unit vectors, and solve the equilibrium conditions. The rope tensions, the normal force, and the weight act on the climber. The coordinates of points A, B, C are given by the problem, $A(3, 0, 4)$, $B(2, 2, 0)$, $C(5, 2, -1)$.

The vector locations of the points A, B, C are:

$$\mathbf{r}_A = 3\mathbf{i} + 0\mathbf{j} + 4\mathbf{k}, \quad \mathbf{r}_B = 2\mathbf{i} + 2\mathbf{j} + 0\mathbf{k}, \quad \mathbf{r}_C = 5\mathbf{i} + 2\mathbf{j} - 1\mathbf{k}.$$

The unit vector parallel to the tension acting between the points A, B in the direction of B is

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|}$$

The unit vectors are

$$\mathbf{e}_{AB} = -0.2182\mathbf{i} + 0.4364\mathbf{j} - 0.8729\mathbf{k},$$

$$\mathbf{e}_{AC} = 0.3482\mathbf{i} + 0.3482\mathbf{j} - 0.8704\mathbf{k},$$

and $\mathbf{e}_N = -0.286\mathbf{i} + 0.429\mathbf{j} + 0.857\mathbf{k}$.

where the last was given by the problem statement. The forces are expressed in terms of the unit vectors,

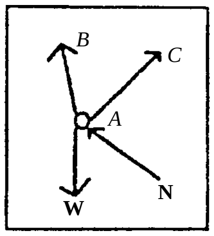
$$\mathbf{T}_{AB} = |\mathbf{T}_{AB}|\mathbf{e}_{AB}, \quad \mathbf{T}_{AC} = |\mathbf{T}_{AC}|\mathbf{e}_{AC}, \quad \mathbf{N} = |\mathbf{N}|\mathbf{e}_N.$$

The weight is

$$\mathbf{W} = 0\mathbf{i} - |\mathbf{W}|\mathbf{j} + 0\mathbf{k} = 0\mathbf{i} - (80)(9.81)\mathbf{j} + 0\mathbf{k} - 0\mathbf{i} - 784.8\mathbf{j} + 0\mathbf{k}.$$

The equilibrium conditions are

$$\sum \mathbf{F} = 0 = \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{N} + \mathbf{W} = 0.$$



Substitute and collect like terms,

$$\sum \mathbf{F}_x = (-0.2182|\mathbf{T}_{AB}| + 0.3482|\mathbf{T}_{AC}| - 0.286|\mathbf{N}|)\mathbf{i} = 0$$

$$\sum \mathbf{F}_y = (0.4364|\mathbf{T}_{AB}| + 0.3482|\mathbf{T}_{AC}| + 0.429|\mathbf{N}| - 784.8)\mathbf{j} = 0$$

$$\sum \mathbf{F}_z = (0.8729|\mathbf{T}_{AB}| + 0.8704|\mathbf{T}_{AC}| - 0.857|\mathbf{N}|)\mathbf{k} = 0$$

We have three linear equations in three unknowns. The solution is:

$\boxed{|\mathbf{T}_{AB}| = 100.7 \text{ N}}$, $\boxed{|\mathbf{T}_{AC}| = 889.0 \text{ N}}$, $\boxed{|\mathbf{N}| = 1005.5 \text{ N}}$.

Problem 3.92 Consider the climber A being helped by his friends in Problem 3.91. To try to make the tensions in the ropes more equal, the friend at B moves to the position (4, 2, 0) m. What are the new tensions in the ropes AB and AC and the magnitude of the force exerted on the climber by the slope?

Solution: Get the unit vectors parallel to the ropes using the coordinates of the end points. Express the tensions in terms of these unit vectors, and solve the equilibrium conditions. The coordinates of points A, B, C are $A(3, 0, 4)$, $B(4, 2, 0)$, $C(5, 2, -1)$. The vector locations of the points A, B, C are:

$$\mathbf{r}_A = 3\mathbf{i} + 0\mathbf{j} + 4\mathbf{k}, \quad \mathbf{r}_B = 4\mathbf{i} + 2\mathbf{j} + 0\mathbf{k}, \quad \mathbf{r}_C = 5\mathbf{i} + 2\mathbf{j} - 1\mathbf{k}.$$

The unit vectors are

$$\mathbf{e}_{AB} = +0.2182\mathbf{i} + 0.4364\mathbf{j} - 0.8729\mathbf{k},$$

$$\mathbf{e}_{AC} = +0.3482\mathbf{i} + 0.3482\mathbf{j} - 0.8704\mathbf{k},$$

$$\mathbf{e}_N = -0.286\mathbf{i} + 0.429\mathbf{j} + 0.857\mathbf{k}.$$

where the last was given by the problem statement. The forces are expressed in terms of the unit vectors,

$$\mathbf{T}_{AB} = |\mathbf{T}_{AB}|\mathbf{e}_{AB}, \quad \mathbf{T}_{AC} = |\mathbf{T}_{AC}|\mathbf{e}_{AC}, \quad \mathbf{N} = |\mathbf{N}|\mathbf{e}_N.$$

The weight is

$$\mathbf{W} = 0\mathbf{i} - |\mathbf{W}|\mathbf{j} + 0\mathbf{k} = 0\mathbf{i} - (80)(9.81)\mathbf{j} + 0\mathbf{k} - 0\mathbf{i} - 784.8\mathbf{j} + 0\mathbf{k}.$$

The equilibrium conditions are

$$\sum \mathbf{F} = 0 = \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{N} + \mathbf{W} = 0.$$

Substitute and collect like terms,

$$\sum \mathbf{F}_x = (+0.281|\mathbf{T}_{AB}| + 0.3482|\mathbf{T}_{AC}| - 0.286|\mathbf{N}|)\mathbf{i} = 0$$

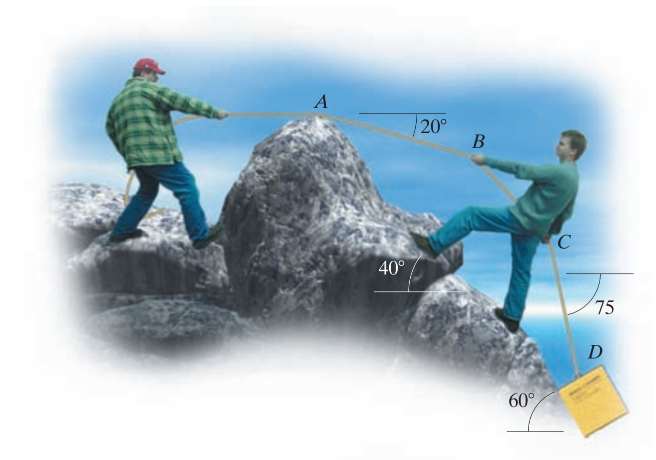
$$\sum \mathbf{F}_y = (0.4364|\mathbf{T}_{AB}| + 0.3482|\mathbf{T}_{AC}| + 0.429|\mathbf{N}| - 784.8)\mathbf{j} = 0$$

$$\sum \mathbf{F}_z = (0.8729|\mathbf{T}_{AB}| + 0.8704|\mathbf{T}_{AC}| - 0.857|\mathbf{N}|)\mathbf{k} = 0$$

The HP-28S hand held calculator was used to solve these simultaneous equations. The solution is:

$$\boxed{|\mathbf{T}_{AB}| = 420.5 \text{ N}}, \quad \boxed{|\mathbf{T}_{AC}| = 532.7 \text{ N}}, \quad \boxed{|\mathbf{N}| = 969.3 \text{ N}}.$$

Problem 3.93 A climber helps his friend up an icy slope. His friend is hauling a box of supplies. If the mass of the friend is 90 kg and the mass of the supplies is 22 kg, what are the tensions in the ropes AB and CD? Assume that the slope is smooth. That is, only normal forces are exerted on the man and the box by the slope.



Solution: Isolate the box. The weight vector is

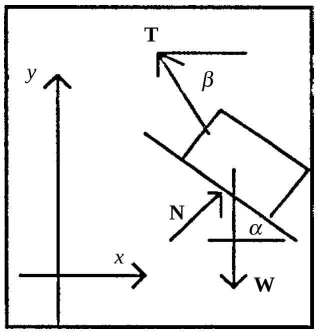
$\mathbf{W}_2 = (22)(9.81)\mathbf{j} = -215.8\mathbf{j}$ (N).

The angle between the normal force and the positive x axis is $(90^\circ - 60^\circ) = 30^\circ$.

The normal force is $\mathbf{N}_B = |\mathbf{N}_B|(0.866\mathbf{i} - 0.5\mathbf{j})$.

The angle between the rope CD and the positive x axis is $(180^\circ - 75^\circ) = 105^\circ$; the tension is:

$\mathbf{T}_2 = |\mathbf{T}_2|(\mathbf{i}\cos 105^\circ + \mathbf{j}\sin 105^\circ) = |\mathbf{T}_2|(-0.2588\mathbf{i} + 0.9659\mathbf{j})$

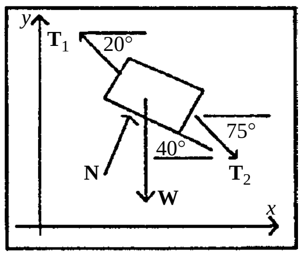


The equilibrium conditions are

$$\sum \mathbf{F}_x = (0.866|\mathbf{N}_B| + 0.2588|\mathbf{T}_2|)\mathbf{i} = 0,$$

$$\sum \mathbf{F}_y = (0.5|\mathbf{N}_B| + 0.9659|\mathbf{T}_2| - 215.8)\mathbf{j} = 0.$$

Solve: $\mathbf{N}_B = 57.8$ N, $|\mathbf{T}_2| = 193.5$ N.



Isolate the friend. The weight is

$\mathbf{W} = -(90)(9.81)\mathbf{j} = -882.9\mathbf{j}$ (N).

The angle between the normal force and the positive x axis is $(90^\circ - 40^\circ) = 50^\circ$. The normal force is:

$\mathbf{N}_F = |\mathbf{N}_F|(0.6428\mathbf{i} + 0.7660\mathbf{j})$.

The angle between the lower rope and the x axis is -75°; the tension is

$\mathbf{T}_2 = |\mathbf{T}_2|(0.2588\mathbf{i} + 0.9659\mathbf{j})$.

The angle between the tension in the upper rope and the positive x axis is $(180^\circ - 20^\circ) = 160^\circ$, the tension is

$\mathbf{T}_1 = |\mathbf{T}_1|(0.9397\mathbf{i} + 0.3420\mathbf{j})$.

The equilibrium conditions are

$$\sum \mathbf{F} = \mathbf{W} + \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{N}_F = 0.$$

From which:

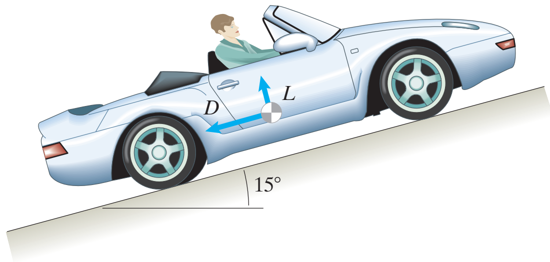
$$\sum \mathbf{F}_x = (0.6428|\mathbf{N}_F| + 0.2588|\mathbf{T}_2| - 0.9397|\mathbf{T}_1|)\mathbf{i} = 0$$

$$\sum \mathbf{F}_y = (-0.7660|\mathbf{N}_F| - 0.9659|\mathbf{T}_2| + 0.3420|\mathbf{T}_1| - 882.9)\mathbf{j} = 0$$

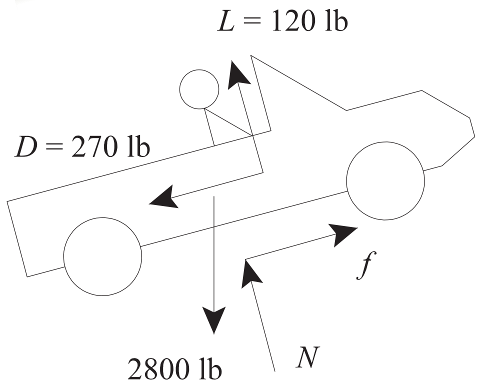
Solve, for $|T_2| = 193.5$ N. The result:

$\boxed{|\mathbf{N}_F| = 1051.6 \text{ N}}$, $\boxed{|\mathbf{T}_1| = 772.6 \text{ N}}$.

Problem 3.94 The 2800-lb car is moving at constant speed on a road with the slope shown. The aerodynamic forces on the car the drag $D = 270$ lb, which is parallel to the road, and the lift $L = 120$ lb, which is perpendicular to the road. Determine the magnitudes of the total normal and friction forces exerted on the car by the road.



Solution: The free-body diagram is shown. If we write the equilibrium equations parallel and perpendicular to the road, we have:



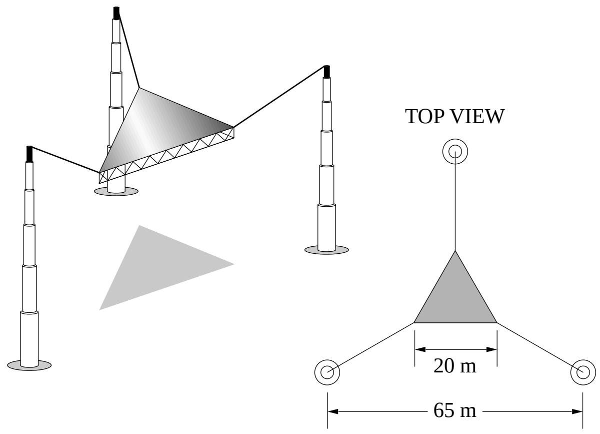
$$\sum F\nwarrow : N - (2800\text{ lb})\cos 15^\circ + (120\text{ lb}) = 0$$

$$\sum F\nearrow : f - (270\text{ lb}) - (2800\text{ lb})\sin 15^\circ = 0$$

Solving, we find

$$\boxed{N = 2580\text{ lb},\ f = 995\text{ lb}}$$

Problem 3.95 An engineer doing preliminary design studies for a new radio telescope envisions a triangular receiving platform suspended by cables from three equally spaced 40-m towers. The receiving platform has a mass of 20 Mg (megagrams) and is 10 m below the tops of the towers. What tension would the cables be subjected to?



Solution: Isolate the platform. Choose a coordinate system with the origin at the center of the platform, with the z axis vertical, and the x,y axes as shown. Express the tensions in terms of unit vectors, and solve the equilibrium conditions. The cable connections at the platform are labeled a, b, c, and the cable connections at the towers are labeled A, B, C. The horizontal distance from the origin (center of the platform) to any tower is given by

$$L = \frac{65}{2\sin(60)} = 37.5 \text{ m}.$$

The coordinates of points A, B, C are

$A(37.5, 0, 10)$, $B(37.5\cos(120^\circ), 37.5\sin(120^\circ).10)$,

$C(37.5\cos(240^\circ), 37.5\sin(240^\circ), 10)$,

The vector locations are:

$\mathbf{r}_A = 37.5\mathbf{i} + 0\mathbf{j} + 10\mathbf{k}$, $\mathbf{r}_B = 18.764\mathbf{i} + 32.5\mathbf{j} + 10\mathbf{k}$,

$\mathbf{r}_C = 18.764\mathbf{i} + -32.5\mathbf{j} + 10\mathbf{k}$.

The distance from the origin to any cable connection on the platform is

$$d = \frac{20}{2\sin(60^\circ)} = 11.547 \text{ m}.$$

The coordinates of the cable connections are

$a(11.547, 0, 0)$, $b(11.547\cos(120^\circ), 11547\sin(120^\circ), 0)$,

$c(11.547\cos(240^\circ), 11.547\sin(240^\circ), 0)$.

The vector locations of these points are,

$\mathbf{r}_a = 11.547\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$, $\mathbf{r}_b = 5.774\mathbf{i} + 10\mathbf{j} + 0\mathbf{k}$,

$\mathbf{r}_c = 5.774\mathbf{i} + 10\mathbf{j} + 0\mathbf{k}$.

The unit vector parallel to the tension acting between the points A, a in the direction of A is by definition

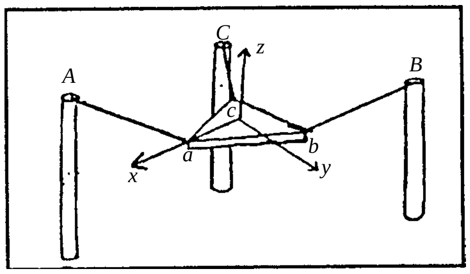
$$\mathbf{e}_{aA} = \frac{\mathbf{r}_A - \mathbf{r}_a}{|\mathbf{r}_A - \mathbf{r}_a|}.$$

Perform this operation for each of the unit vectors to obtain

$$\mathbf{e}_{aA} = +0.9333\mathbf{i} + 0\mathbf{j} - 0.3592\mathbf{k}$$

$$\mathbf{e}_{bB} = -0.4667\mathbf{i} + 0.8082\mathbf{j} - 0.3592\mathbf{k}$$

$$\mathbf{e}_{cC} = -0.4667\mathbf{i} + 0.8082\mathbf{j} + 0.3592\mathbf{k}$$



The tensions in the cables are expressed in terms of the unit vectors,

$\mathbf{T}_{aA} = |\mathbf{T}_{aA}|\mathbf{e}_{aA}$, $\mathbf{T}_{bB} = |\mathbf{T}_{bB}|\mathbf{e}_{bB}$, $\mathbf{T}_{cC} = |\mathbf{T}_{cC}|\mathbf{e}_{cC}$.

The weight is $\mathbf{W} = 0\mathbf{i} - 0\mathbf{j} - (20000)(9.81)\mathbf{k} = 0\mathbf{i} + 0\mathbf{j} - 196200\mathbf{k}$.

The equilibrium conditions are

$$\sum \mathbf{F} = 0 = \mathbf{T}_{aA} + \mathbf{T}_{bB} + \mathbf{T}_{cC} + \mathbf{W} = 0,$$

from which:

$$\sum \mathbf{F}_x = (0.9333|\mathbf{T}_{aA}| - 0.4666|\mathbf{T}_{bB}| - 0.4666|\mathbf{T}_{cC}|)\mathbf{i} = 0$$

$$\sum \mathbf{F}_y = (0|\mathbf{T}_{aA}| + 0.8082|\mathbf{T}_{bB}| - 0.8082|\mathbf{T}_{cC}|)\mathbf{j} = 0$$

$$\sum \mathbf{F}_z = (0.3592|\mathbf{T}_{aA}| - 0.3592|\mathbf{T}_{bB}|$$

$$+ 0.3592|\mathbf{T}_{cC} - 196200|)\mathbf{k} = 0$$

The commercial package **TK Solver Plus** was used to solve these equations. The results:

$\boxed{|\mathbf{T}_{aA}| = 182.1 \text{ kN}}$, $\boxed{|\mathbf{T}_{bB}| = 182.1 \text{ kN}}$, $\boxed{|\mathbf{T}_{cC}| = 182.1 \text{ kN}}$.

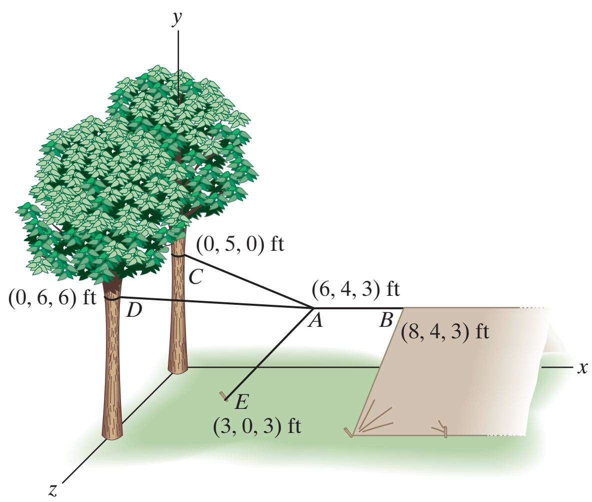
Check: For this geometry, where from symmetry all cable tensions may be assumed to be the same, only the z-component of the equilibrium equations is required:

$$\sum F_z = 3|\mathbf{T}|\sin\theta - 196200 = 0,$$

where $\theta = \tan^{-1}\left(\frac{10}{37.5 - 11.547}\right) = 21.07^\circ$,

from which each tension is $|\mathbf{T}| = 182.1$ kN. *check.*

Problem 3.96 To support the tent, the tension in the rope AB must be 35 lb. What are the tensions in the ropes AC, AD, and AE?



Solution: We develop the following position vectors

$\mathbf{r}_{AB} = (2\mathbf{i})$ ft

$\mathbf{r}_{AC} = (-6\mathbf{i} + \mathbf{j} - 3\mathbf{k})$ ft

$\mathbf{r}_{AD} = (-6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ ft

$\mathbf{r}_{AE} = (-3\mathbf{i} - 4\mathbf{j})$ ft

If we divide by the respective magnitudes we can develop the unit vectors that are parallel to these position vectors.

$\mathbf{e}_{AB} = 1.00\mathbf{i}$

$\mathbf{e}_{AC} = -0.885\mathbf{i} + 0.147\mathbf{j} - 0.442\mathbf{k}$

$\mathbf{e}_{AD} = -0.857\mathbf{i} + 0.286\mathbf{j} + 0.429\mathbf{k}$

$\mathbf{e}_{AE} = -6.00\mathbf{i} - 0.800\mathbf{j}$

The equilibrium equation is

$$T_{AB}\mathbf{e}_{AB} + T_{AC}\mathbf{e}_{AC} + T_{AD}\mathbf{e}_{AD} + T_{AE}\mathbf{e}_{AE} = 0.$$

If we break this up into components, we have

$\sum F_x : T_{AB} - 0.885T_{AC} - 0.857T_{AD} - 0.600T_{AE} = 0$

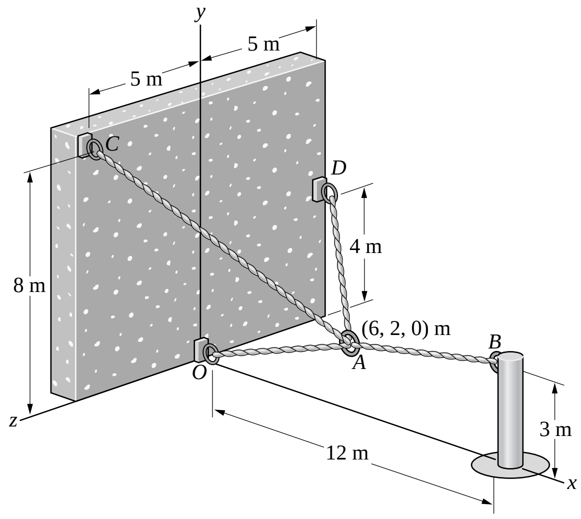
$\sum F_y : 0.147T_{AC} + 0.286T_{AD} - 0.800T_{AE} = 0$

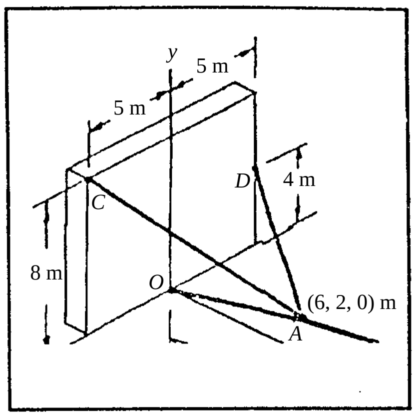
$\sum F_z : -0.442T_{AC} + 0.429T_{AD} = 0$

If we set $T_{AB} = 35$ lb, we cans solve for the other tensions. The result is

$$\boxed{T_{AC} = 16.7 \text{ lb}, T_{AD} = 17.2 \text{ lb}, T_{AE} = 9.21 \text{ lb}}$$

Problem 3.97 Cable AB is attached to the top of the vertical 3-m post, and its tension is 50 kN. What are the tensions in cables AO, AC, and AD?





Solution: Get the unit vectors parallel to the cables using the coordinates of the end points. Express the tensions in terms of these unit vectors, and solve the equilibrium conditions. The coordinates of points A, B, C, D, O are found from the problem sketch: The coordinates of the points are $A(6, 2, 0)$, $B(12, 3, 0)$, $C(0, 8, 5)$, $D(0, 4, -5)$, $O(0, 0, 0)$.

The vector locations of these points are:

$$\mathbf{r}_A = 6\mathbf{i} + 2\mathbf{j} + 0\mathbf{k}, \quad \mathbf{r}_B = 12\mathbf{i} + 3\mathbf{j} + 0\mathbf{k}, \quad \mathbf{r}_C = 0\mathbf{i} + 8\mathbf{j} + 5\mathbf{k},$$

$$\mathbf{r}_D = 0\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}, \quad \mathbf{r}_O = 0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}.$$

The unit vector parallel to the tension acting between the points A, B in the direction of B is by definition

$$\mathbf{e}_{AB} = \frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|}.$$

Perform this for each of the unit vectors

$$\mathbf{e}_{AB} = +0.9864\mathbf{i} + 0.1644\mathbf{j} + 0\mathbf{k}$$

$$\mathbf{e}_{AC} = -0.6092\mathbf{i} + 0.6092\mathbf{j} + 0.5077\mathbf{k}$$

$$\mathbf{e}_{AD} = -0.7442\mathbf{i} + 0.2481\mathbf{j} - 0.6202\mathbf{k}$$

$$\mathbf{e}_{AO} = -0.9487\mathbf{i} - 0.3162\mathbf{j} + 0\mathbf{k}$$

The tensions in the cables are expressed in terms of the unit vectors,

$$\mathbf{T}_{AB} = |\mathbf{T}_{AB}|\mathbf{e}_{AB} = 50\mathbf{e}_{AB}, \quad \mathbf{T}_{AC} = |\mathbf{T}_{AC}|\mathbf{e}_{AC},$$

$$\mathbf{T}_{AD} = |\mathbf{T}_{AD}|\mathbf{e}_{AD}, \quad \mathbf{T}_{AO} = |\mathbf{T}_{AO}|\mathbf{e}_{AO}.$$

The equilibrium conditions are

$$\sum \mathbf{F} = 0 = \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{T}_{AO} = 0.$$

Substitute and collect like terms,

$$\sum \mathbf{F}_x = (0.9864(50) - 0.6092|\mathbf{T}_{AC}| - 0.7422|\mathbf{T}_{AD}|$$

$$- 0.9487|\mathbf{T}_{AO}|)\mathbf{i} = 0$$

$$\sum \mathbf{F}_y = (0.1644(50) + 0.6092|\mathbf{T}_{AC}| + 0.2481|\mathbf{T}_{AD}|$$

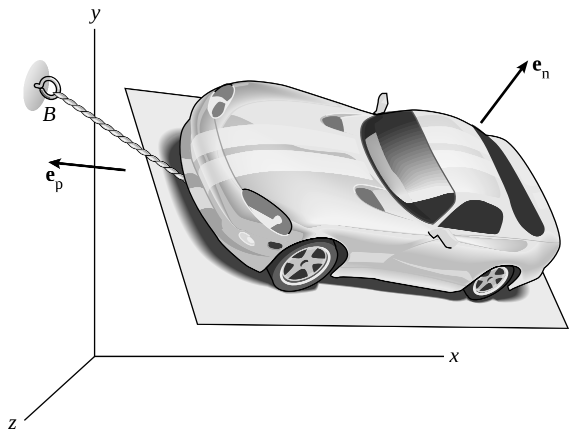
$$- 0.3162|\mathbf{T}_{AO}|)\mathbf{j} = 0$$

$$\sum \mathbf{F}_z = (+0.5077|\mathbf{T}_{AC}| - 0.6202|\mathbf{T}_{AD}|)\mathbf{k} = 0.$$

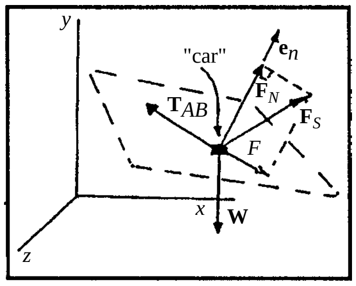
This set of simultaneous equations in the unknown forces may be solved using any of several standard algorithms. The results are:

$$|\mathbf{T}_{AO}| = 43.3 \text{ kN}, \quad |\mathbf{T}_{AC}| = 6.8 \text{ kN}, \quad |\mathbf{T}_{AD}| = 5.5 \text{ kN}.$$

Problem 3.98* The 1350-kg car is at rest on a plane surface with its brakes locked. The unit vector $\mathbf{e}_n = 0.231\mathbf{i} + 0.923\mathbf{j} + 0.308\mathbf{k}$ is perpendicular to the surface. The y axis points upward. The direction cosines of the cable from A to B are $\cos\theta_x = -0.816$, $\cos\theta_y = 0.408$, $\cos\theta_z = -0.408$, and the tension in the cable is 1.2 kN. Determine the magnitudes of the normal and friction forces the car's wheels exert on the surface.



Solution: Assume that all forces act at the center of mass of the car. The vector equation of equilibrium for the car is



$$\mathbf{F}_S + \mathbf{T}_{AB} + \mathbf{W} = 0.$$

Writing these forces in terms of components, we have

$$\mathbf{W} = -mg\mathbf{j} = -(1350)(9.81) = -13240\mathbf{j} \text{ N},$$

$$\mathbf{F}_S = \mathbf{F}_{Sx}\mathbf{i} + \mathbf{F}_{Sy}\mathbf{j} + F_{Sz}\mathbf{k},$$

and $\mathbf{T}_{AB} = \mathbf{T}_{AB}\mathbf{e}_{AB}$,

where

$$\mathbf{e}_{AB} = \cos\theta_x\mathbf{i} + \cos\theta_y\mathbf{j} + \cos\theta_z\mathbf{k} = -0.816\mathbf{i} + 0.408\mathbf{j} - 0.408\mathbf{k}.$$

Substituting these values into the equations of equilibrium and solving for the unknown components of $\mathbf{F}_S$, we get three scalar equations of equilibrium. These are:

$$F_{Sx} - T_{ABx} = 0, \quad F_{Sy} - T_{ABy} - W = 0,$$

and $F_{Sz} - T_{ABz} = 0$.

Substituting in the numbers and solving, we get

$$F_{Sx} = 979.2 \text{ N}, \quad F_{Sy} = 12,754 \text{ N},$$

and $F_{Sz} = 489.6$ N.

The next step is to find the component of $\mathbf{F}_S$ normal to the surface. This component is given by

$$F_N = \mathbf{F}_N \cdot \mathbf{e}_n = F_{Sx}\mathbf{e}_{ny} + F_{Sx}\mathbf{e}_{ny} + F_{Sz}\mathbf{e}_{nz}.$$

Substitution yields

$\boxed{F_N = 12149 \text{ N}}$.

From its components, the magnitude of $\mathbf{F}_S$ is $F_S = 12800$ N. Using the Pythagorean theorem, the friction force is

$$\boxed{f = \sqrt{F_S^2 - F_N^2} = 4033 \text{ N.}}$$

Problem 3.99* The brakes of the car in Problem 3.98 are released, and the car is held in place on the plane surface by the cable AB. The car's front wheels are aligned so that the tires exert no friction forces parallel to the car's longitudinal axis. The unit vector $\mathbf{e}_p = -0.941\mathbf{i} + 0.131\mathbf{j} + 0.314\mathbf{k}$ is parallel to the plane surface and aligned with the car's longitudinal axis. What is the tension in the cable?

Solution: Only the cable and the car's weight exert forces in the direction parallel to $\mathbf{e}_p$. Therefore

$$\mathbf{e}_p \cdot (\mathbf{T} - mg\mathbf{j}) = 0: \quad (-0.941\mathbf{i} + 0.131\mathbf{j} + 0.314\mathbf{k})$$

$$\cdot\, [T(-0.816\mathbf{i} + 0.408\mathbf{j} - 0.408\mathbf{k}) - mg\mathbf{j}] = 0,$$

$$(0.941)(0.816)T$$

$$+ (0.131)(0.408T - mg) + (0.314)(-0.408T) = 0.$$

Solving, we obtain $T = 2.50$ kN.