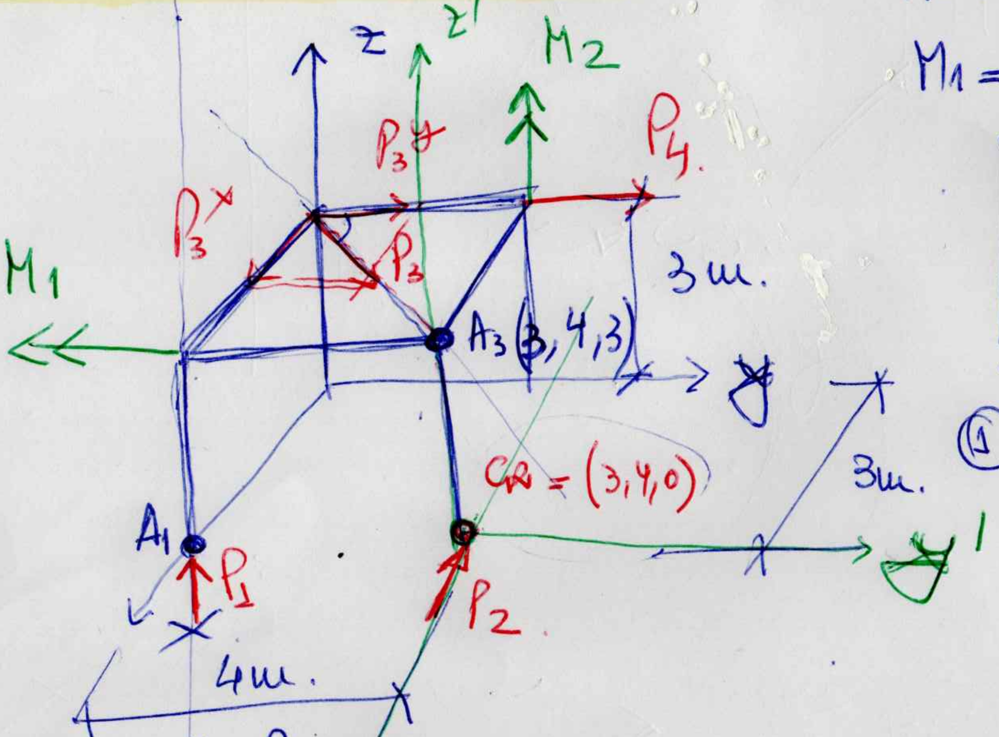
REDUCIR EL SISTEMA A CR

(1)

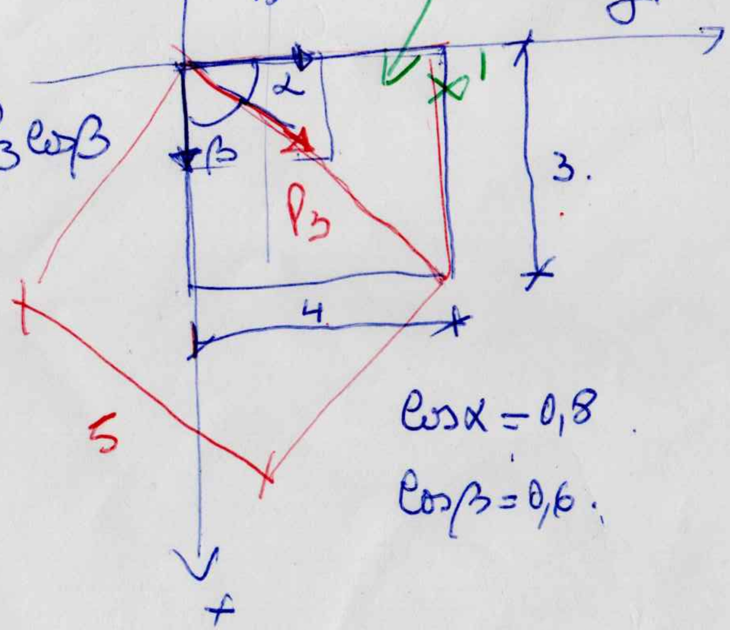
$P_1 = 10kN$; $P_2 = 20kN$; $P_3 = 30kN$; $P_4 = 40kN$.

$M_1 = 20kNm$ $M_2 = 30kNm$



$P_3^y = P_3 \cdot \cos\alpha$

$P_3^x = P_3 \cos\beta$



$\cos\alpha = 0{,}8$

$\cos\beta = 0{,}6$

(1) $\vec{R}_R = \sum_{i=1}^{n} \vec{P}_i$

$$\vec{M}_{RR}^{CR} = \sum_{i=1}^{n} \vec{P}_i \times (\vec{CR} - \vec{A_i}) + \sum_{j=1}^{m} \vec{M}_j$$

(1). $\vec{P}_1 = 0\check{i} + 0\check{j} + 10kN\,\check{k}$

$\vec{P}_2 = -20kN\,\check{i} + 0\check{j} + 0\,\check{k}$

$\vec{P}_3 = 18kN\,\check{i} + 24kN\,\check{j} + 0\check{k}$

$\vec{P}_4 = 0\check{i} + 40kN\,\check{j} + 0\check{k}$

$$\vec{R}_R = -2kN\,\check{i} + 64kN\,\check{j} + 10\check{k}$$

MOMENTOS DE TRASLACIÓN

$\vec{M}_{\vec{P}_1}^{CR} = \vec{P}_1 \times (\vec{CR} - \vec{A}_1)$

$CR = (3,4,0)$

$A_1 = (3,0,0)$

$$\vec{M}_{\vec{P}_1}^{CR} = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 0 & 0 & 10 \\ 0 & 4 & 0 \end{vmatrix} = (-40kNm\,\check{i} + 0\check{j} + 0\check{k})\,kNm$$

$\vec{M}_{\vec{P}_2}^{CR} = 0$

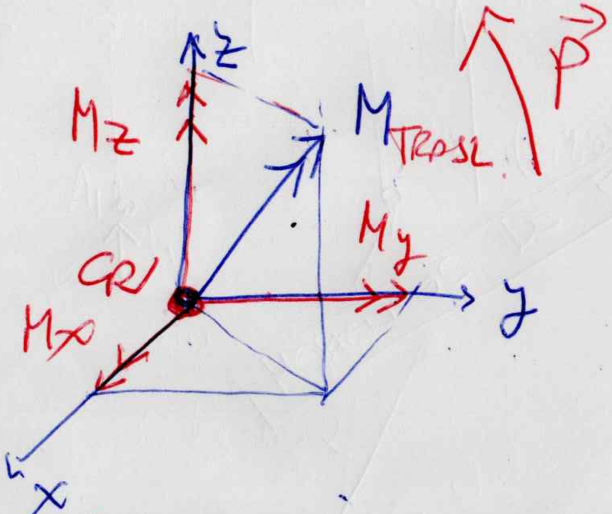
$$\vec{M}^{CR}_{\vec{P}_3} = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 18 & 24 & 0 \\ 0 & 0 & -3 \end{vmatrix} =$$

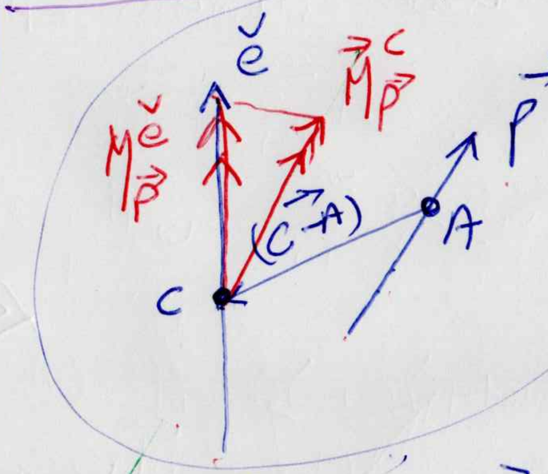
$CR = (3,4,0)$

$A_3 = (3,4,3)$

$$\vec{M}^{CR}_{\vec{P}_3} = \vec{P}_3 \times (\overrightarrow{CR - A_3})$$

$$\boxed{\vec{M}^{CR}_{\vec{P}_3} = -72\,kNm\,\check{i} + 54\,kNm\,\check{j} + 0\,\check{k}}$$



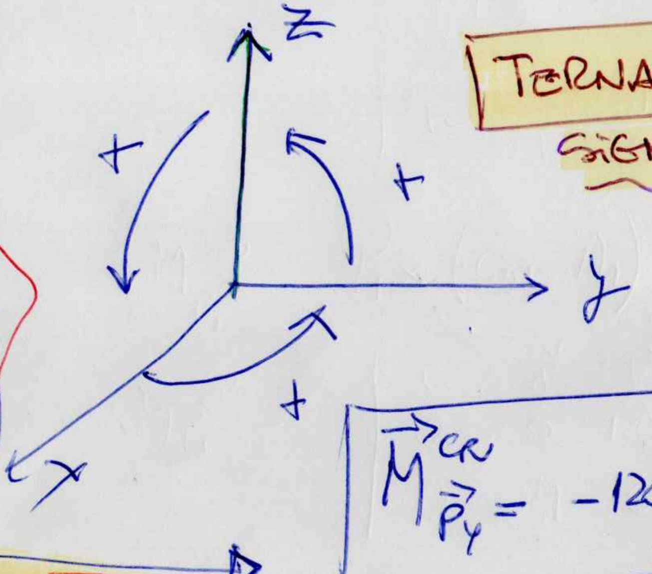


MOMENTO DE UNA FUERZA RESPECTO DE UN EJE

$$\vec{M}^{\check{e}}_{\vec{P}} = \left[\vec{P} \times (C-A)\right] \cdot \check{e} \cdot \check{e}$$

TERNA DERECHA

SIGNOS GIRO



$M^{\vec{P}_4}_{x} = -|P_4| \times 3m$

$M^{\vec{P}_4}_{y} = \cancel{P_4 \cdot 0}$ NULO. $P_4 \parallel y$

$M^{\vec{P}_4}_{z} = -|P_4| \cdot 3m$

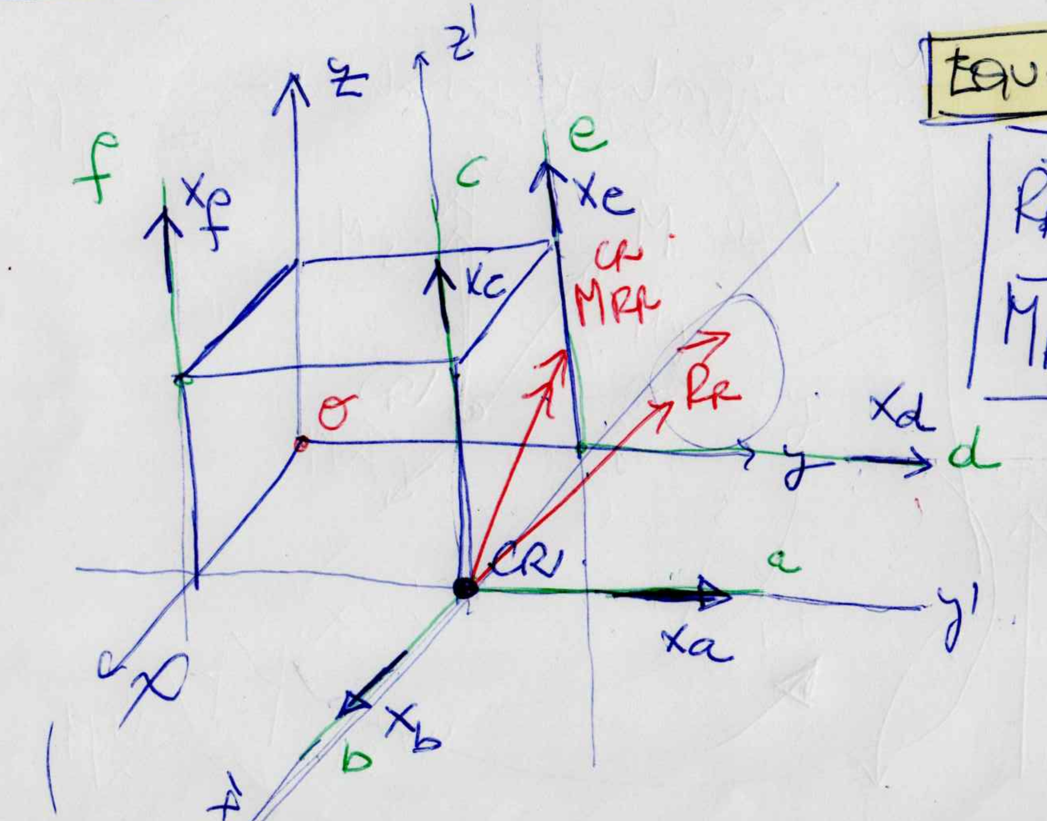
MOMENTO DE UNA FUERZA RESPECTO DE EJES

$$\boxed{\vec{M}^{CR}_{\vec{P}_4} = -120\,kNm\,\check{i} + 0\,\check{j} - 120\,kNm\,\check{k}}$$

$\vec{M_1} = 0\check{i} - 20kNm\,\check{j} + 0\check{k}$

$\vec{M_2} = 0\check{i} + 0\check{j} + 30\check{k}$

$\vec{M}^{CR}_{RR} = -232kNm\,\check{i} + 34kNm\,\check{j} - 90kNm\,\check{k}$ (3)



EQUILIBRIO

EQUILIBRAR CON 6 FUERZAS CUYAS DIRECCIONES SON LAS DADAS: a, b, c, d, e.

$\vec{R_{RR}} = \vec{0}$ → $R_x = 0$, $R_y = 0$, $R_z = 0$ (1)

$\vec{M^{C}_{RR}} = \vec{0}$ → $M^C_x = 0$, $M^C_y = 0$, $M^C_z = 0$ (2)

$X_f = -58kN$

(1)

$R_x = -2kN + X_b = 0 \rightarrow$ $X_b = 2kN$

$R_y = 64kN + X_a + X_b = 0$ $X_a = -34kN$

$R_z = 10kN + X_c + X_e + X_f = 0$

$X_c = 59,33kN$

(2)

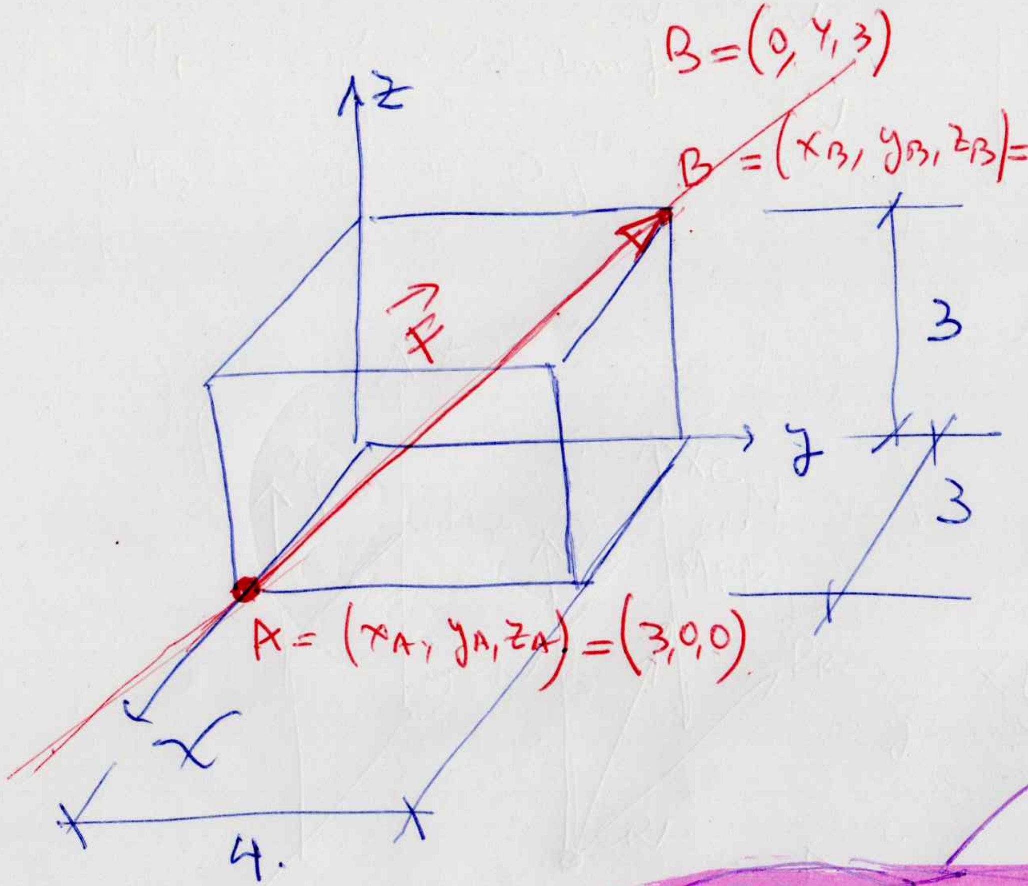
$M^{CR}_x = -232 - X_f \cdot 4m = 0 \rightarrow$

$X_e = -11,33kN$

$M^{CR}_y = 34 + X_e \cdot 3m = 0$

$M^{CR}_z = -90 - X_d \cdot 3m = 0$

$X_d = -30kN$



$$\vec{F} = |F| \cdot n_F$$

$$\breve{n}_F = \frac{(\overrightarrow{B-A})}{|(\overrightarrow{B-A})|} = \frac{(x_B; y_B; z_B) - (x_A, y_A, z_A)}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}$$

$$\breve{n}_F = \frac{(-3, 4, 3)}{\sqrt{(-3)^2 + 4^2 + 3^2}} = \frac{-3}{\sqrt{34}}\breve{i} + \frac{4}{\sqrt{34}}\breve{j} + \frac{3}{\sqrt{34}} \ldots$$

DETERMINAR UNA DIRECCIÓN UNITARIA

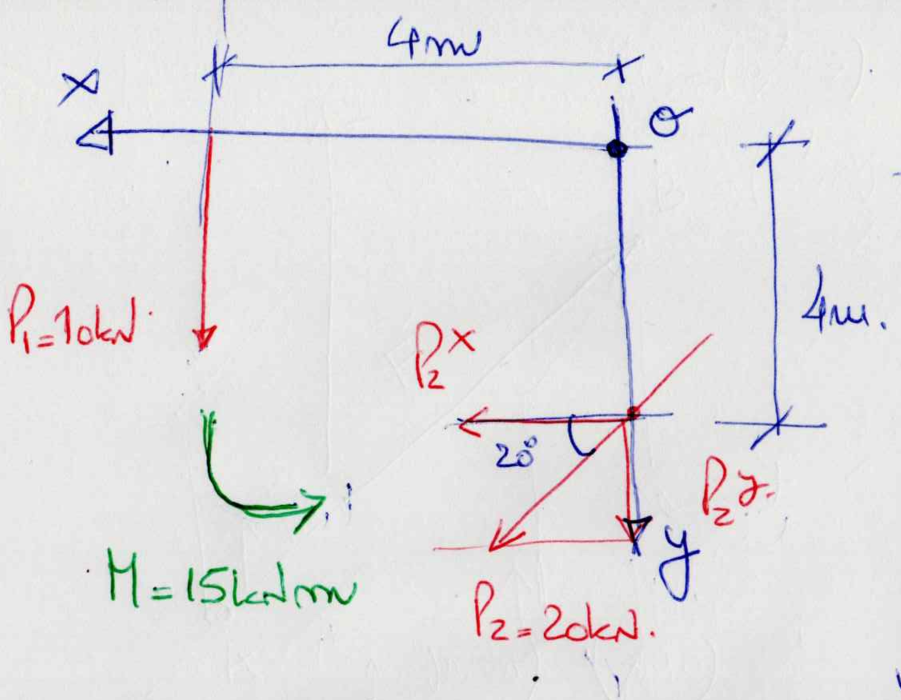
↓

Cosenos directores.

$$\vec{F} = |\vec{F}| \cdot \breve{n}_F = |\vec{F}| \left(-\frac{3}{\sqrt{34}}\breve{i} + \frac{4}{\sqrt{34}}\breve{j} + \frac{3}{\sqrt{34}}\breve{k} \right)$$ → VECTOR FUERZA

→ Módulo de la Fuerza

1°) Hallar el Binomio de Reducción.



$\Sigma F_x = 20kN \cdot \cos 20° = 18{,}79kN.$

$\Sigma F_y = 10kN + 20kN \cdot \text{sen}\, 20° = 16{,}84kN.$

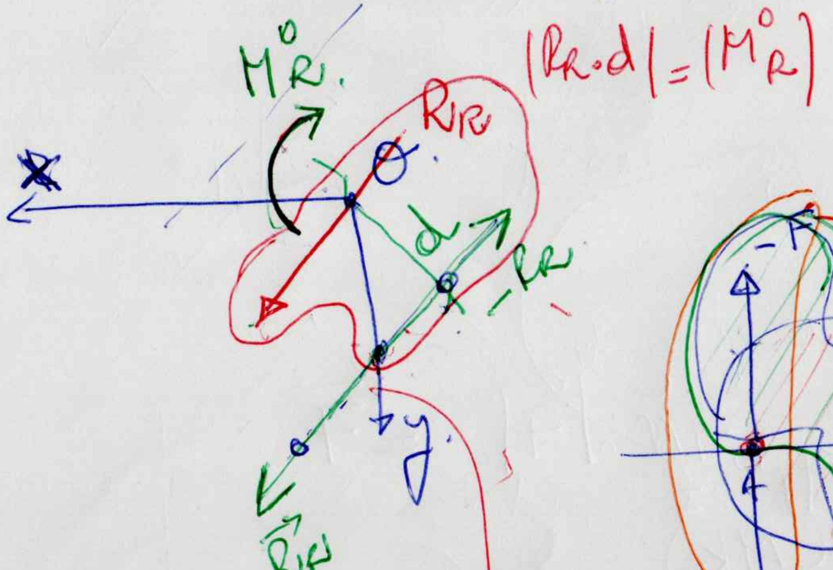
$\vec{R}_R = (18{,}79kN,\ 16{,}84kN,\ 0).$

ESFUERZO PLANO

$M^O_R = 15kNm + 10kN \cdot 4m - 18{,}79 \cdot 4m.$

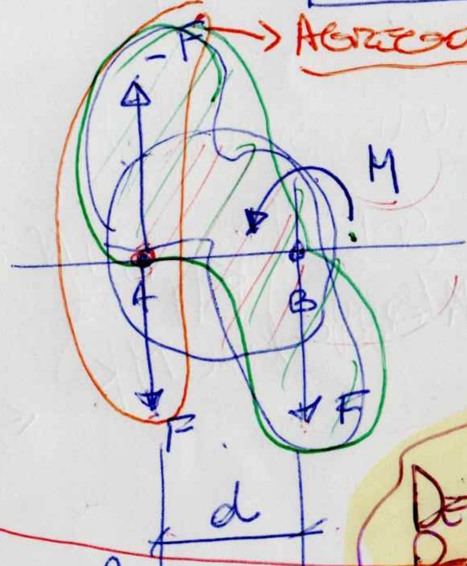
M_z

$M^O_R = -20{,}16\,kNm$ → Componente en Z



→ Agrego sistema nulo.

$F \cdot d = M$



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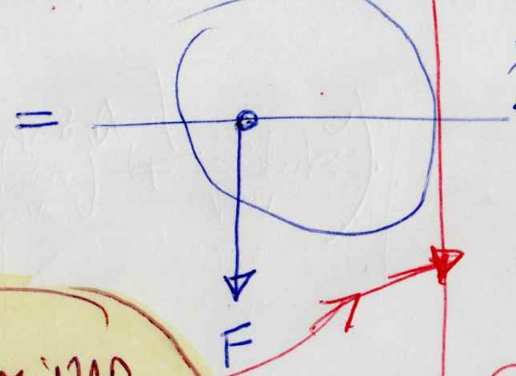
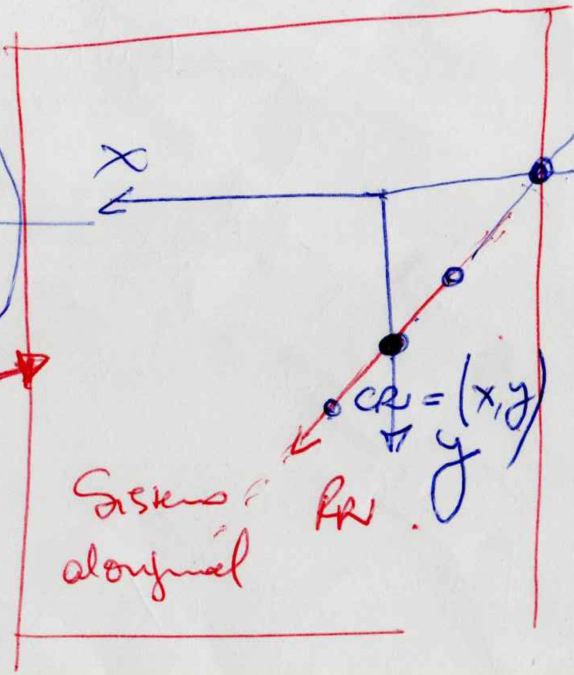


Determinar Resultante única



$M^{CR} = 0$

$M^{CRO} = \vec{R} \times (CR - O) + M^O_R$

$$\vec{M}_{CR} = \vec{R} \times (\overrightarrow{CR - O}) + M_R^O \qquad O = (0,0,0) \quad CR = (x, y)$$

(6)

$$\vec{M}_{CR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ R_x & R_y & 0 \\ x & y & 0 \end{vmatrix} + M_R^O = 0$$

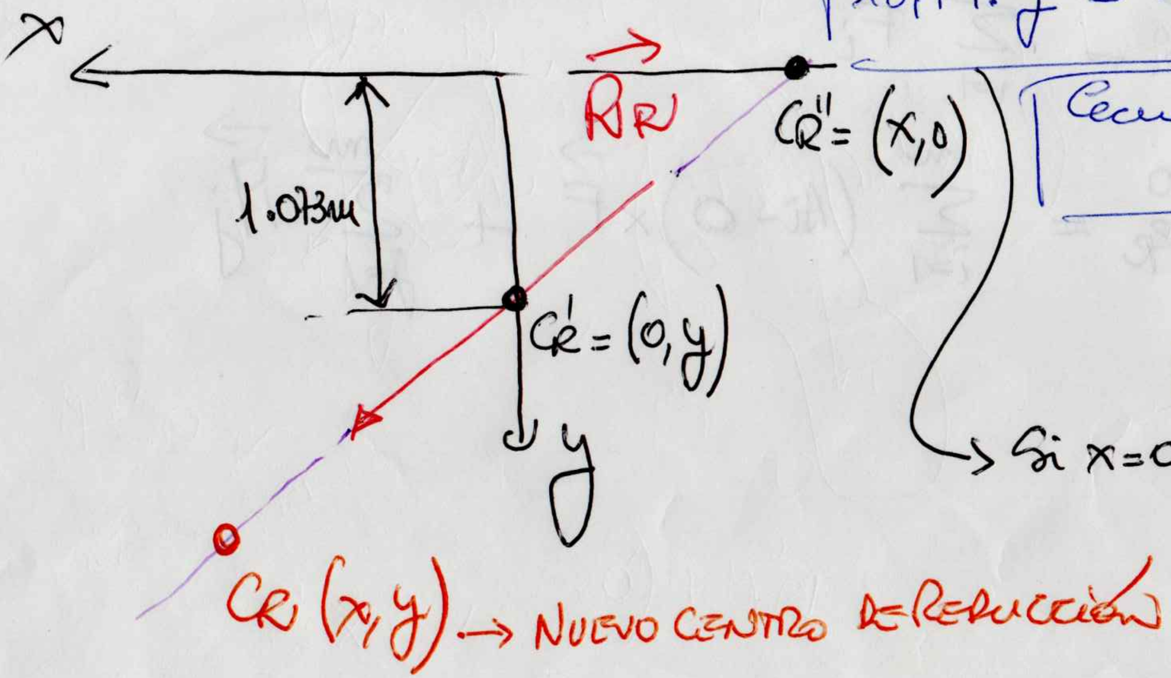
$$\vec{M}_{CR} = 0\vec{i} + 0\vec{j} + (R_x \cdot y - R_y \cdot x)\vec{k} + M_R^O \vec{k} = 0$$

$$18{,}79 \cdot y - 16{,}84 \cdot x - 20{,}16 = 0$$

Ecuación de una recta

⇓

Recta de Acción de la fuerza.

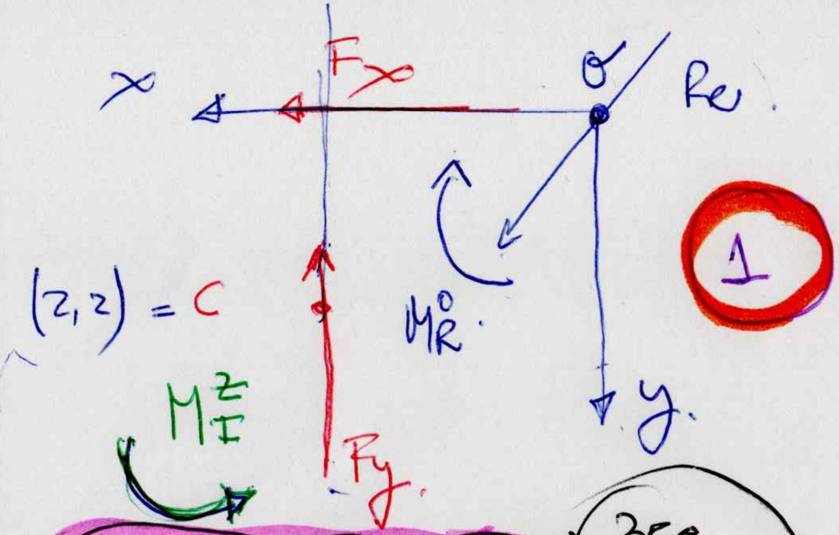


Si $x = 0$ $\quad y = \dfrac{20{,}16}{18{,}79} \Rightarrow$ $y = 1.073\,m$

CON DAR UN VALOR ESTÁ DEFINIDO UN PUNTO DE LA RECTA DE ACCIÓN

EQUILIBRAR

$\Sigma F_x = 0 \quad \Sigma F_y = 0$

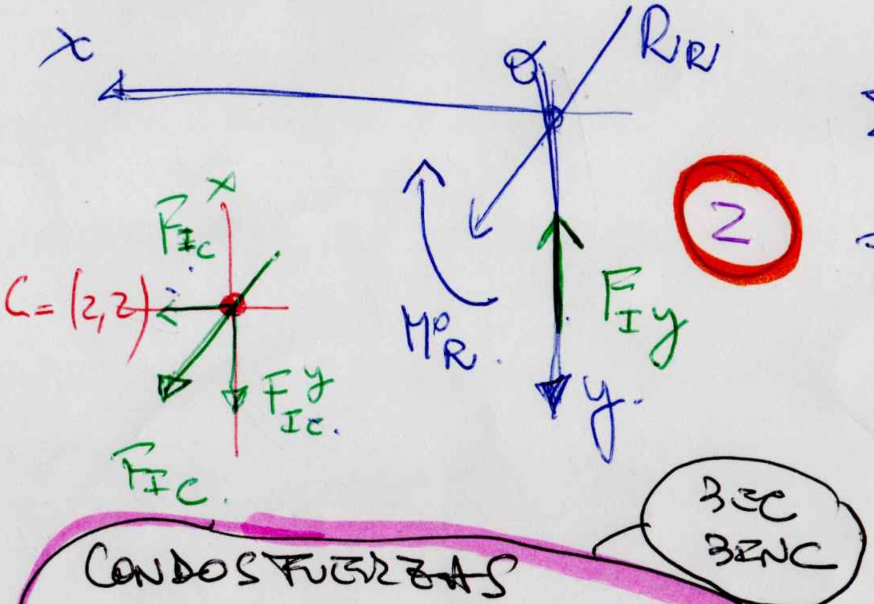


CON DOS FUERZAS Y UN PAR

$\Sigma F_x = 18{,}79\,kN + F_x = 0 \Rightarrow$ $F_x = -18{,}79\,kN$

$\Sigma F_y = 16{,}84\,kN - F_y = 0 \Rightarrow$ $F_y = 16{,}84\,kN$

$\Sigma M^O = \underbrace{M^O_R}_{-20{,}16} + M^z_I - F_y \cdot 2m = 0$ $M^z_I = 53{,}84\,kNm$



CON DOS FUERZAS UNA coincide CON EL EJE Y Y LA OTRA PASA POR ("C")

$\Sigma F_x = 18{,}79\,kN + F^x_{IC} = 0 \Rightarrow$ $F^x_{IC} = -18{,}79\,kN$

$\Sigma F_y = 16{,}84\,kN - F_{Iy} + F^y_{IC} = 0$

$\Sigma M^O = \underbrace{M^O_R}_{-20{,}16} + F^y_{IC} \cdot 2m - F^x_{IC} \cdot 2m = 0$

$F^y_{IC} = -8{,}71\,kN$

$F_{Iy} = 8{,}13\,kN$