CIRCUNFERENCIA DE DEFORMACIONES A PARTIR DE LA DE TENSIONES:

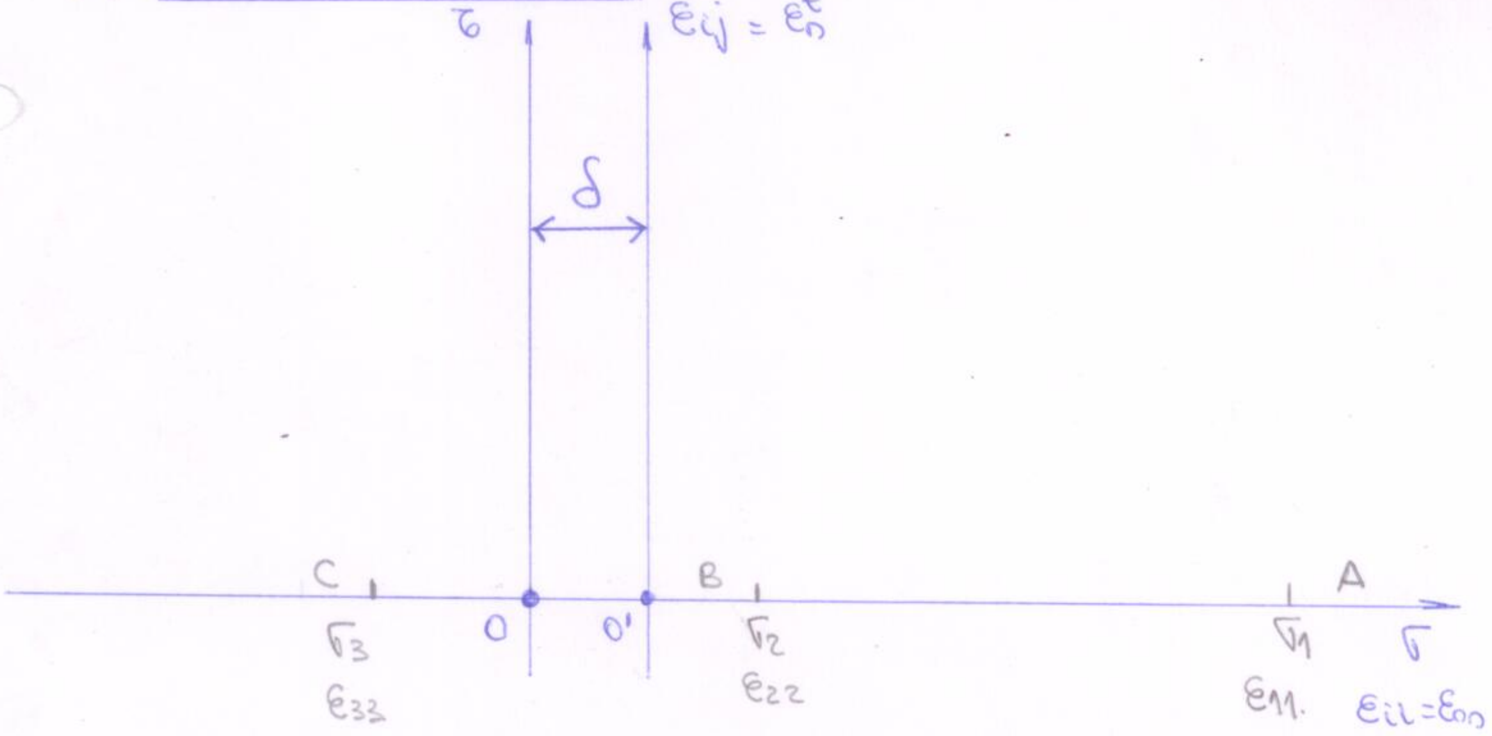
1º) DEFINICIÓN DE ESCALAS:

$$\text{ESC } \sigma = \text{ESC } \tau = \frac{[F]/[L]^2}{[L]} \rightarrow \underline{\text{EJ:}} \ \frac{\text{MPa}}{\text{cm}} \ ; \ \frac{\text{KN/cm}^2}{\text{cm}} \qquad (1)$$

$$\text{ESC } \varepsilon = \frac{1}{[L]} \Bigg\} \ \underline{\text{1º INCÓGNITA}} \qquad (2)$$

$$\text{SIEMPRE} \quad \text{ESCALA} = \frac{\text{OBJETO}}{\text{DIBUJO}} = \frac{\text{MAGNITUD}}{\text{DIBUJO}} = \frac{\text{RESULTADO}}{\text{DIBUJO}}. \qquad (3)$$

2º) ESQUEMA BÁSICO:



3º) 1ª RELACIÓN ENTRE ESCALAS:

$$\overline{AB} \cdot ESC\,\sigma = (\sigma_1 - \sigma_2) \rightarrow \overline{AB} = \frac{(\sigma_1 - \sigma_2)}{ESC\,\sigma} \quad (4)$$

$$\overline{AB} \cdot ESC\,\varepsilon = (\varepsilon_1 - \varepsilon_2) \rightarrow \overline{AB} = \frac{(\varepsilon_1 - \varepsilon_2)}{ESC\,\varepsilon} \quad (5)$$

PERO (4) = (5).

$$\overline{AB} = \frac{(\sigma_1 - \sigma_2)}{ESC\,\sigma} = \frac{(\varepsilon_1 - \varepsilon_2)}{ESC\,\varepsilon}$$

$$ESC\,\varepsilon = \frac{(\varepsilon_1 - \varepsilon_2)}{(\sigma_1 - \sigma_2)} ESC\,\sigma \quad (6)$$

"LAS ESCALAS SON DIRECTAMENTE PROPORCIONALES"

4º) RELACIÓN ENTRE ESCALAS ENTRE LOS PARÁMETROS QUE CARACTERIZAN AL MATERIAL:

LA (6) NO ES APLICABLE PORQUE YO QUIERO OBTENER LA CIRC. DE DEF. A PARTIR DE LA DE ~~DEFORMACIONES~~ TENSIONES, SALVO QUE CONOZCA ε_1, ε_2, (O CUALQUIER OTRO PAR) Y A PARTIR DE AHÍ DETERMINARLAS.

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \mu\sigma_2 - \mu\sigma_3). \quad (7a)$$

$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \mu\sigma_1 - \mu\sigma_3). \quad (7b)$$

$$(7a) - (7b) = \varepsilon_1 - \varepsilon_2 = \frac{1}{E}\left[\sigma_1 - \mu\sigma_2 - \mu\sigma_3 - (\sigma_2 - \mu\sigma_1 - \mu\sigma_3)\right] =$$

$$= \frac{1}{E}\left[\sigma_1 - \mu\sigma_2 - \cancel{\mu\sigma_3} - \sigma_2 + \mu\sigma_1 + \cancel{\mu\sigma_3}\right] = \frac{1}{E}\left[\sigma_1(1+\mu) - \sigma_2(1+\mu)\right] =$$

$$= \frac{1}{E}\left[(\sigma_1-\sigma_2)(1+\mu)\right] \rightarrow$$

$$\rightarrow \varepsilon_1 - \varepsilon_2 = \frac{1+\mu}{E} \cdot (\sigma_1 - \sigma_2) \qquad (8)$$

SE REEMPLAZA (8) EN (6):

$$\text{ESC } \varepsilon = \frac{(1+\mu)}{E} \cdot \frac{(\sigma_1-\sigma_2)}{(\sigma_1-\sigma_2)} \cdot \text{ESC } \sigma$$

$$\text{ESC } \varepsilon = \frac{1+\mu}{E} \cdot \text{ESC } \sigma \qquad (9)$$

5e) DETERMINACIÓN DEL DESPLAZAMIENTO 'δ' ENTRE LOS EJES σ y $\varepsilon_{ij} = \varepsilon_n^t$:

$$\delta = \overline{OO'} = \overline{AO} - \overline{AO'} \qquad (10)$$

DONDE:

$$\overline{AO} = \frac{\sigma_1}{\text{ESC } \sigma} \qquad (11a)$$

$$\overline{AO'} = \frac{\varepsilon_{11}}{\text{ESC } \varepsilon} = \frac{E}{(1+\mu)} \cdot \frac{\varepsilon_{11}}{\text{ESC } \sigma} \qquad (11b)$$

LUEGO

$$\delta = \overline{OO'} = \overline{AO} - \overline{AO'} = \frac{\sigma_1}{\text{ESC } \sigma} - \frac{E}{(1+\mu)} \cdot \frac{\varepsilon_{11}}{\text{ESC } \sigma} \qquad (12)$$

PERO:

$$\varepsilon_{11} = \frac{1}{E}\left(\sigma_1 - \mu\sigma_2 - \mu\sigma_3\right) \quad (70)$$

LA CUAL REEMPLAZADO EN (72).

$$\delta = \frac{1}{ESC\,\sigma}\cdot\left[\sigma_1 - \frac{E}{(1+\mu)}\varepsilon_{11}\right]$$

$$\delta = \frac{1}{ESC\,\sigma}\cdot\left[\sigma_1 - \frac{\cancel{E}}{(1+\mu)}\cdot\frac{1}{\cancel{E}}\left(\sigma_1 - \mu\sigma_2 - \mu\sigma_3\right)\right]$$

$$\delta = \frac{1}{ESC\,\sigma}\cdot\frac{(1+\mu)\,\sigma_1 - (\sigma_1 - \mu\sigma_2 - \mu\sigma_3)}{(1+\mu)}$$

$$\delta = \frac{1}{ESC\,\sigma}\cdot\frac{\cancel{\sigma_1} + \mu\sigma_1 - \cancel{\sigma_1} + \mu\sigma_2 + \mu\sigma_3}{(1+\mu)}$$

$$\delta_\sigma = \frac{\mu}{(1+\mu)}\cdot\frac{(\sigma_1 + \sigma_2 + \sigma_3)}{ESC\,\sigma}.$$

2º INCÓGNITA

APLICACIÓN:

$$\begin{cases} \sigma_1 = +67.08 \ MN/m^2 \\ \sigma_2 = 0.00 \ \text{"} \\ \sigma_3 = -67.08 \ \text{"} \end{cases} \qquad \begin{cases} \varepsilon_1 = 4.1925 \cdot 10^{-4} \\ \varepsilon_2 = 0.00 \\ \varepsilon_3 = -4.1925 \cdot 10^{-4} \end{cases}$$

$$\begin{cases} E = 2.10^5 \ \text{"} \\ \mu = 0.25 \\ G = 8.10^4 \ \text{"} \end{cases}$$

$$\text{Esc. } \varepsilon = \frac{1+\mu}{E} \cdot \text{ESC } \sigma = \frac{1+0.25}{2.10^5 \frac{MN}{m^2}} \cdot \frac{10 \ MN/m^2}{cm}$$

$$\text{Esc } \sigma = \frac{10 MN/m^2}{1 cm} \qquad = \boxed{\frac{6{,}25 \cdot 10^{-5}}{cm}}$$

$$\delta = \frac{\mu}{1+\mu} \cdot \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{\text{Esc } \sigma}$$

$$\delta = \frac{0.25}{1+0.25} \cdot \frac{0}{10 \frac{MN/m^2}{cm}} = 0.$$

~~1 cm — 6,25 . 10^-5~~

$$\begin{cases} 10 \ MN/m^2 & — 1 cm. \\ 67{,}08 \ \text{"} & — \boxed{6{,}708 cm} \end{cases} \qquad \begin{matrix} 6{,}25 \cdot 10^{-5} & — 1 cm \\ 4{,}1925 \cdot 10^{-4} & — \boxed{6{,}708 cm} \end{matrix}$$

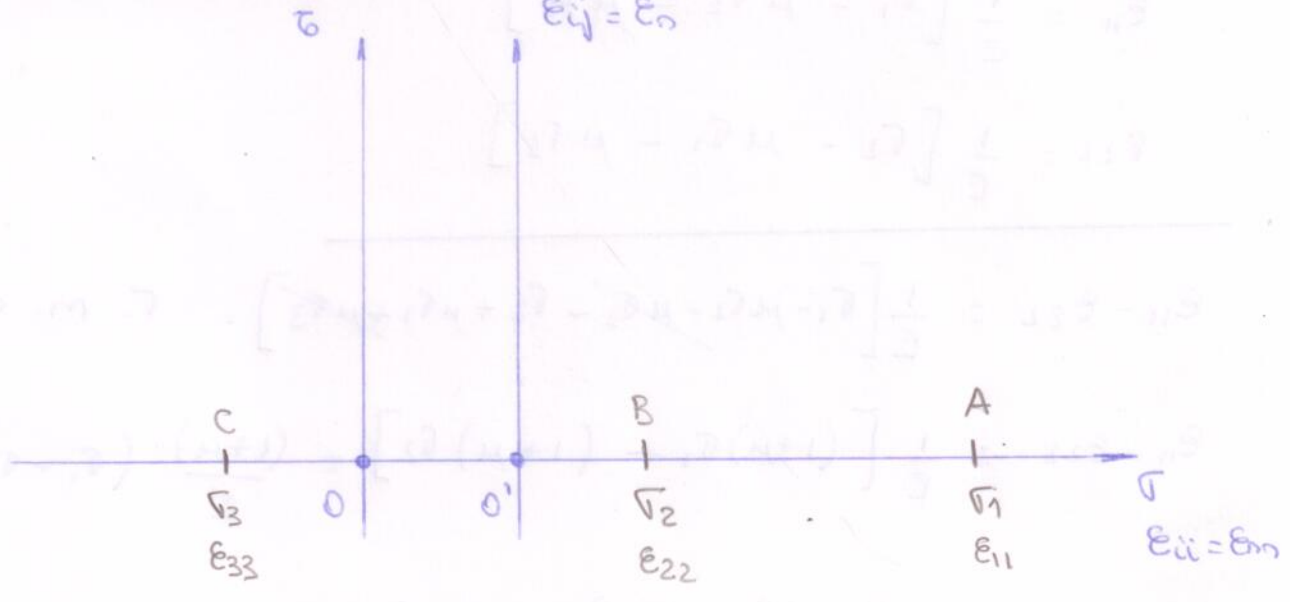
CIRCUNFERENCIA DE TENSIONES A PARTIR DE LA CIRCUNFERENCIA DE DEFORMACIONES:

1º) ESCALAS:

ESCALA DATO $\longrightarrow$ $ESC\ \varepsilon = \dfrac{1}{[L]}$ (1)

ESCALA INCÓGNITA $\longrightarrow$ $ESC\ \sigma = \dfrac{[F/L^2]}{[L]} = ESC\ \tau$ (2)

2º) ESQUEMA BÁSICO:



3º) 1ª RELACIÓN ENTRE ESCALAS.

$\overline{AB} \cdot ESC\ \varepsilon = (\varepsilon_{11} - \varepsilon_{22})$ (3)

$\overline{AB} \cdot ESC\ \sigma = (\sigma_1 - \sigma_2)$ (4)

$$\overline{AB} = \frac{(\varepsilon_{11} - \varepsilon_{22})}{ESC\ \varepsilon} = \frac{(\sigma_1 - \sigma_2)}{ESC\ \sigma}$$

$$\cancel{(\sigma_1 - \sigma_2)} = \frac{ESC\ \varepsilon}{\cancel{ESC\ \sigma}} \frac{(\sigma_1 - \sigma_2)}{(\varepsilon_{11} - \varepsilon_{22})} = ESC\ \sigma \quad (5)$$

"LAS ESCALAS SON DIMENSIONALMENTE PROPORCIONALES"

4º) 2º RELACIÓN ENTRE ESCALAS:

UTILIZANDO LOS PARÁMETROS QUE CARACTERIZAN AL MATERIAL:

↳ LA (5) NO ES DIRECTAMENTE APLICABLE A MENOS QUE 1º SE CALCULEN σ_1 Y σ_2 A PARTIR DE ε_{11}, ε_{22} Y ε_{33}.

$$\varepsilon_{11} = \frac{1}{E}\left[\sigma_1 - \mu\sigma_2 - \mu\sigma_3\right]$$

$$\varepsilon_{22} = \frac{1}{E}\left[\sigma_2 - \mu\sigma_1 - \mu\sigma_3\right]$$

$$\varepsilon_{11} - \varepsilon_{22} = \frac{1}{E}\left[\sigma_1 - \mu\sigma_2 - \cancel{\mu\sigma_3} - \sigma_2 + \mu\sigma_1 + \cancel{\mu\sigma_3}\right] \quad \text{r. m. a m.}$$

$$\varepsilon_{11} - \varepsilon_{22} = \frac{1}{E}\left[(1+\mu)\sigma_1 - (1+\mu)\sigma_2\right] = \frac{(1+\mu)}{E}(\sigma_1 - \sigma_2)$$

$$(\sigma_1 - \sigma_2) = \frac{E}{(1+\mu)}(\varepsilon_{11} - \varepsilon_{22}). \quad (6)$$

↳ SE REEMPLAZA LA (6) EN LA (5):

$$ESC\ \sigma = ESC\ \varepsilon \frac{(\sigma_1 - \sigma_2)}{(\varepsilon_{11} - \varepsilon_{22})} = \frac{ESC\ \varepsilon}{\cancel{(\varepsilon_{11} - \varepsilon_{22})}} \frac{E}{(1+\mu)} \cancel{(\varepsilon_{11} - \varepsilon_{22})}$$

$$ESC\ \sigma = \frac{E}{(1+\mu)} \cdot ESC\ \varepsilon. \quad (7)$$

5º) Determinación del desplazamiento 'δ' entre los ejes σ y $\varepsilon_{ij} = \varepsilon_{nt}$;

$$\delta = OO' = \overline{OA} - \overline{O'A'} \qquad (8)$$

$$\overline{OA} = \frac{\sigma_1}{ESC\,\sigma} = \frac{\sigma_1}{\frac{E}{(1+\mu)} \cdot ESC\,\varepsilon} = \frac{(1+\mu)}{E} \cdot \frac{\sigma_1}{ESC\,\varepsilon} \qquad (9a)$$

$$\overline{O'A'} = \frac{\varepsilon_{11}}{ESC\,\varepsilon} \qquad (9b)$$

• Reemplazando (9a) y (9b) en (8):

$$\delta = \frac{\sigma_1}{ESC\,\sigma} - \frac{\varepsilon_{11}}{ESC\,\varepsilon} = \frac{(1+\mu)}{E} \cdot \frac{\sigma_1}{ESC\,\varepsilon} - \frac{\varepsilon_{11}}{ESC\,\varepsilon} \qquad (10)$$

• De la (10) se la aplicará de 2 maneras.

$\boxed{I}$ Considerando: $\varepsilon_{11} = \frac{1}{E}\left[\sigma_1 - \mu\sigma_2 - \mu\sigma_3\right]$

$$\delta = \frac{1+\mu}{E}\,\frac{\sigma_{11}}{ESC\,\varepsilon} - \frac{1}{E}\,\frac{\sigma_1 - \mu\sigma_2 - \mu\sigma_3}{ESC\,\varepsilon}$$

$$\delta = \frac{1}{E \cdot ESC\,\varepsilon}\left[(1+\mu)\sigma_1 - \sigma_1 + \mu\sigma_2 + \mu\sigma_3\right]$$

$$\delta = \frac{1}{E \cdot ESC\,\varepsilon}\left[\cancel{\sigma_1} + \mu\sigma_1 - \cancel{\sigma_1} + \mu\sigma_2 + \mu\sigma_3\right] = \frac{\mu}{E\,esc}(\sigma_1 + \sigma_2 + \sigma_3)$$

$$\delta = \frac{\mu}{E}\,\frac{(\sigma_1+\sigma_2+\sigma_3)}{ESC\ \varepsilon} = \frac{\mu}{E}\,\frac{I_{1\sigma}}{ESC\ \varepsilon} \qquad (11).$$

$\boxed{\text{II}}$ CONSIDERANDO:

$$\varepsilon_{11} = \frac{1}{E}\left[\sigma_1 - \mu\sigma_2 - \mu\sigma_3\right] = \frac{1}{E}\left[\sigma_1 - \mu(\sigma_2+\sigma_3)\right] \qquad (12a)$$

$$\varepsilon_{22} = \frac{1}{E}\left[\sigma_2 - \mu\sigma_1 - \mu\sigma_3\right] \rightarrow \sigma_2 = E\varepsilon_{22} + \mu\sigma_1 + \mu\sigma_3 \qquad (12b)$$

$$\varepsilon_{33} = \frac{1}{E}\left[\sigma_3 - \mu\sigma_1 - \mu\sigma_2\right] \rightarrow \mu\sigma_2 = \sigma_3 - \mu\sigma_1 - E\varepsilon_{33} \qquad (12c)$$

(12a) - (12b) r. m. a m.:

$$\sigma_2 - \mu\sigma_2 = E\varepsilon_{22} + E\varepsilon_{33} + \mu\sigma_1 + \mu\sigma_3 + \mu\sigma_1 - \sigma_3$$

$$(1-\mu)\sigma_2 = E(\varepsilon_{22} + \varepsilon_{33}) + 2\mu\sigma_1 - (1-\mu)\sigma_3.$$

$$(1-\mu)\sigma_2 + (1-\mu)\sigma_3 = E(\varepsilon_{22}+\varepsilon_{33}) + 2\mu\sigma_1$$

$$\sigma_2 + \sigma_3 = \frac{E}{(1-\mu)}(\varepsilon_{22}+\varepsilon_{33}) + \frac{2\mu}{(1-\mu)}\sigma_1 \qquad (13)$$

• se reemplaza la (13) en la (12a):

$$\varepsilon_{11} = \frac{1}{E}\left[\sigma_1 - \frac{\mu}{(1-\mu)}E(\varepsilon_{22}+\varepsilon_{33}) - \frac{2\mu^2}{(1-\mu)}\sigma_1\right]$$

$$E\varepsilon_{11} + \frac{\mu}{(1-\mu)}E(\varepsilon_{22}+\varepsilon_{33}) = \left[1 - \frac{2\mu^2}{(1-\mu)}\right]\sigma_1$$

$$E\varepsilon_{11} + \frac{\mu}{(1-\mu)} E(\varepsilon_{22}+\varepsilon_{33}) = \frac{1-\mu-2\mu^2}{(1-\mu)}\sigma_1 =$$

$$= \frac{(1+\mu)(1-2\mu)}{(1-\mu)}\sigma_1$$

$$1-\mu-2\mu^2 = (1+\mu)(1-2\mu) = 1-2\mu+\mu-2\mu^2 =$$

$$= 1-\mu-2\mu^2.$$

$$\frac{E(1-\mu)}{(1+\mu)(1-2\mu)}\left[\varepsilon_{11} + \frac{\mu}{1-\mu}(\varepsilon_{22}+\varepsilon_{33})\right] = \sigma_1$$

$$\sigma_1 = \frac{E}{(1+\mu)(1-2\mu)}\left[(1-\mu)\varepsilon_{11} + \mu(\varepsilon_{22}+\varepsilon_{33})\right] \quad (14)$$

- SE REEMPLAZA LA (14) EN (10):

$$\delta = \left[\frac{(1+\mu)}{E}\sigma_1 - \varepsilon_{11}\right]\cdot\frac{1}{ESCE}.$$

$$\delta = \frac{1}{ESCE}\left\{\frac{\cancel{(1+\mu)}}{\cancel{E}}\frac{\cancel{E}}{\cancel{(1+\mu)}(1-2\mu)}\left[(1-\mu)\varepsilon_{11}+\mu(\varepsilon_{22}+\varepsilon_{33})\right] - \varepsilon_{11}\right\}$$

$$\delta = \frac{1}{ESCE}\left\{\frac{1}{1-2\mu}\left[\varepsilon_{11}-\mu\varepsilon_{11}+\mu\varepsilon_{22}+\mu\varepsilon_{33}\right] - \varepsilon_{11}\right\} =$$

$$= \frac{1}{ESCE}\left\{\frac{\varepsilon_{11}-\mu\varepsilon_{11}+\mu\varepsilon_{22}+\mu\varepsilon_{33}-(1-2\mu)\varepsilon_{11}}{(1-2\mu)}\right\} =$$

$$= \frac{1}{ESCE}\left\{\frac{\cancel{\varepsilon_{11}}-\mu\varepsilon_{11}+\mu\varepsilon_{22}+\mu\varepsilon_{33}\cancel{-\varepsilon_{11}}+2\mu\varepsilon_{11}}{(1-2\mu)}\right\} \rightarrow$$

$$\delta = \frac{1}{ESC\,\varepsilon}\left[\frac{\mu}{1-2\mu}\left(\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}\right)\right]$$

$$\delta = \frac{\mu}{1-2\mu} \cdot \frac{\left(\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}\right)}{ESC\,\varepsilon} = \frac{\mu}{1-2\mu} \cdot \frac{I_{1\varepsilon}}{ESC\,\varepsilon} \qquad (15)$$

EJEMPLO:

$$\begin{cases} \sigma_1 = 60 \text{ MPa} \\ \sigma_2 = 20 \\ \sigma_3 = -100 \end{cases}$$

$$\varepsilon_1 = \frac{1}{2 \cdot 10^5}\left[60 - 0{,}25(20 - 100)\right] = 4 \cdot 10^{-4}$$

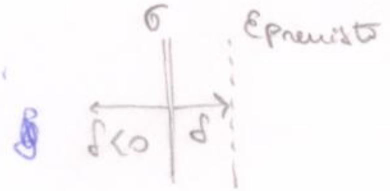
$$\varepsilon_2 = \frac{1}{2 \cdot 10^5}\left[20 - 0{,}25(60 - 100)\right] = 1{,}5 \cdot 10^{-4}$$

$$\varepsilon_3 = \frac{1}{2 \cdot 10^5}\left[-100 - 0{,}25(60 + 20)\right] = -6 \cdot 10^{-4}$$

$$\begin{cases} E = 2 \cdot 10^5 \text{ MPa} \\ \mu = 0{,}25 \\ G = 8 \cdot 10^4 \text{ MPa} \end{cases}$$

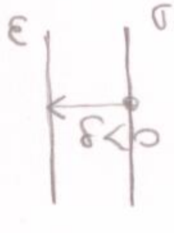
$$\delta_\sigma = \frac{\mu}{1+\mu} \cdot \frac{I_{1\sigma}}{\text{esc}\,\sigma} = \frac{0{,}25}{1+0{,}25} \cdot \frac{(60 + 20 - 100)}{10 \text{ MPa/cm}} = \underline{-0{,}40 \text{ cm}}$$

$$\text{esc}\,\sigma = \frac{10 \text{ MPa}}{\text{cm}} \qquad \text{esc}\,\varepsilon = \frac{1+\mu}{E} \cdot \text{esc}\,\sigma = \frac{1+0{,}25}{2 \cdot 10^5 \text{ MPa}} \cdot \frac{10 \text{ MPa}}{\text{cm}}$$



$$\underline{\text{esc}\,\varepsilon = \frac{6{,}25 \cdot 10^{-5}}{\text{cm}}}$$

$$\delta_\varepsilon = \frac{\mu}{1-2\mu} \cdot \frac{I_{1\varepsilon}}{\text{esc}\,\varepsilon} = \frac{0{,}25}{1-2 \cdot 0{,}25} \cdot \frac{(4 \cdot 10^{-4} + 1{,}5 \cdot 10^{-4} - 6 \cdot 10^{-4})}{6{,}25 \cdot \frac{10^{-5}}{\text{cm}}}$$



$$\underline{\delta_\varepsilon = -0{,}40 \text{ cm}}$$

$$\text{esc}\,\sigma = \frac{E}{1+\mu} \cdot \text{esc}\,\varepsilon = \frac{2 \cdot 10^5 \text{ MPa}}{1+0{,}25} \cdot \frac{6{,}25 \cdot 10^{-5}}{\text{cm}} =$$

$$\underline{\text{esc}\,\sigma = \frac{10 \text{ MPa}}{\text{cm}}}$$