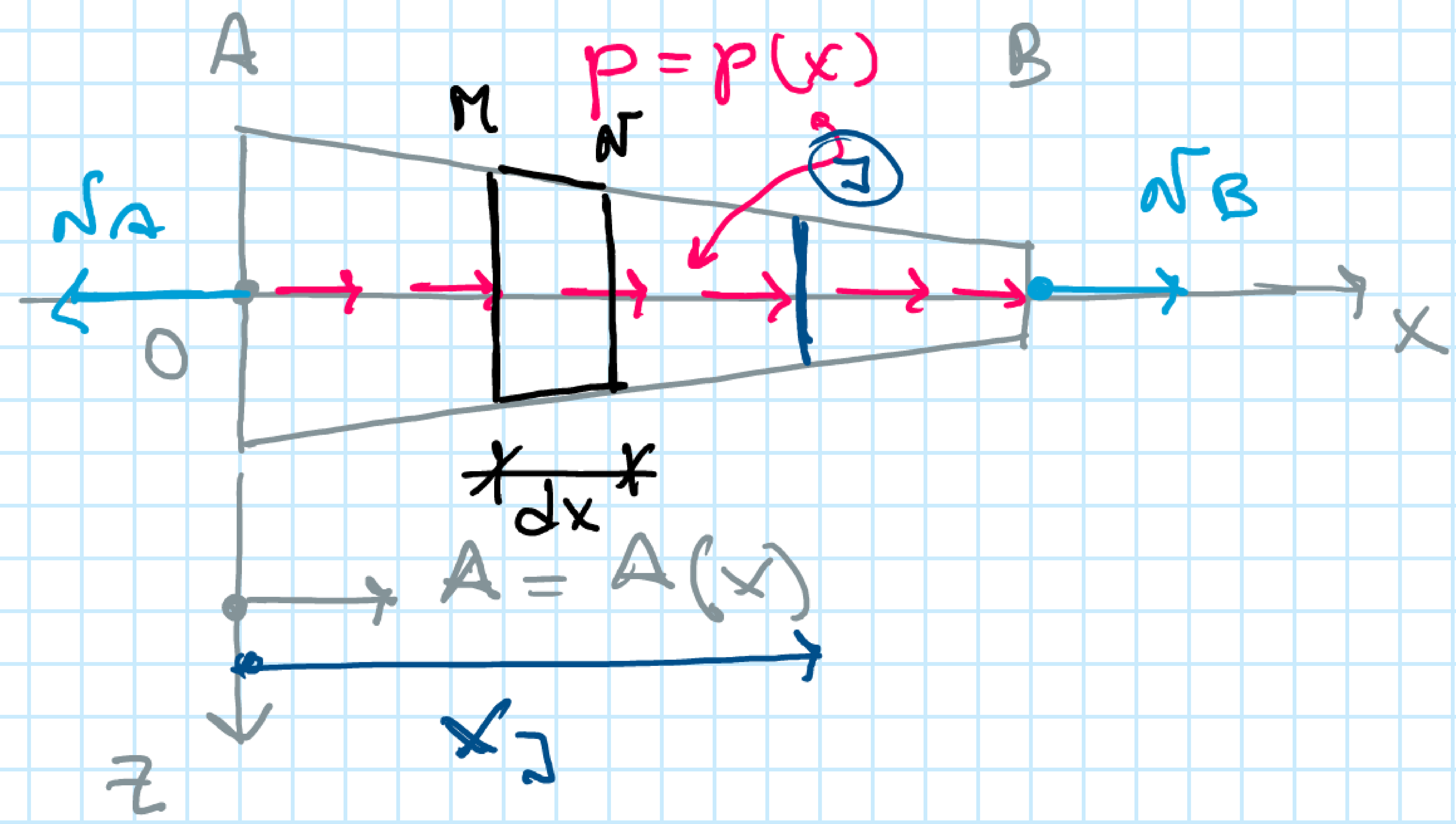
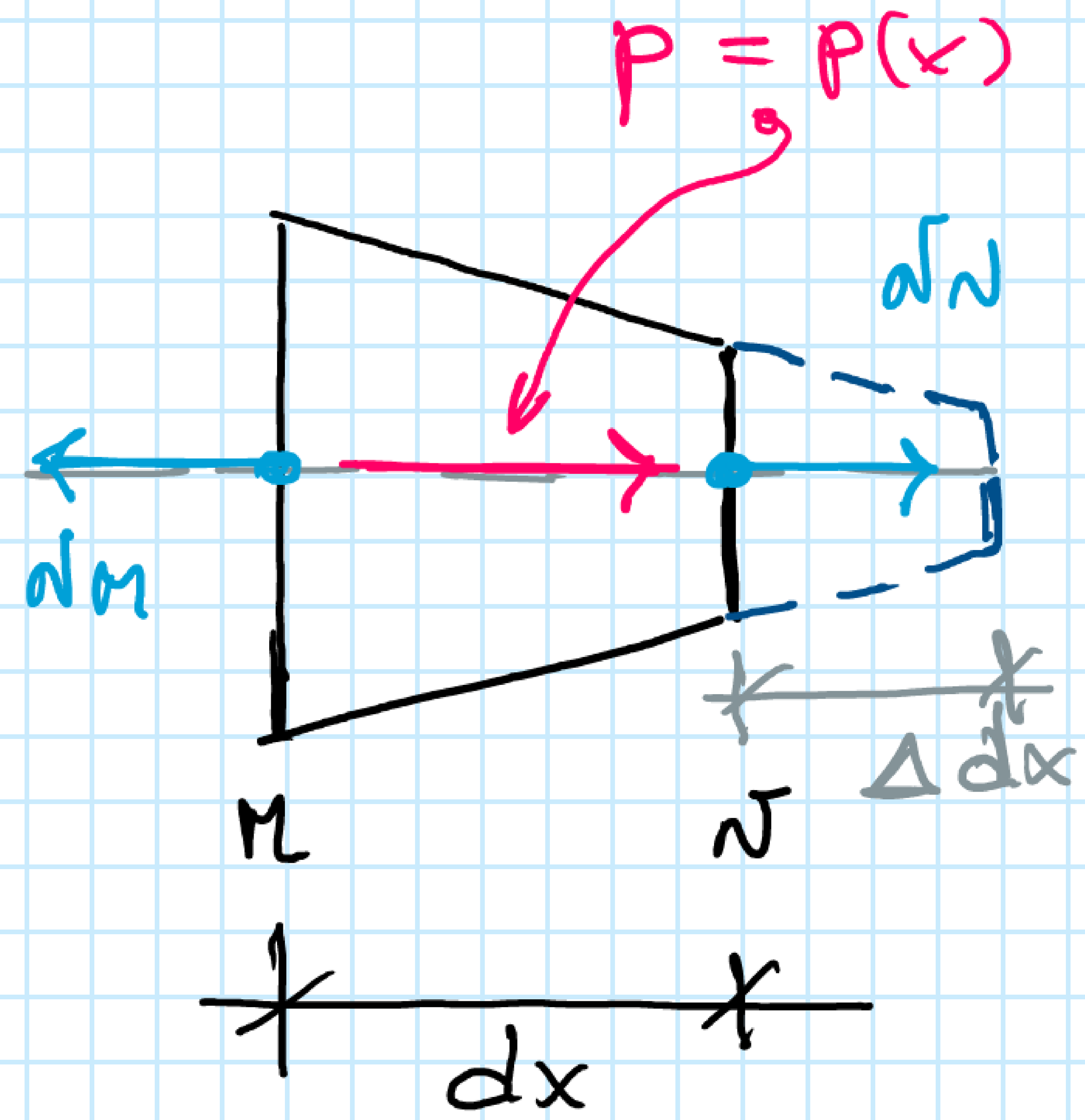
01.09 - DESPLAZAMIENTOS Y DEFORMACIONES:

lunes, 20 de septiembre de 2021 13:01



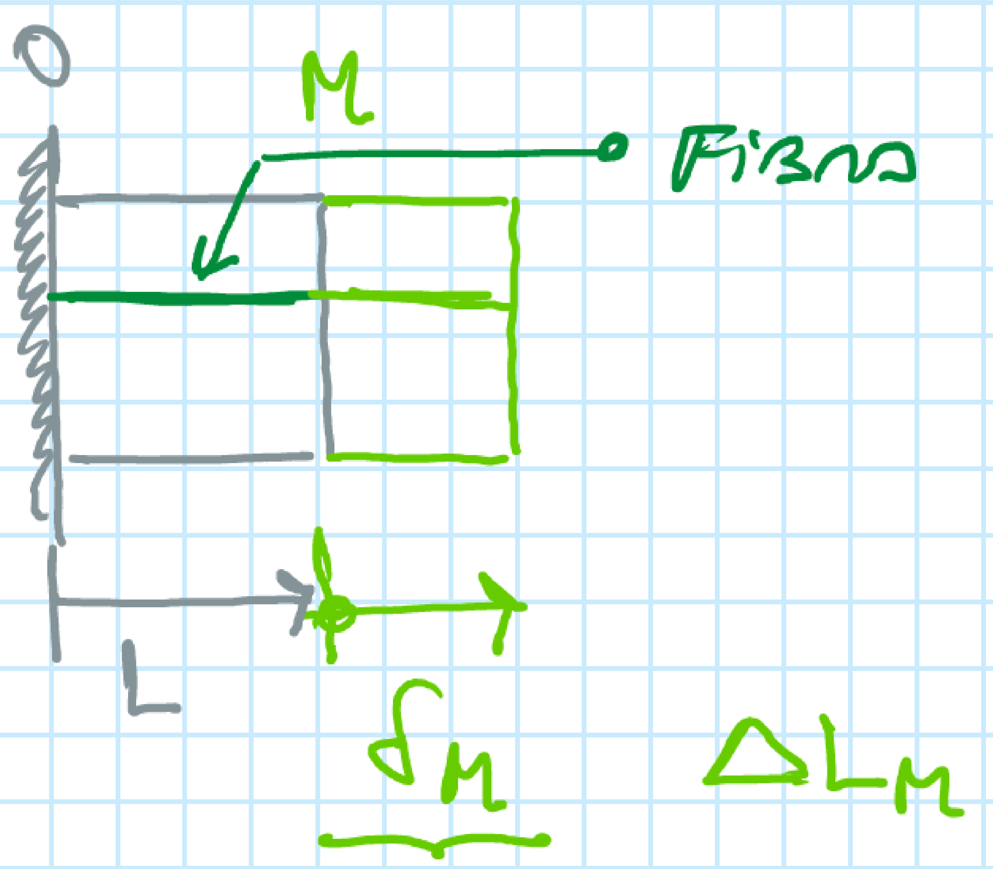


$$\frac{\Delta dx(x)}{dx} = \varepsilon_{xx}(x) \; ; \quad \sigma_x(x) = E\,\varepsilon_{xx}(x) \rightarrow \varepsilon_{xx}(x) = \frac{\sigma_x(x)}{E}$$

$$\underbrace{\Delta dx(x) = \varepsilon_{xx}(x)\cdot dx}\; = \frac{\sigma_x(x)}{E}\cdot dx \; ; \quad \sigma_x(x) = \frac{N(x)}{A(x)}$$

$$\boxed{\Delta dx(x) = \frac{N(x)}{E\cdot A(x)}\cdot dx.}$$

$$\boxed{\Delta L(x) = \delta(x) = \int \Delta dx(x) = \int \frac{N(x)}{EA(x)}\, dx.}$$



δ: DESPLAZAMIENTOS DE LOS PUNTOS

FIBRA $\overline{OM}$ = LONGITUD INICIAL = L = L_0

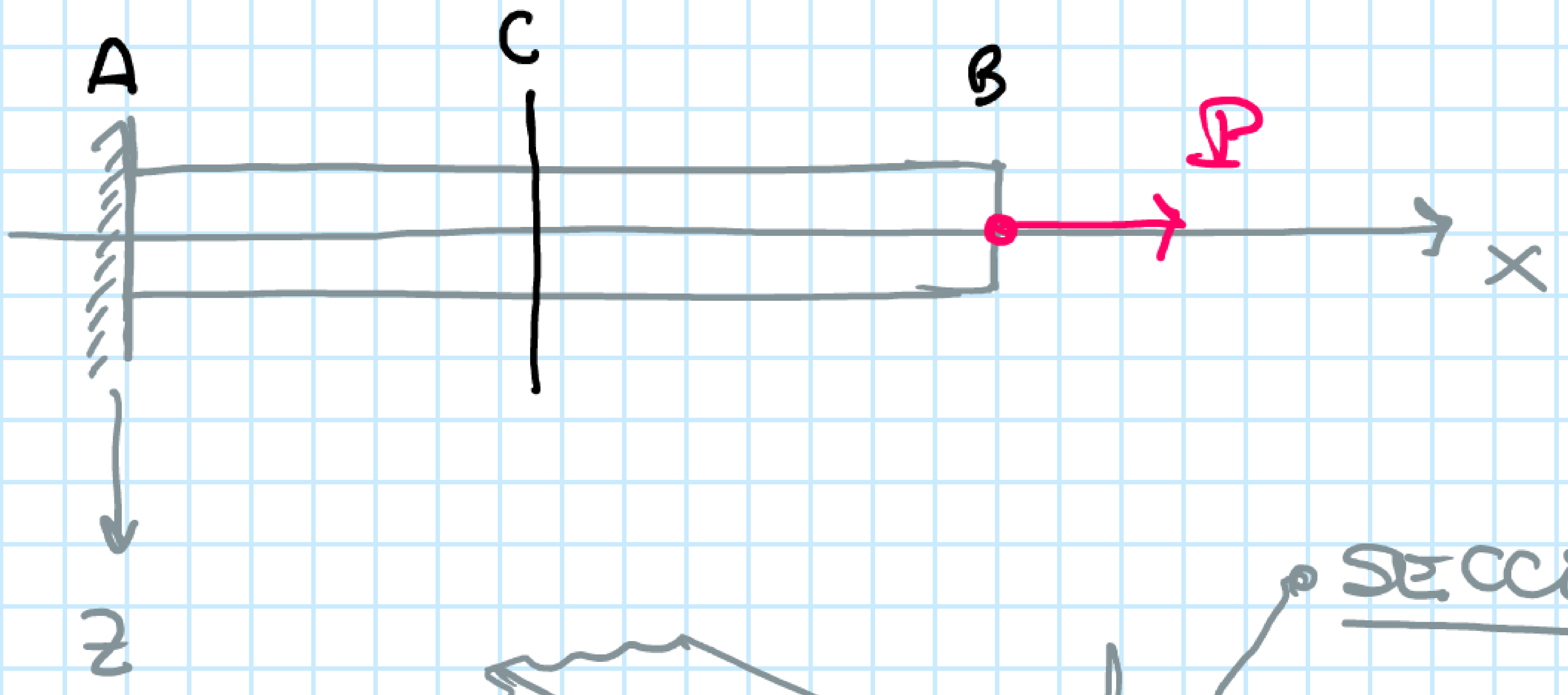
" FINAL = $L + \Delta L$ = L_f.

$$\boxed{\varepsilon_{xx} = \frac{d\delta(x)}{dx} = \frac{d[\Delta L(x)]}{dx} = \frac{N(x)}{E\cdot A(x)}}$$

VÁLIDO SI LA CARGA DEFORMANTE ES LA CARGA FUERZA.

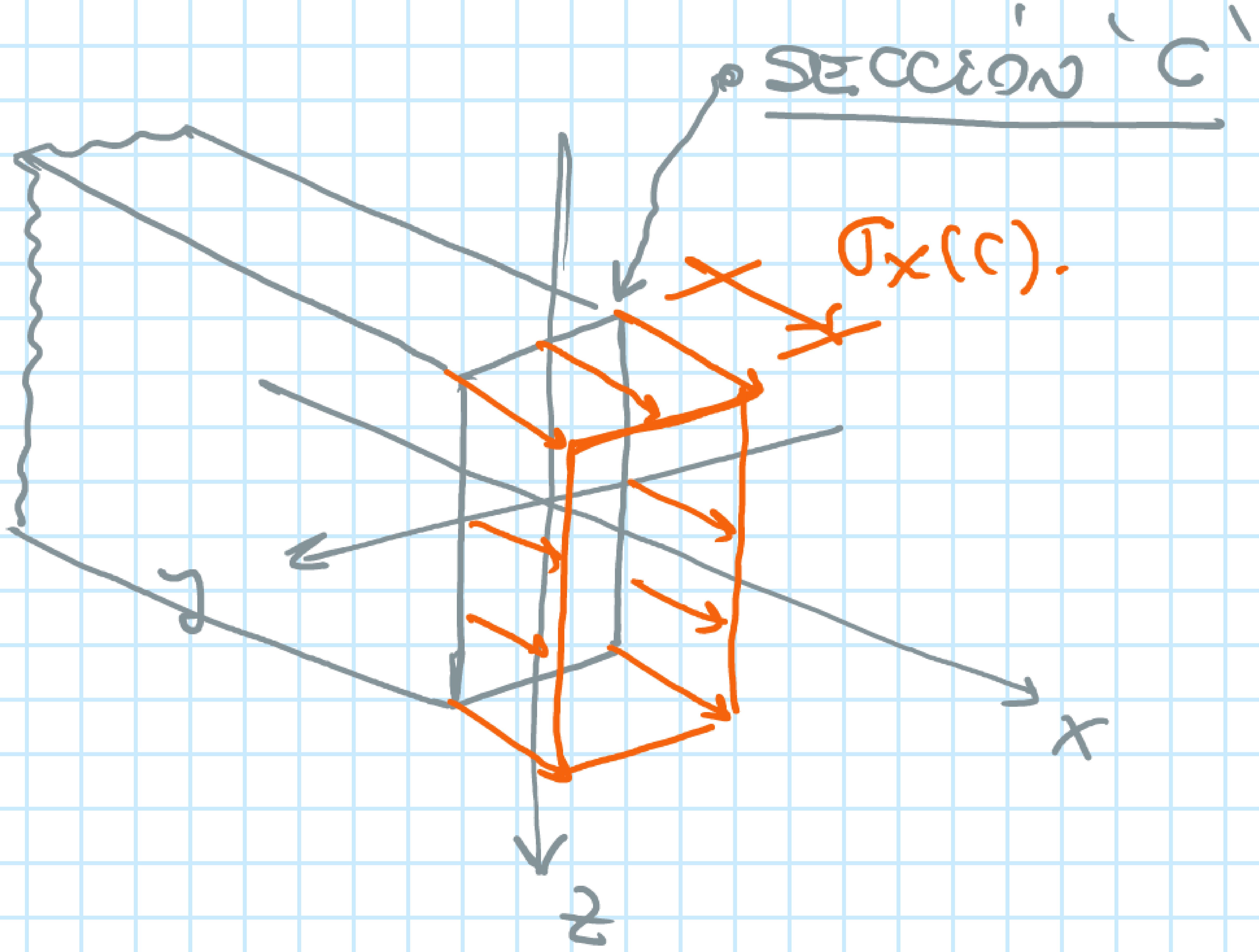
01.10 - VOLUMEN DE TENSIONES:

lunes, 20 de septiembre de 2021 13:21



$$N_C = P.$$

$$\sigma_x(C) = \frac{N_C}{A_C}$$



$$N_C = \int_A \sigma_x \, dA$$

01.11 - RESUMEN DE EXPRESIONES:

lunes, 20 de septiembre de 2021 13:27

1) MAGNITUDES ESTÁTICAS:

$$N = N(x)$$

$$\sigma_x = \frac{N(x)}{A(x)}$$

2) - MAGNITUDES CINEMÁTICAS

$$\delta(x) = \Delta L(x) = \int \frac{N(x)}{E\,A(x)}\,dx. \qquad (I)$$

$$\varepsilon_{xx}(x) = \frac{d\delta(x)}{dx} = \frac{d\Delta L(x)}{dx}.$$

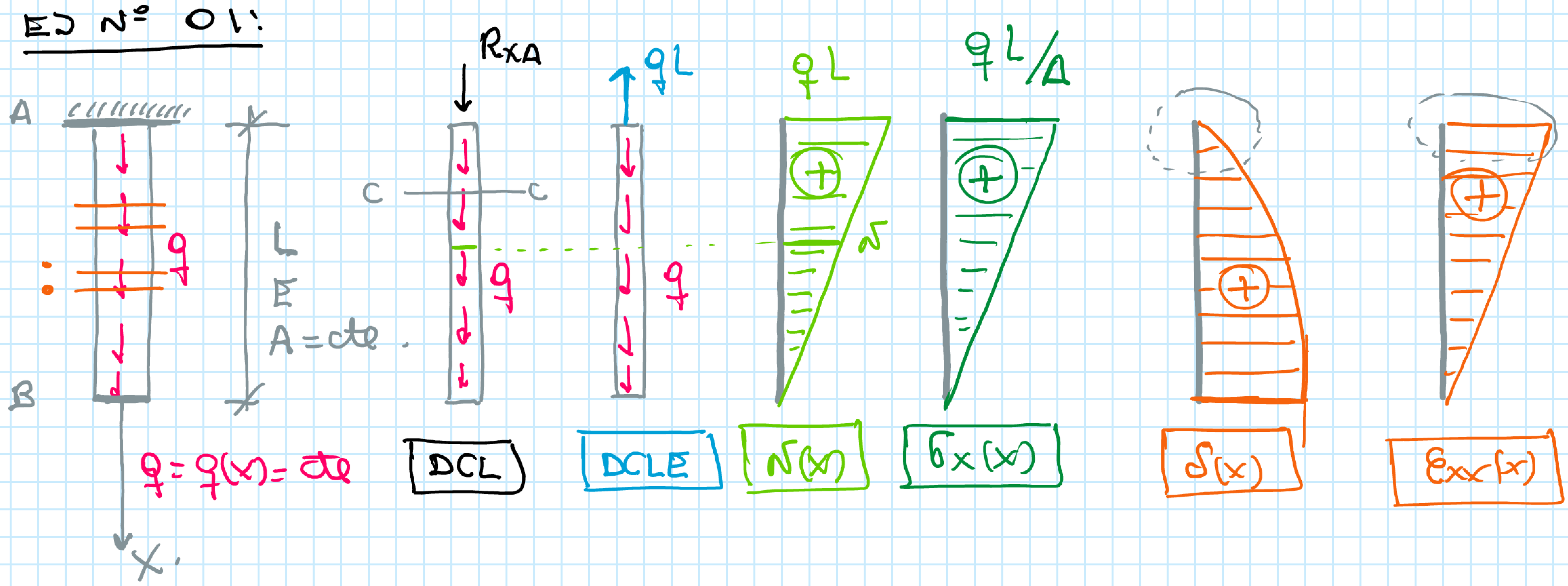
$$\varepsilon_{xx}(x) = \frac{N(x)}{E\,A(x)} \qquad (II)$$

(I) y (II) SON VÁLIDAS P/ LA CAUSA FUERZA.

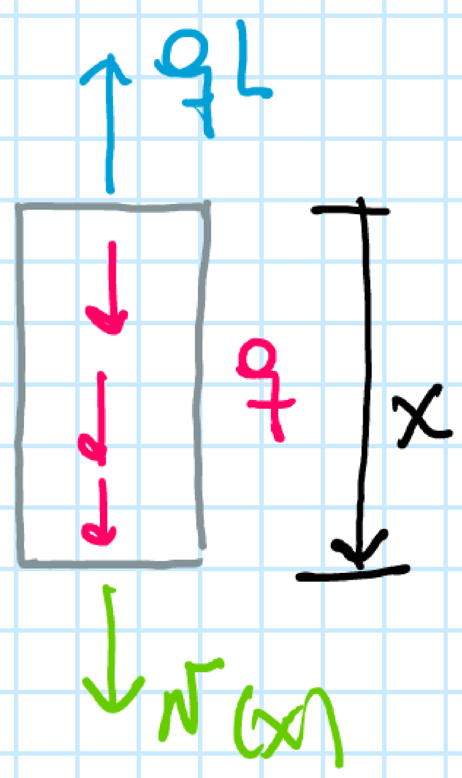
01.12 - EJERCICIOS:

lunes, 20 de septiembre de 2021 13:31

EJ Nº 01:



RVE $\quad \Sigma F_x = 0 \qquad q \cdot L + R_{xA} = 0 \rightarrow \boxed{R_{xA} = -qL}$ ↑.



$$\Sigma F_x = 0 \qquad N(x) + q \cdot x - qL = 0.$$

$$N(x) = qL - qx \rightarrow \boxed{N(x) = q(L-x)}.$$

$$\begin{cases} N(x=0) = qL \\ N(x=L) = 0. \end{cases}$$

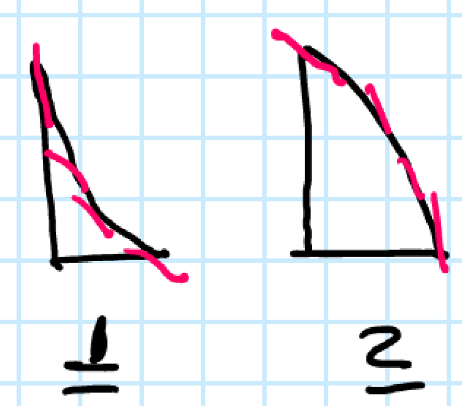
TENSIONES:

$$\sigma_x(x) = \frac{N(x)}{A(x)} = \frac{N(x)}{A} = \frac{q(L-x)}{A}$$

DESPLAZAMIENTOS:

$$\delta(x) = \Delta L(x) = \int \frac{N(x)}{EA(x)}\,dx = \frac{1}{EA}\int q(L-x)\,dx = \frac{q}{EA}\int (L-x)\,dx.$$

$$\delta(x) = \Delta L(x) = \frac{q}{EA}\left[Lx - \frac{x^2}{2}\right]$$



$$\delta(x=0) = 0.$$

$$\delta(x=L) = \frac{q}{EA}\left[L^2 - \frac{L^2}{2}\right] = \boxed{\frac{1}{2}\frac{q}{EA}L^2}$$

DEFORMACIONES:

$$\varepsilon_{xx}(x) = \frac{d\delta(x)}{dx} = \frac{q}{EA}\left[L - x\right]$$

$$\varepsilon_{xx}(0) = \frac{qL}{EA}$$

$$\varepsilon_{xx}(L) = 0$$

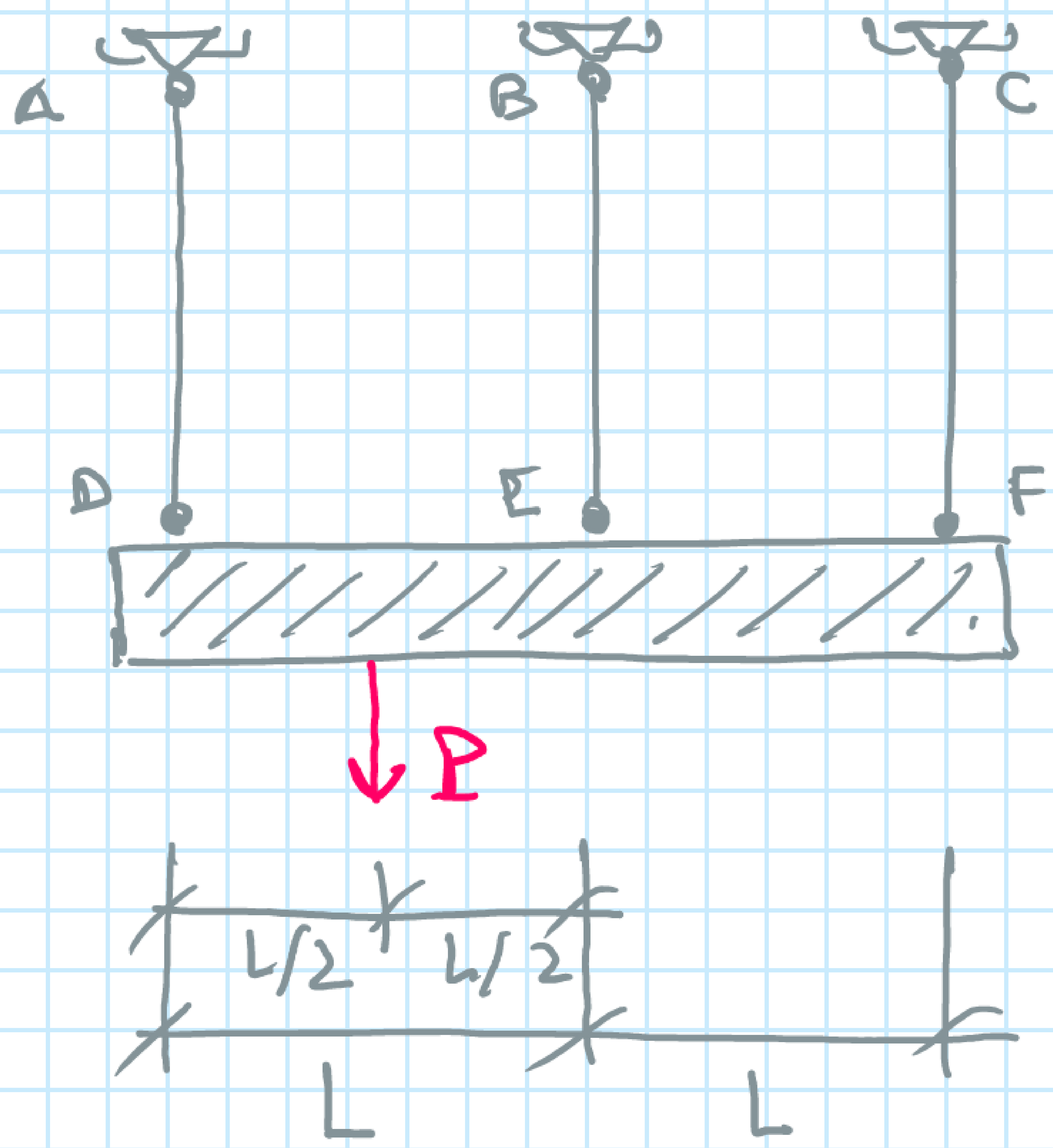
ALTERNATIVA:

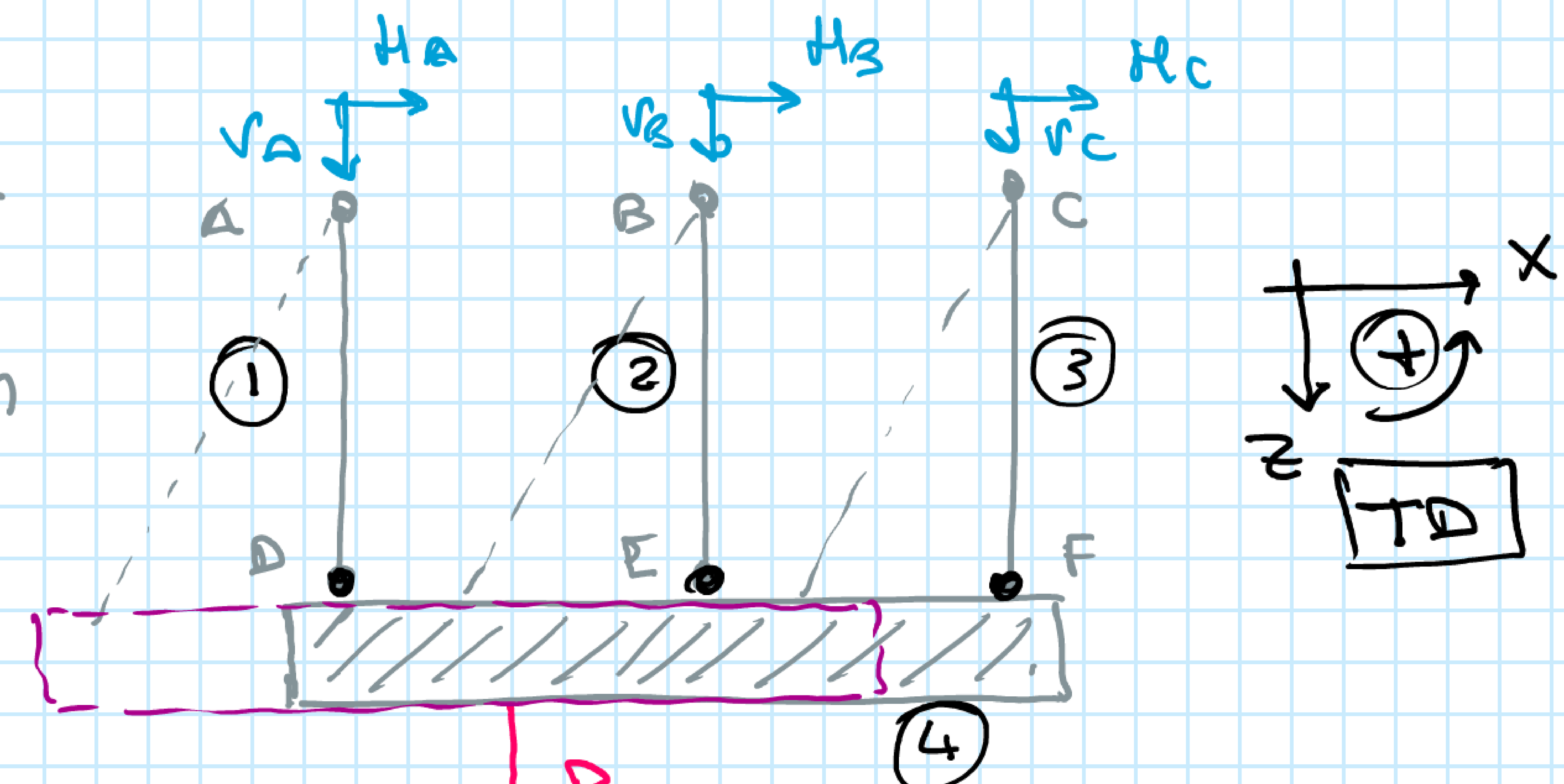
q $\quad P$ en el extremo libre.

01.12 -

lunes, 20 de septiembre de 2021 13:55

EJ N° 02:





$$\begin{cases} GL = N + 2 = 4 + 2 = 6 \\ CV = 6 = 2 + 2 + 2 \end{cases}$$

ECS DE EQUILIBRIO RELATIVO: 'EER'

$\Sigma M^{D}_{Fi,sup} = 0 \qquad H_A \cdot h = 0 \rightarrow \boxed{H_A = 0}$

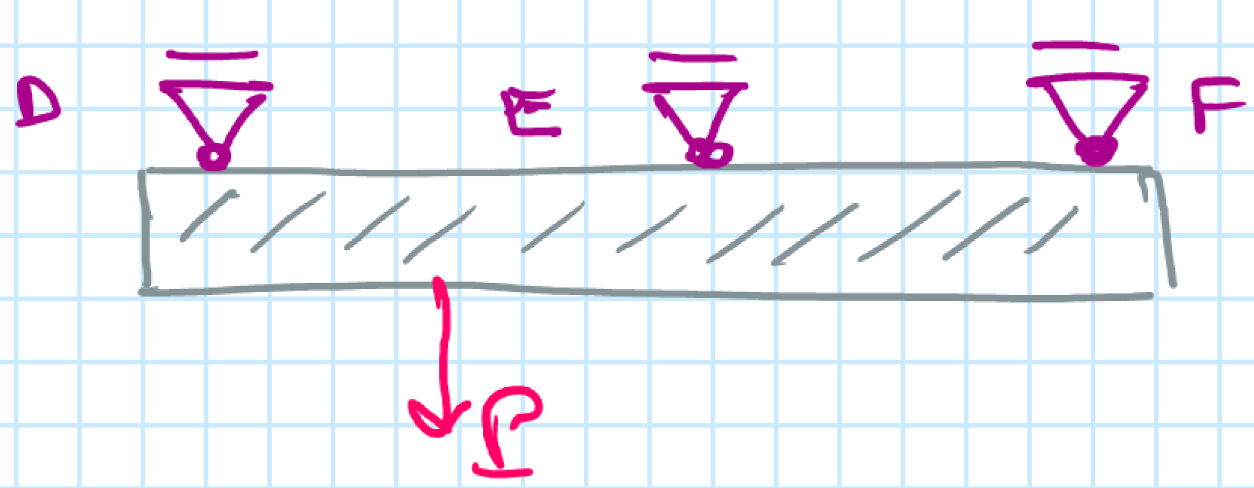
$\Sigma M^{E}_{Fi,sup} = 0 \qquad H_B \cdot h = 0 \rightarrow \boxed{H_B = 0}$

$\Sigma M^{F}_{Fi,sup} = 0 \qquad H_C \cdot h = 0 \rightarrow \boxed{H_C = 0}$

EEA:

$$\left.\begin{aligned} \Sigma F_V = 0 \qquad & P + V_A + V_B + V_C = 0. \\ \Sigma M^{F}_{Fi} = 0 \qquad & P \cdot \tfrac{3}{2} L + V_A \cdot 2L + V_B \cdot L = 0. \end{aligned}\right\}$$

$\Sigma F_H = 0 \qquad H_A + H_B + H_C = 0 \rightarrow 0 = 0$



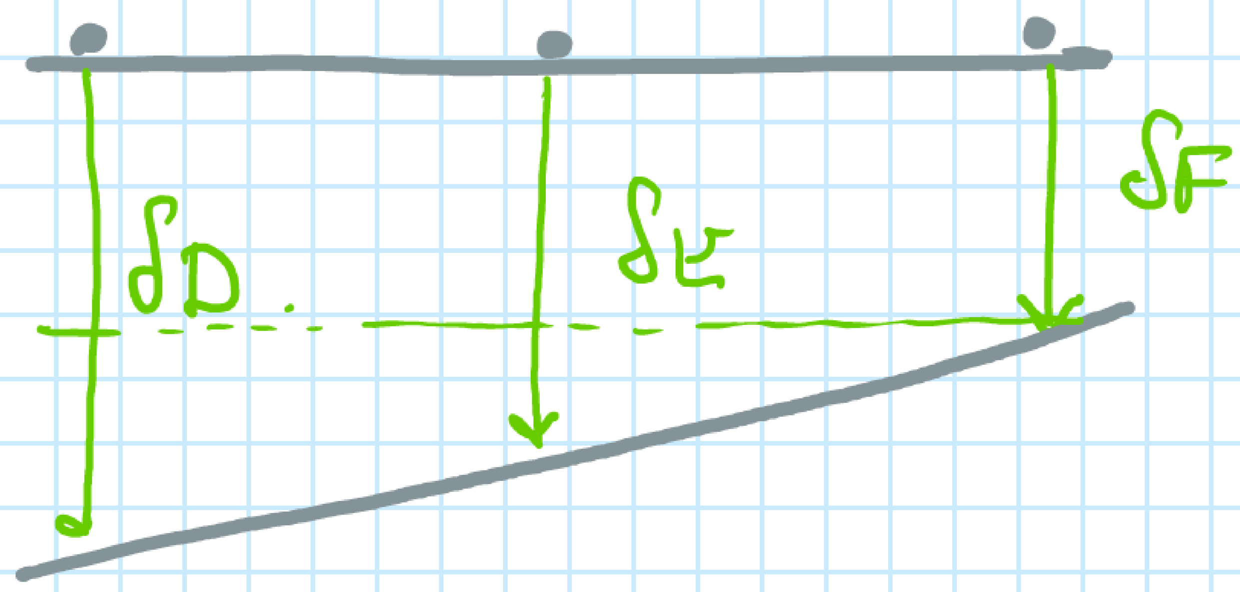
$\begin{cases} \text{P/ CARGAS HORIZONTALES} \rightarrow \text{HIPOESTÁTICA.} \\ \text{P/ CARGAS VERTICALES} \rightarrow \text{HIPERESTÁTICO.} \end{cases}$

01.12 -

lunes, 20 de septiembre de 2021 13:55

ECS DE COMPATIBILIDAD DE LOS DESPLAZAMIENTOS.

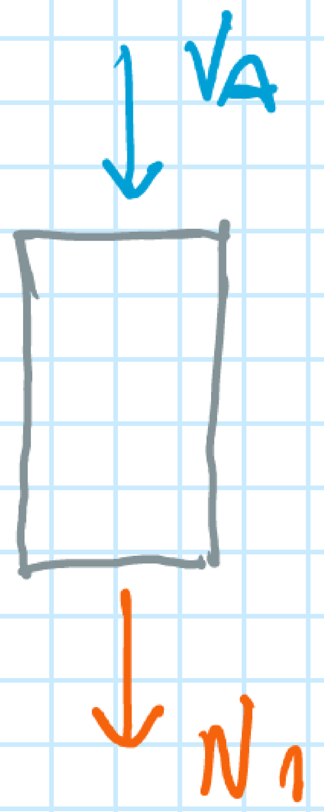
$$\boxed{\frac{\delta_D - \delta_F}{2L} = \frac{\delta_E - \delta_F}{L}} \leftarrow$$



$$\delta_D = \int \frac{N(x)_1}{E_1 A_1} \cdot dx = \frac{N_1}{E_1 A_1} \underbrace{\int dx}_{h} = \frac{N_1 h}{E_1 A_1}$$

$$\delta_E = \frac{N_2 h}{E_2 A_2} \qquad \delta_F = \frac{N_3 h}{E_3 A_3}.$$

SUPONEMOS QUE $\begin{cases} E_1 = E_2 = E_3 = E \\ A_1 = A_2 = A_3 = A \end{cases}$



$\Sigma F_x = 0 \qquad V_A + N_1(x) = 0 \rightarrow \boxed{N_1(x) = -V_A.}$

$$\delta_D = -\frac{V_A h}{EA} \quad ; \quad \delta_E = -\frac{V_B h}{EA} \quad ; \quad \delta_F = -\frac{V_C h}{EA}.$$

$$\frac{\delta_D - \delta_F}{2L} = \frac{\delta_E - \delta_F}{L} \leftarrow$$

$$\frac{-\frac{V_A h}{EA} - \left(-\frac{V_C h}{EA}\right)}{2L} = \frac{-\frac{V_B h}{EA} - \left(-\frac{V_C h}{EA}\right)}{L}$$

$$\cancel{\frac{h}{EA}}\left(V_C - V_A\right)\frac{1}{2\cancel{L}} = \cancel{\frac{h}{EA}}\left(V_C - V_B\right)\frac{1}{\cancel{L}}$$

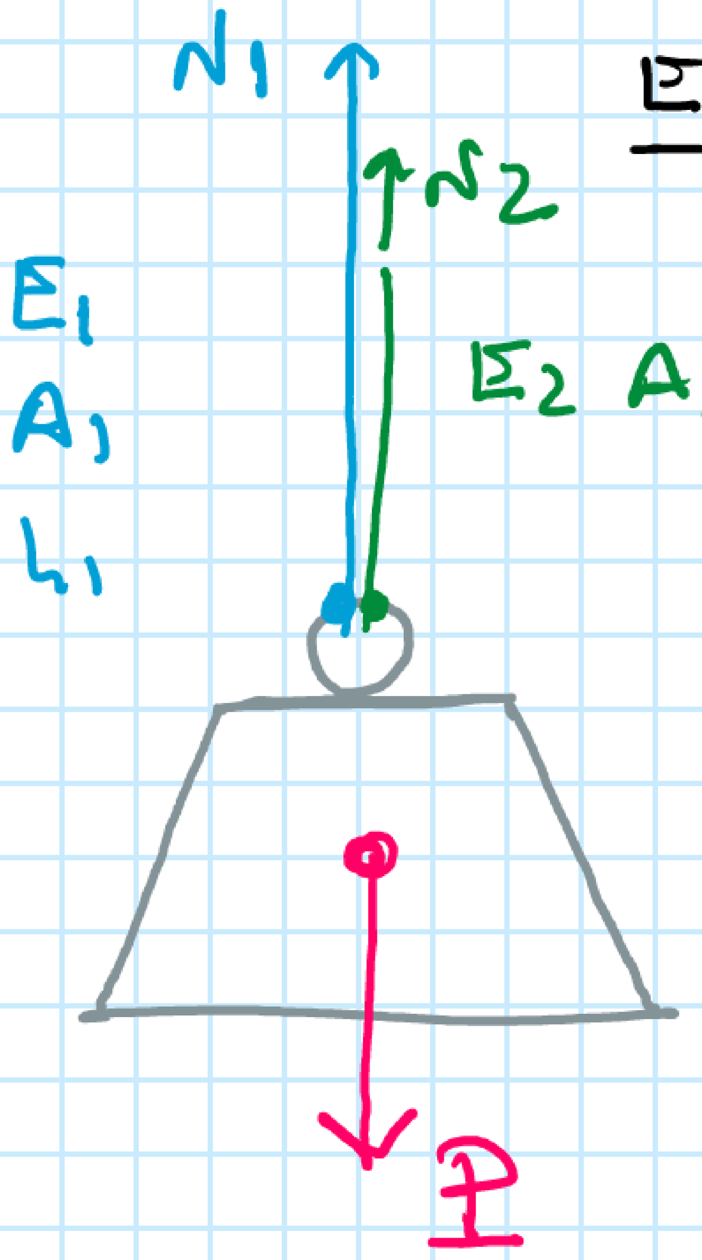
$$\frac{1}{2}V_C - \frac{1}{2}V_A = V_C - V_B \rightarrow V_B - \frac{1}{2}V_A = V_C - \frac{1}{2}V_C = \frac{V_C}{2}.$$

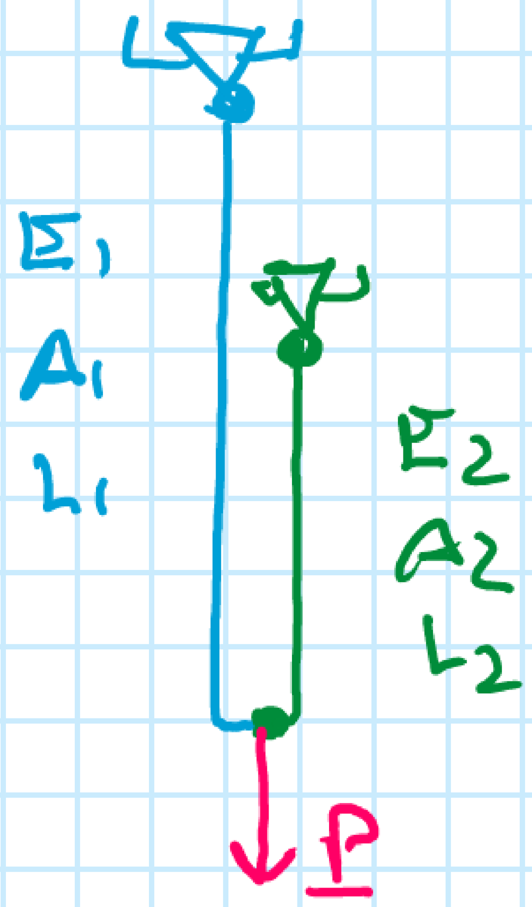
$$\boxed{V_C = 2V_B - V_A}$$

01.12 -

lunes, 20 de septiembre de 2021 14:22

EJ Nº 03





$$\Sigma F_x = 0 \qquad P - N_1 - N_2 = 0.$$

$$\Delta L_1 = \Delta L_2$$

$$\frac{N_1 L_1}{E_1 A_1} = \frac{N_2 L_2}{E_2 A_2}$$

$$N_1 = \frac{N_2}{\underbrace{\frac{E_2 A_2}{L_2}}_{K_2}} \cdot \underbrace{\frac{E_1 A_1}{L_1}}_{K_1}$$

$$\begin{cases} N_1 = N_2 \cdot \dfrac{K_1}{K_2} & K_1 > K_2 \rightarrow \boxed{N_1 > N_2} \\ K_1 < K_2 \rightarrow N_1 < N_2. \end{cases}$$

$$\delta(x) = \Delta L(x) = \int \frac{N(x)}{EA}\, dx \quad \text{si} \quad \left.\begin{matrix} N = cte. \\ E = cte \\ A = cte \end{matrix}\right\} \quad \delta(x) = \frac{N L}{EA.} \rightarrow$$

$$\rightarrow \delta(x) = \frac{N}{\underbrace{\frac{EA}{L}}_{K}} = \frac{N}{K} \qquad \text{si} \begin{cases} K \uparrow \rightarrow \delta(x) \downarrow \\ K \downarrow \rightarrow \delta(x) \uparrow \end{cases}$$

$$N = K\, \delta(x) \longrightarrow$$

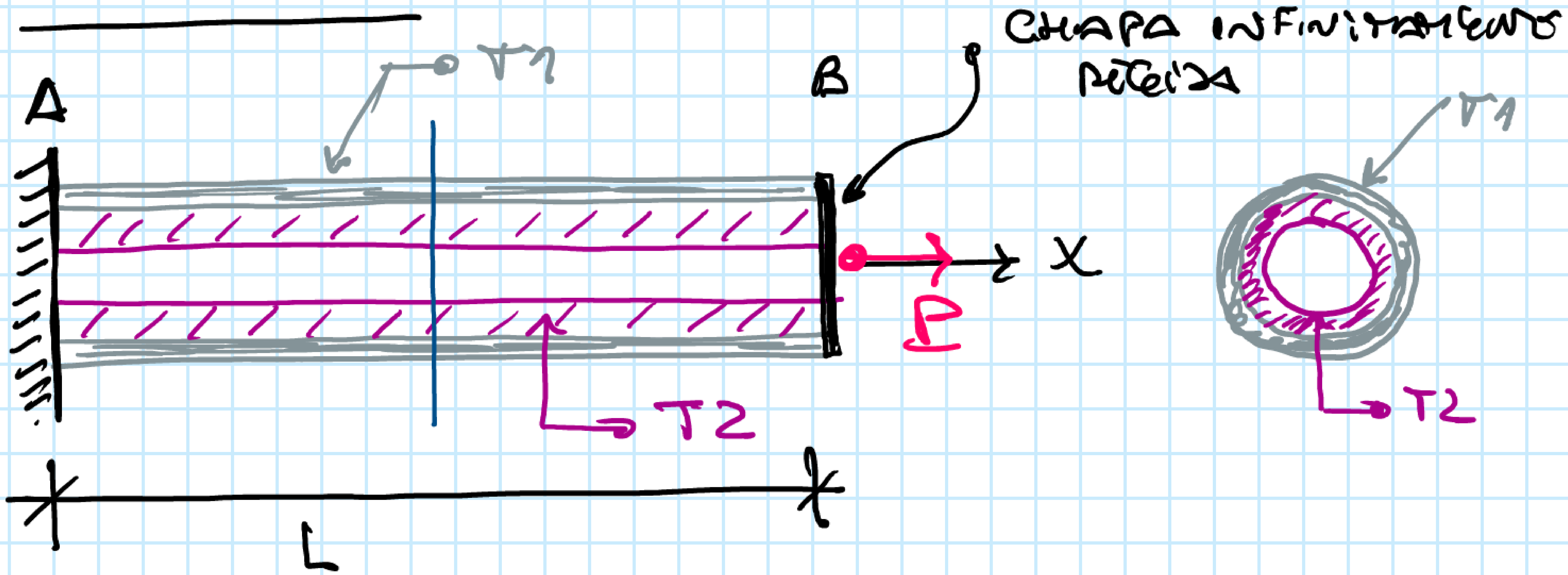
$$K = \frac{EA}{L} = \text{RIGIDEZ A LA SOLICITACIÓN AXIL.}$$

EN ESTRUCTURAS HIPERESTÁTICAS, LOS ELEMENTOS ESTRUCTURALES MÁS RÍGIDOS TOMAN MAYORES ESFUERZOS.

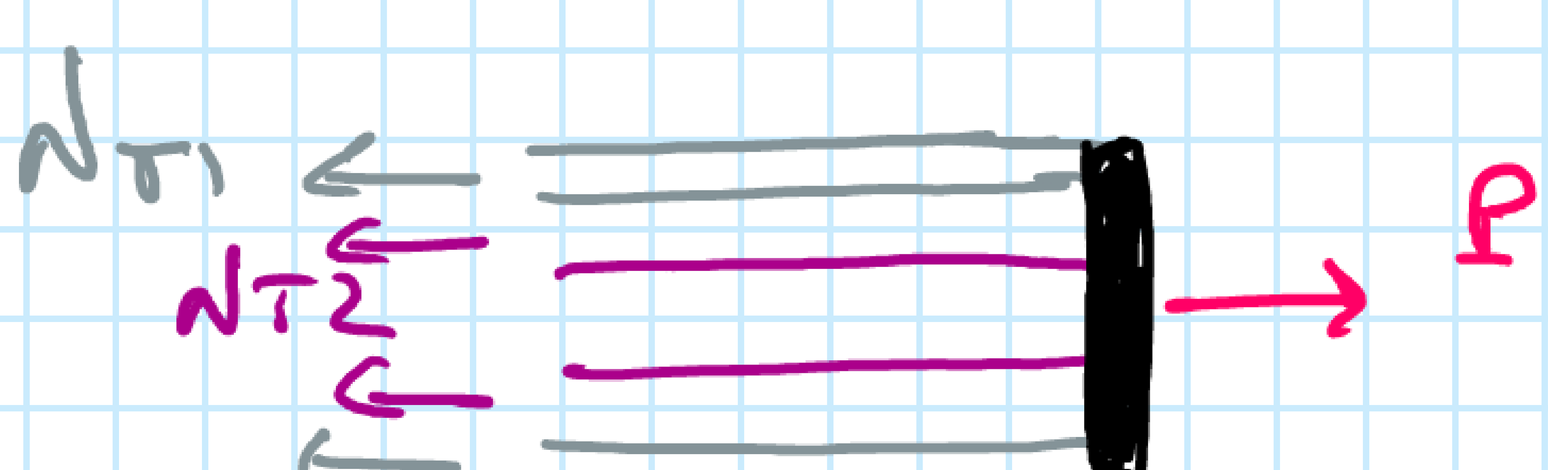
01.12 -

lunes, 20 de septiembre de 2021 14:35

EJ. Nº 04 :



$\Sigma F_x = 0 \qquad R_{xA} + P = 0 \rightarrow \boxed{R_{xA} = -P} \leftarrow$



$$\boxed{P - N_{T1} - N_{T2} = 0}$$

HIPERESTÁTICO POR VÍNCULO INTERNO

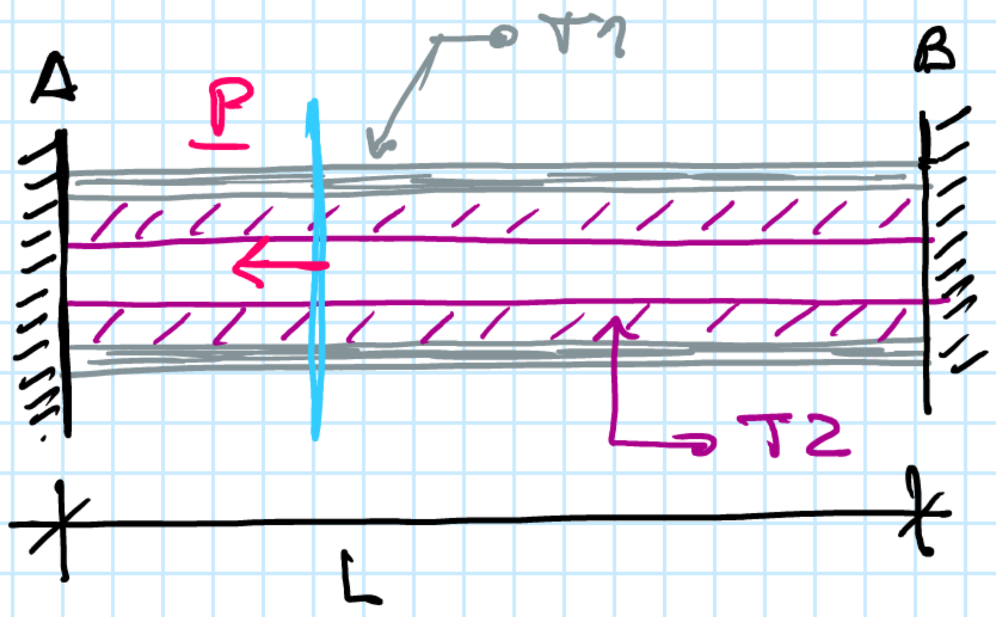
$\Delta L_1 = \Delta L_2$.

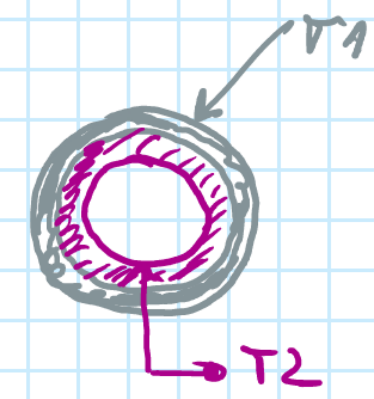
$$\frac{N_{T1} \cancel{L}}{E_1 A_1} = \frac{N_{T2} \cancel{L}}{E_2 A_2} \rightarrow \boxed{N_{T1} = \frac{N_{T2}}{E_2 A_2} \cdot E_1 A_1}$$

01.12 -

lunes, 20 de septiembre de 2021 14:35

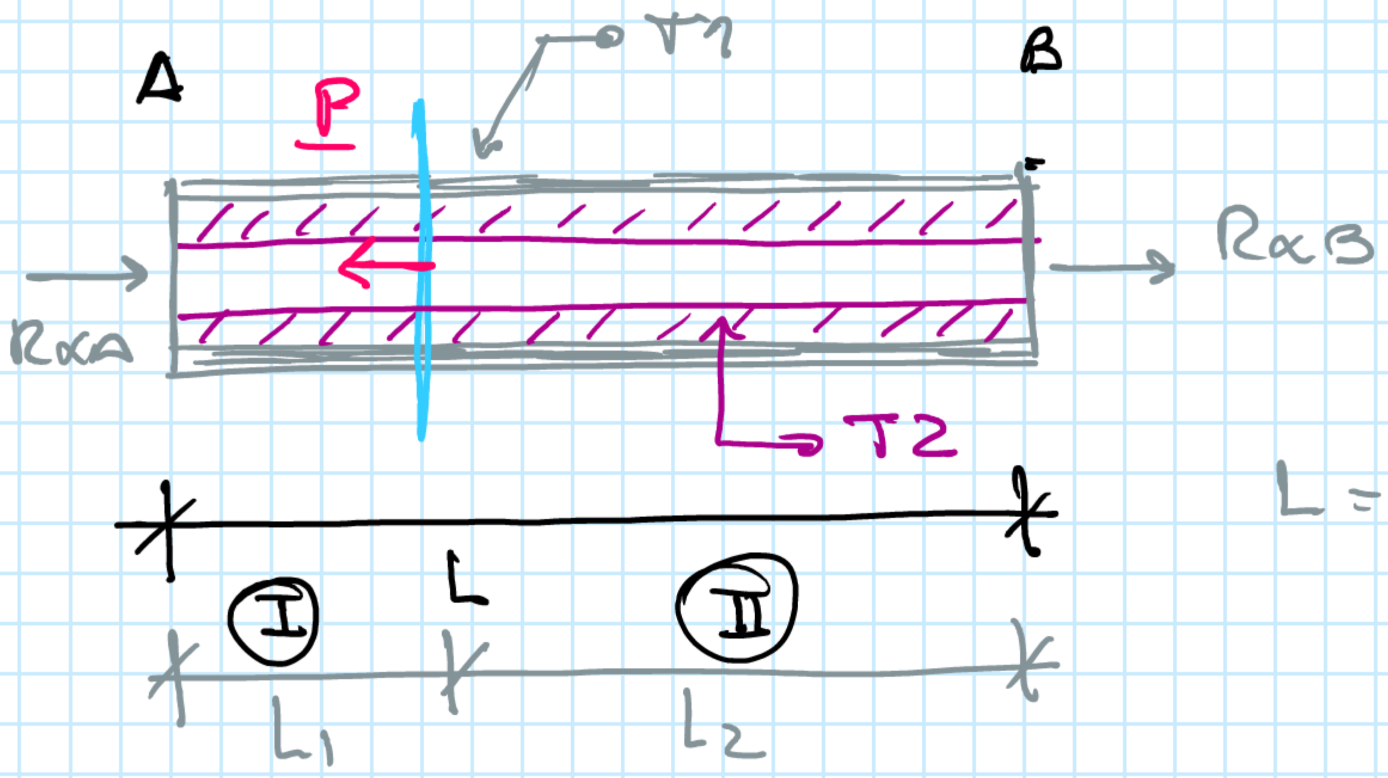
EJ. Nº 05:





Hiperestaticidad por vínculo externo e interno.

$\Sigma F_x = 0 \qquad R_{xA} - P + R_{xB} = 0.$



$L = L_1 + L_2.$

$$L_{f1} + L_{f2} = L = L_{o1} + L_{o2}$$

$$\underbrace{(L_{f1} - L_{o1})} + (L_{f2} - L_{o2}) = 0.$$

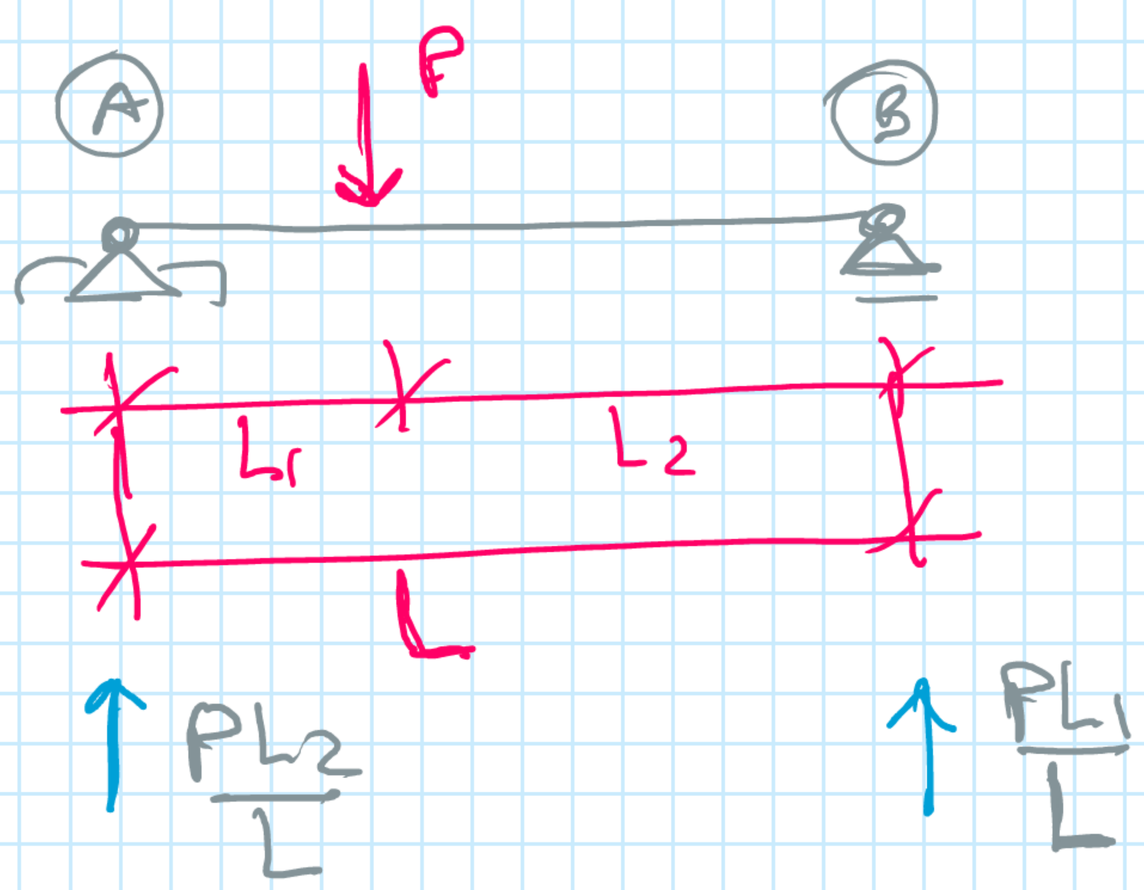
$$\Delta L_1 + \Delta L_2 = 0.$$

$$\Delta L_1 = -\frac{R_{xA}\, L_1}{EA]_m} \quad ; \quad \Delta L_2 = -\frac{(R_{xA} - P)\, L_2}{EA]_m}.$$

$$-\frac{R_{xA}\, L_1}{\cancel{EA]_m}} - \frac{(R_{xA} - P)\, L_2}{\cancel{EA]_m}} = 0.$$

$$-R_{xA}(L_1 + L_2) + P L_2 = 0. \rightarrow \boxed{R_{xA} = \frac{P L_2}{L_1 + L_2} = \frac{P L_2}{L}} \rightarrow$$

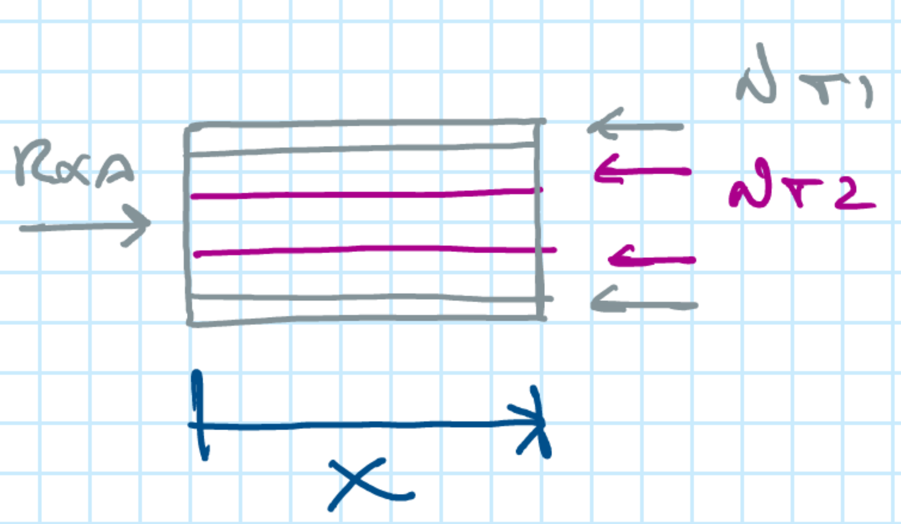
$$R_{xA} - P + R_{xB} = 0. \rightarrow \boxed{R_{xB} = \frac{P L_1}{L_1 + L_2} = \frac{P L_1}{L}} \rightarrow$$



Tramo 1

Ec. de equilibrio:

$$R_{xA} - N_{T1} - N_{T2} = 0.$$



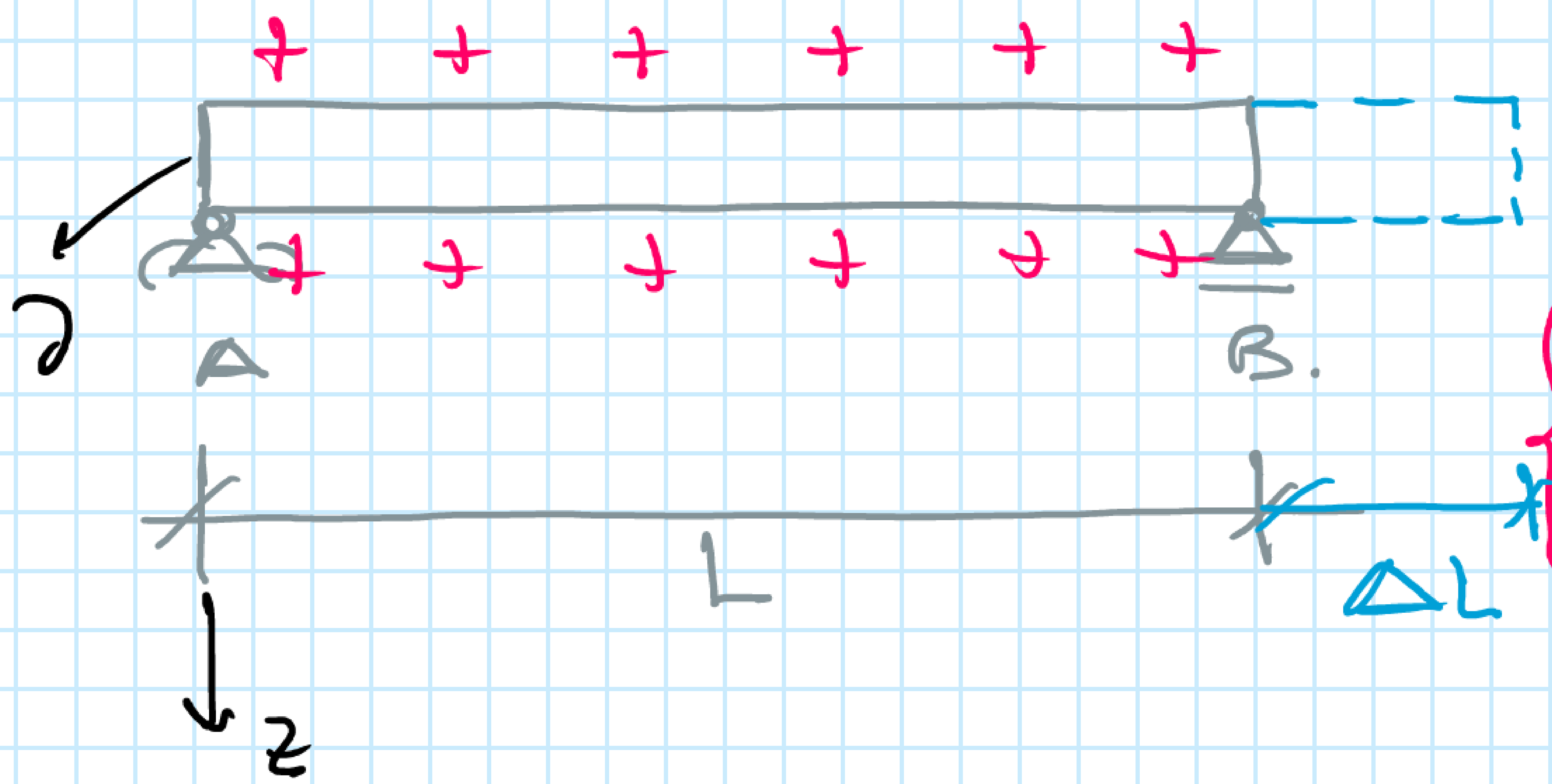
Ec. de compatibilidad:

$$\Delta L_{T1}]_1 = \Delta L_{T2}]_1$$

$$\frac{N_{T1,1} \cdot \cancel{x}}{E_1 A_1} = \frac{N_{T2,1} \cdot \cancel{x}}{E_2 A_2} \rightarrow \boxed{N_{T1,1} = N_{T2,1} \frac{E_1 A_1}{E_2 A_2}}$$

01.13 - VARIACIÓN DE TEMPERATURA:

lunes, 20 de septiembre de 2021 15:00



Variaciones de temperatura ΔT

$\downarrow$

UNIFORMES.

$$\begin{cases} \Delta T > 0 \rightarrow (+) \\ \Delta T < 0 \rightarrow (-) \end{cases}$$

$$\Delta L_x = \alpha \cdot \Delta T \cdot L_{0x}$$

- L_{0x}: Long. inicial de la barra.
- ΔT a la que se somete la barra
- α: Coeficiente de dilatación térmica

$$[\alpha] = \frac{1}{°C}$$

Sistemas isostáticos $\rightarrow$ libremente dilatables.

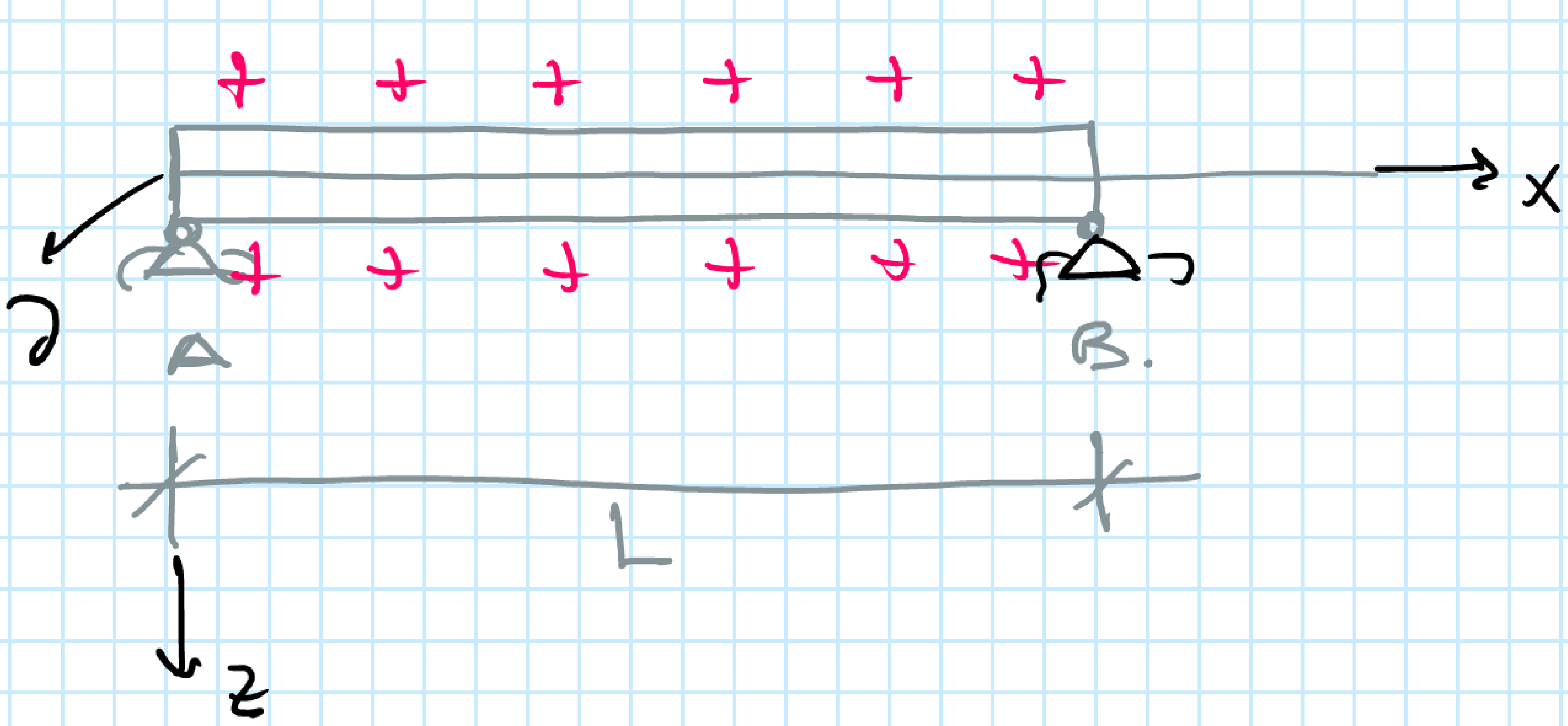
$$\varepsilon_{xx} = \frac{\Delta L}{L_0} = \alpha \cdot \Delta T.$$

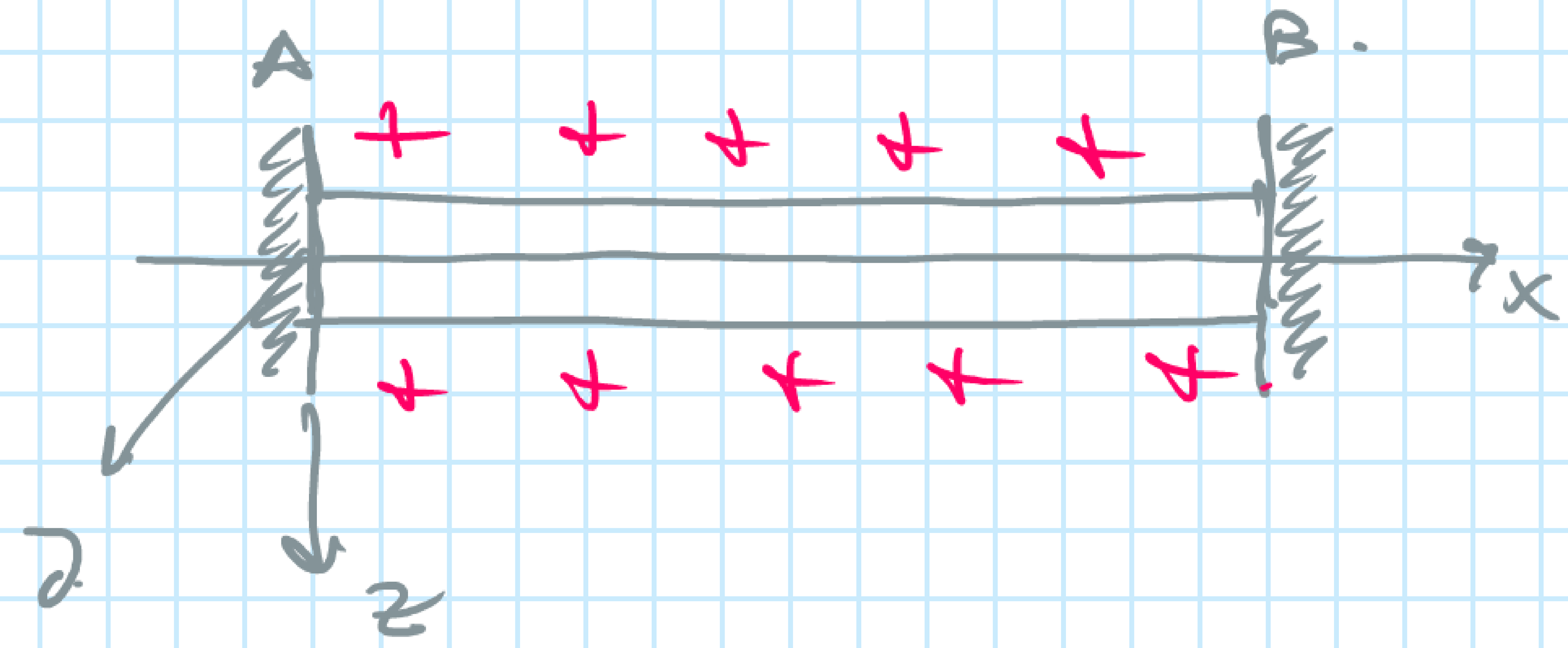
$$\Delta L_y = \alpha \cdot \Delta T \cdot L_{0y} \rightarrow \varepsilon_{yy} = \alpha \cdot \Delta T$$

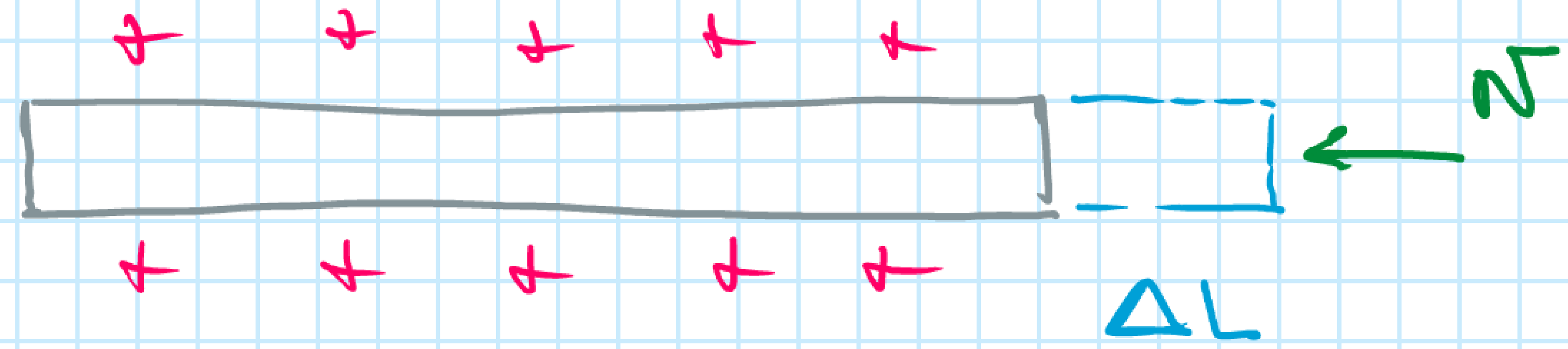
$$\Delta L_z = \alpha \, \Delta T \, L_{0y} \rightarrow \varepsilon_{zz} = \alpha \, \Delta T$$

01.13 - VARIACIÓN DE TEMPERATURA:

lunes, 20 de septiembre de 2021 15:00







$$\Delta T \rightarrow \Delta L\big]_{\Delta T} = \alpha \cdot \Delta T \cdot L_o$$

$$N \rightarrow \Delta L\big]_{N} = \frac{N \cdot L_o}{EA}$$

$$\Delta L\big]_{\Delta T} + \Delta L\big]_{N} = 0.$$

$$\alpha \Delta T L_o + \frac{N L_o}{EA} = 0.$$

$$N = - \alpha \Delta T \cancel{L_o} \cdot \frac{EA}{\cancel{L_o}} \rightarrow \boxed{N = -\alpha \Delta T \cdot EA}$$

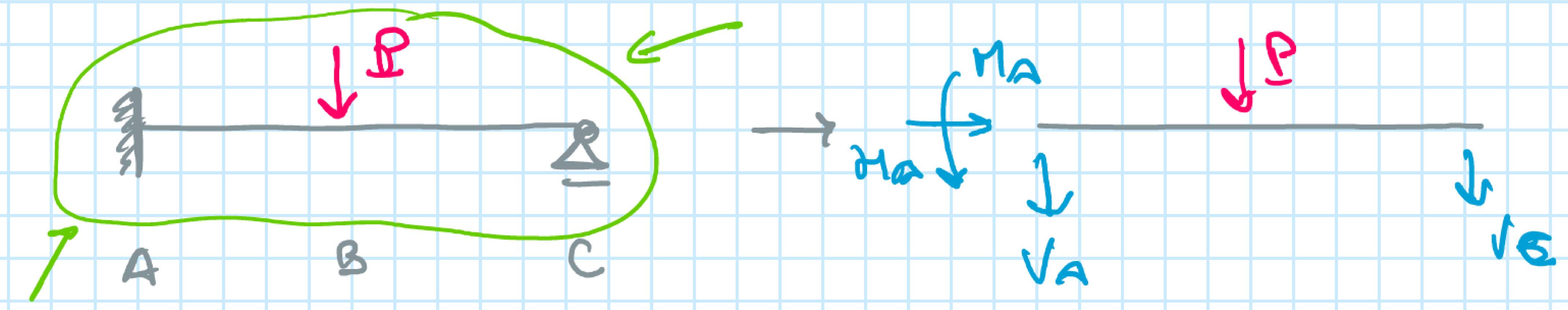
$E > 0$; $A > 0$; $\alpha > 0$.

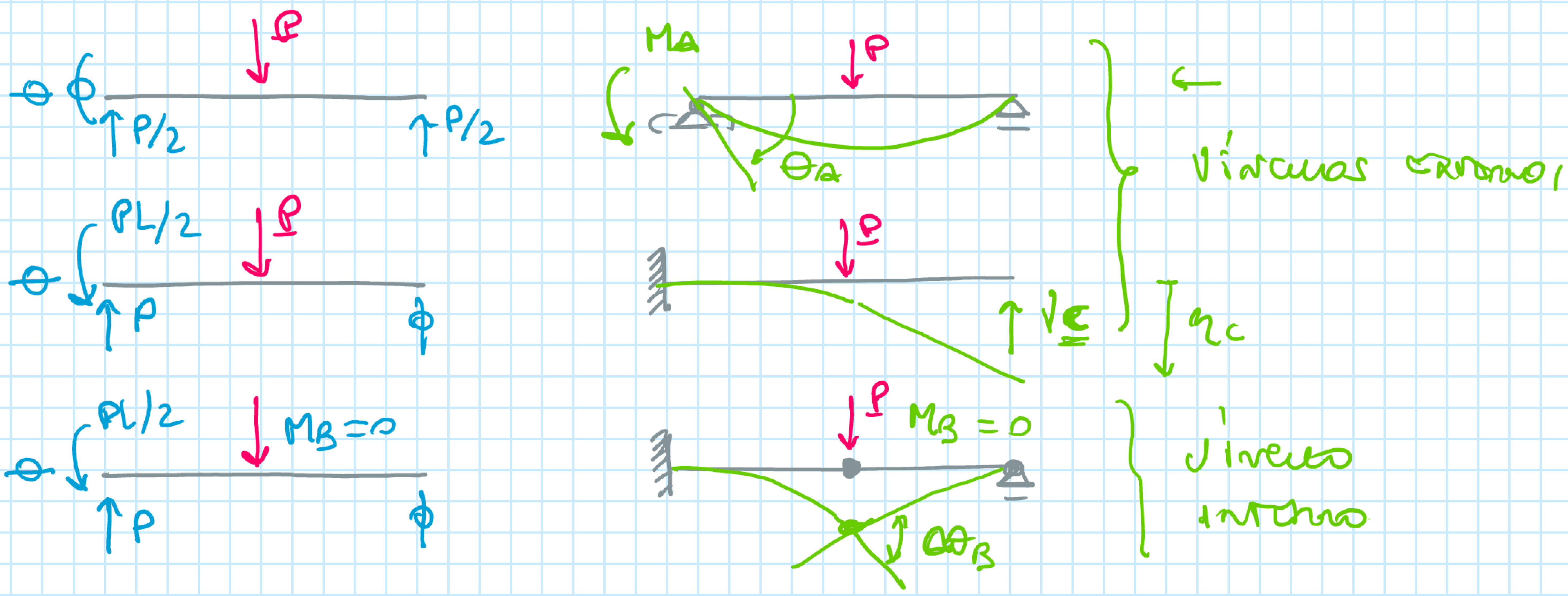
- Si $\Delta T > 0 \rightarrow N < 0$ compresión.
- Si $\Delta T < 0 \rightarrow N > 0$ Tracción.

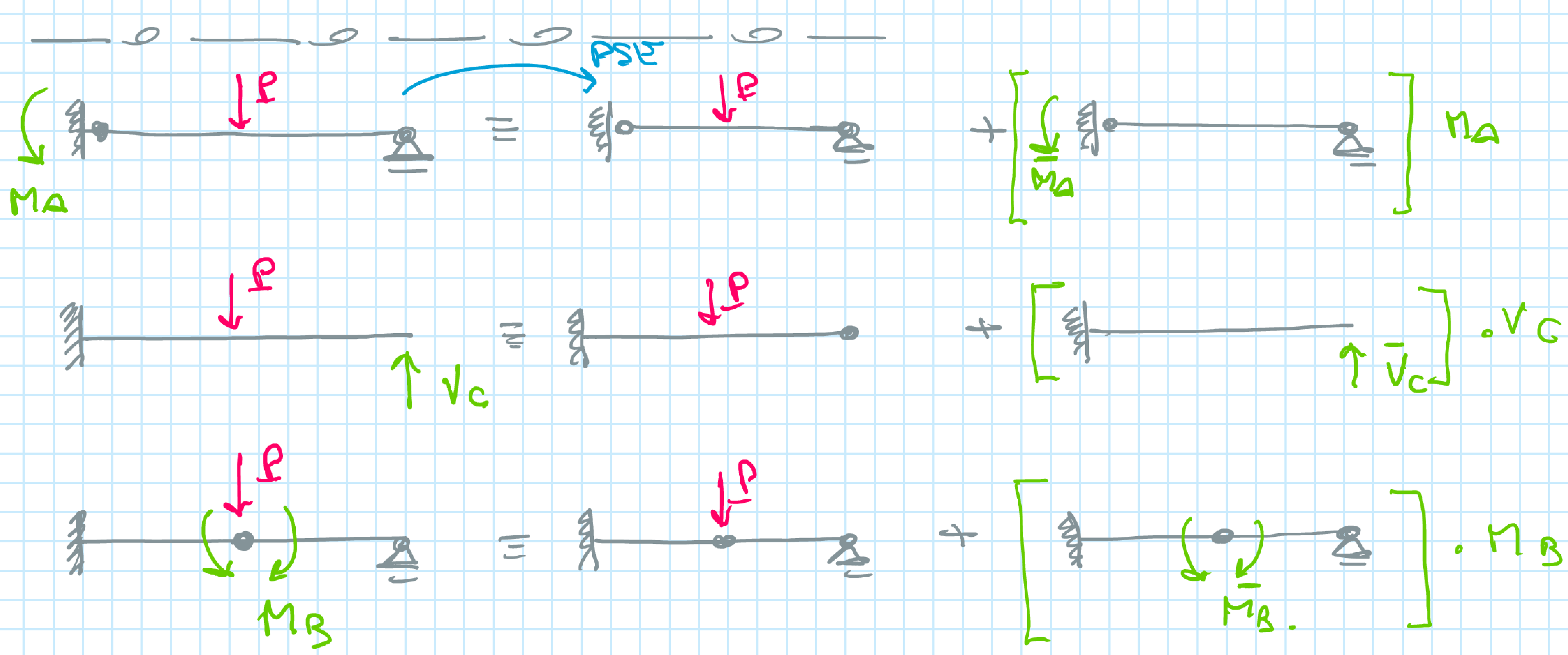
ΔT	ISOSTÁTICO (LIBREM. DILATABLE)	HIPERESTÁTICO
RVE	$= 0$	$\neq 0$
SI ≡ RVI.	$= 0$.	$\neq 0$
Tensiones	$= 0$.	$\neq 0$.
$\delta(x)$	$\neq 0$	$\neq 0$ / $= 0$
ε_x	$\neq 0$.	$\neq 0$ / $= 0$

01.14 - MÉTODO DE LAS INCÓGNITAS ESTÁTICAS - MIE:

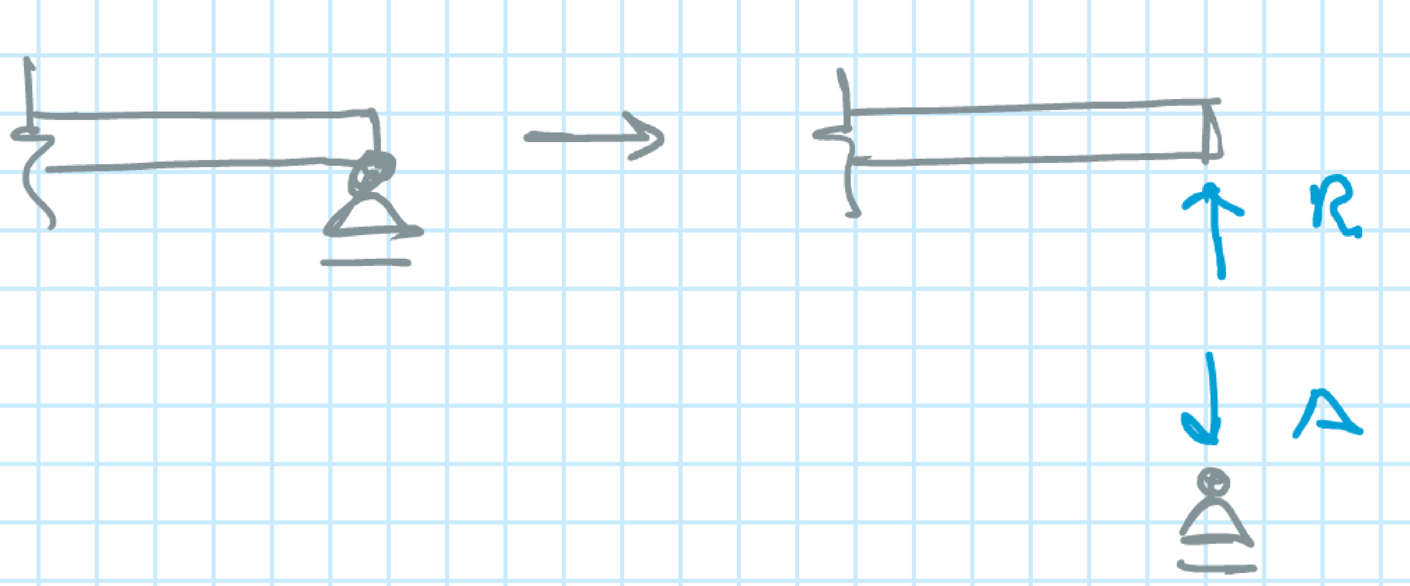
lunes, 20 de septiembre de 2021 15:15







PRV + PA,R.



ECS DE COMPATIBILIDAD DE LOS DESPLAZAMIENTOS:

$$
\begin{cases}
\theta_A^H = \theta^0_{A,CD} + \theta^0_{A,\bar{M}_A} \cdot M_A = 0 \\
\eta_C^H = \eta^0_{C,CD} + \eta^0_{C,\bar{V}_C} \cdot V_C = 0 \\
\Delta\theta_B^H = \Delta\theta^0_{B,CD} + \Delta\theta^0_{B,\bar{M}_B} \cdot M_B = 0.
\end{cases}
$$

→ Teorema de los trabajos virtuales.

↓

Aplicado a 'SA' → luego lo vemos a continuación.