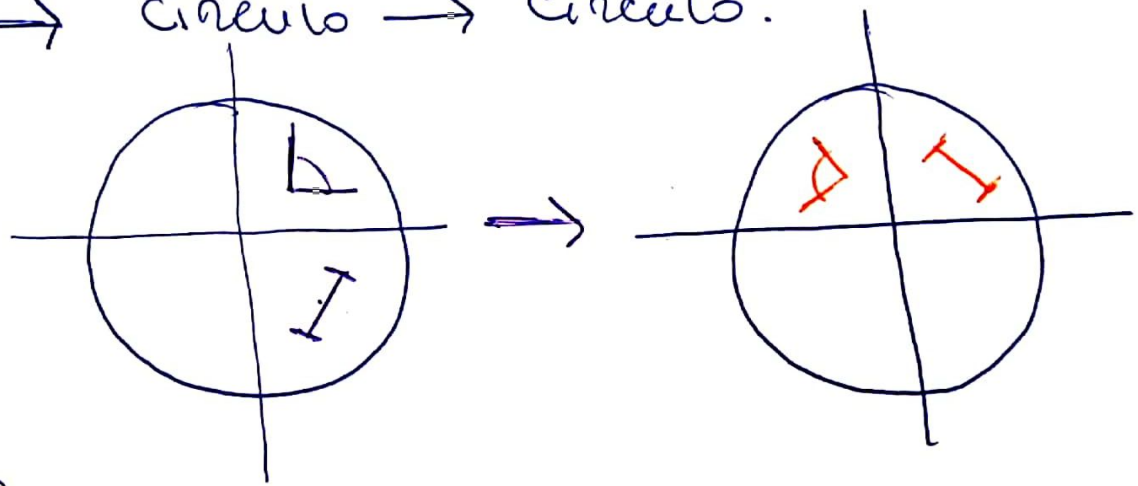
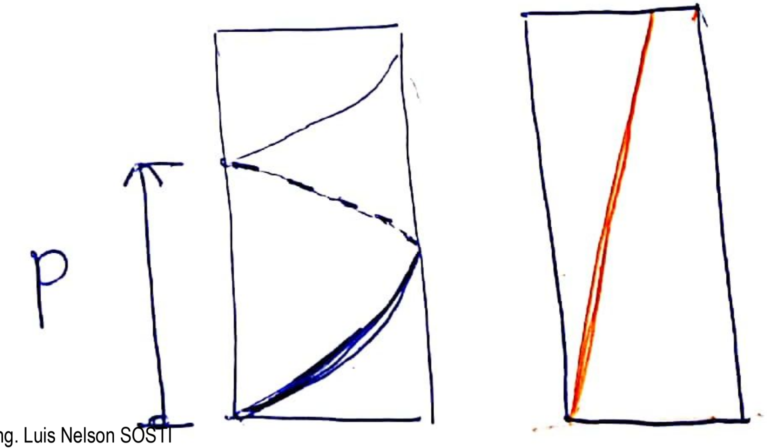
1) SECCIONES PLANAS ⟶ NO SE ALABEAN.

2) " MANTENÍAN SU FORMA } ⟶ CÍRCULO ⟶ CÍRCULO.



3) " GIRABAN ALREDEDOR DEL EJE } ⟶ I) GENERATRICES SE TRANSFORMABAN EN HÉLICES DE PASO MUY GRANDE.

II) LAS SECCIONES EXPERIMENTAN GIROS RELATIVOS UNAS RESPECTO DE LAS OTRAS.



1°) 5°) 6°) ECS. EQUIVALENCIA:

LAS SECCIONES SE MANTIENEN PLANAS → NO SE ALABEAN → LAS FIBRAS VARÍAN SU LONGITUD ???

2 COSAS

I) $\varepsilon = 0$ → $\Delta L = 0$

II) $\varepsilon = \text{CTES}$ → $\Delta L_i = \text{CTES.}$

1° CASO

$\varepsilon = 0$

$\sigma_x = E \cdot \underbrace{\varepsilon_x}_{=0} \rightarrow \sigma_x = 0 \rightarrow$ $\left.\begin{array}{l} 1°) \\ 5°) \\ 6°) \end{array}\right\}$ SE SATISFACEN

2º CASO

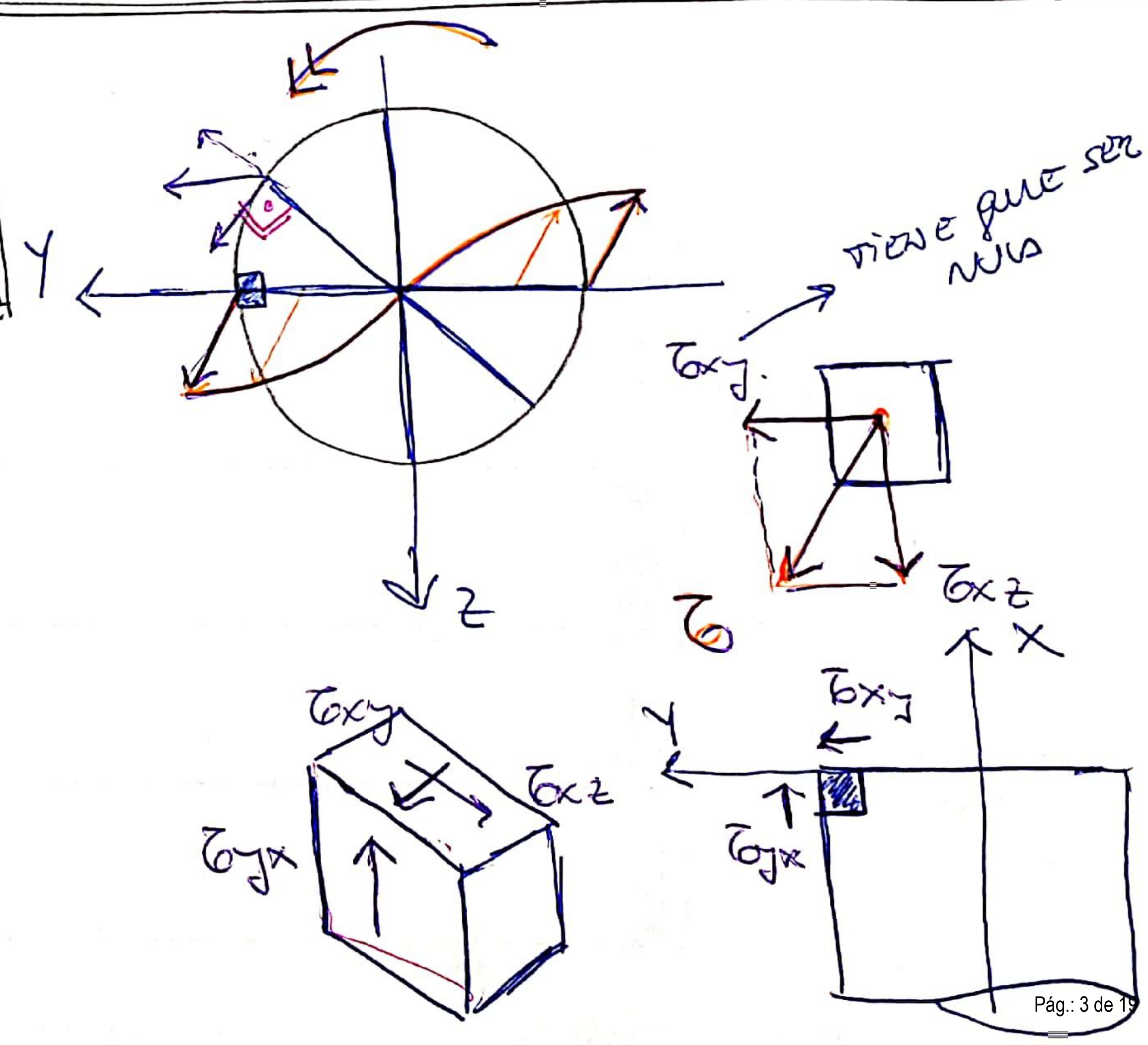
$\varepsilon_x = cte \rightarrow \sigma_x = E \cdot \varepsilon_x = cte \rightarrow$ 1º) $0 = \int_A \sigma_x \cdot dA = \sigma_x \int_A dA$

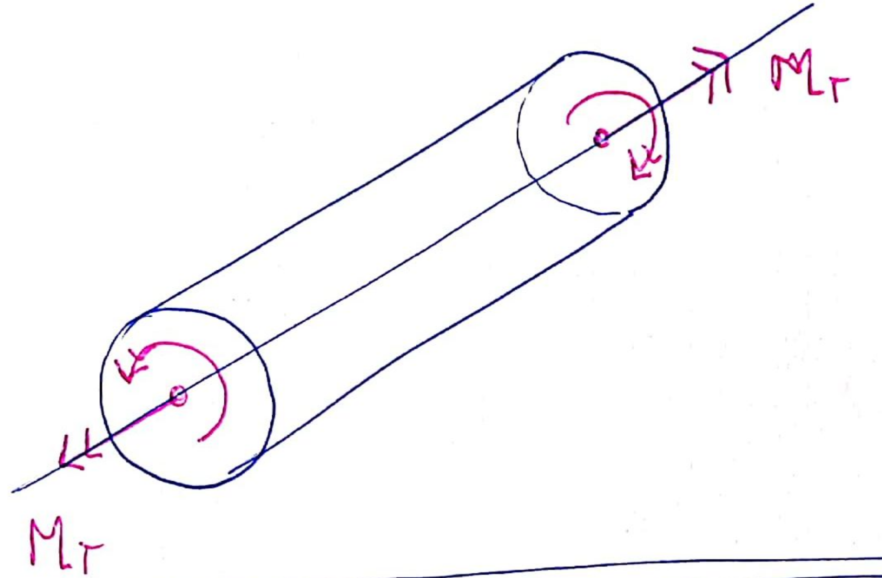
$\boxed{\sigma_x = 0} \longleftarrow 0 = \int_A dA$???

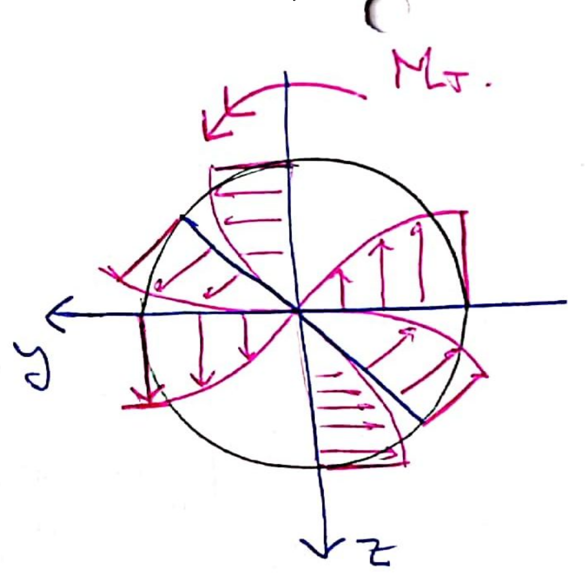
$N = 0$	$M_x \neq 0$
$Q_y = 0$	$M_y = 0$
$Q_z = 0$	$M_z = 0$

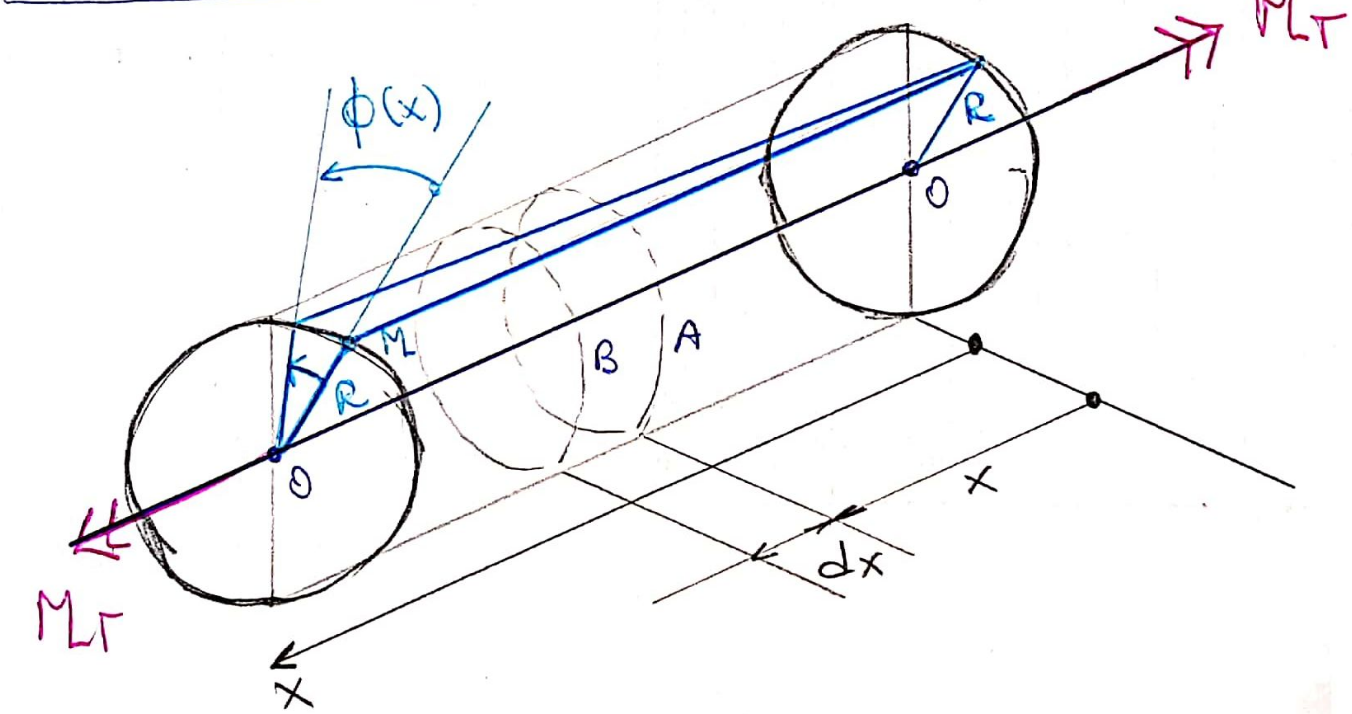
2º) $Q_y = \int_A \tau_{xy} \cdot dA = 0$

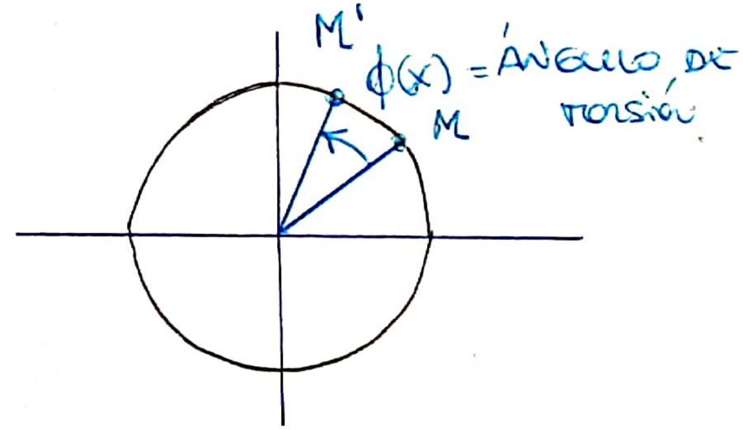
3º) $Q_z = \int_A \tau_{xz}\, dA = 0$

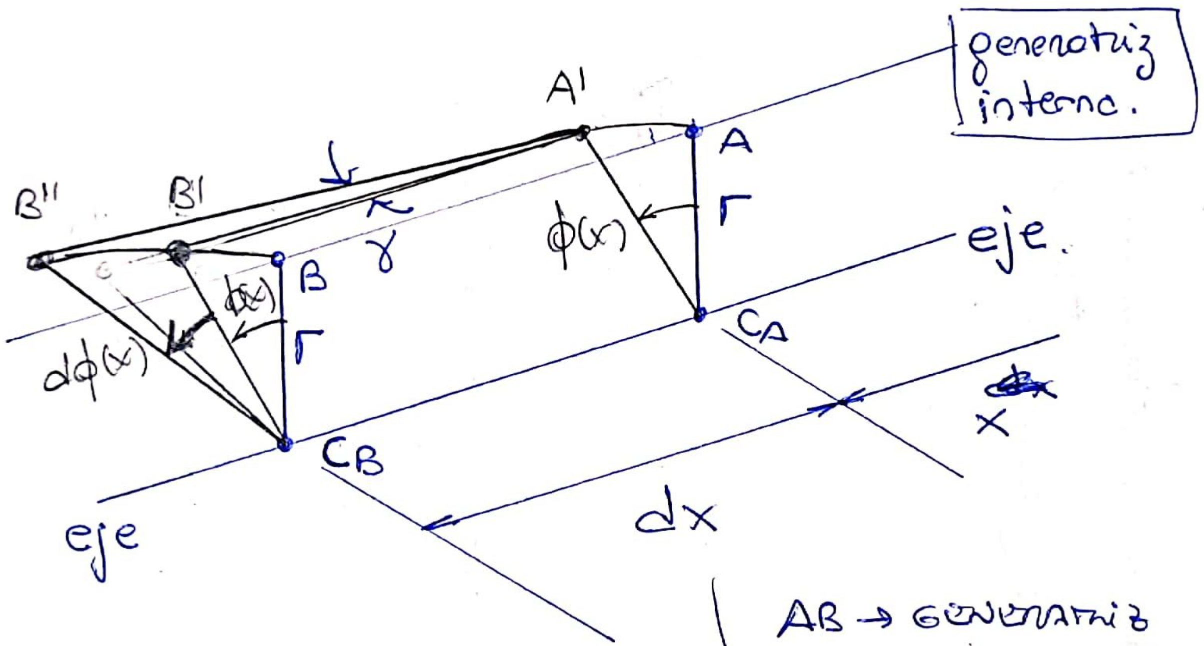



M_T
M_T


M_T.
y
z


M_T
M_T
$\phi(x)$
M
R
O
B
A
R
O
x
dx
x


M'
M
$\phi(x)$ = ÁNGULO DE TORSIÓN



$$\phi_A = \phi(x)$$

$$\phi_B = \phi(x) + d\phi(x)$$

$$\phi_B - \phi_A = d\phi(x)$$

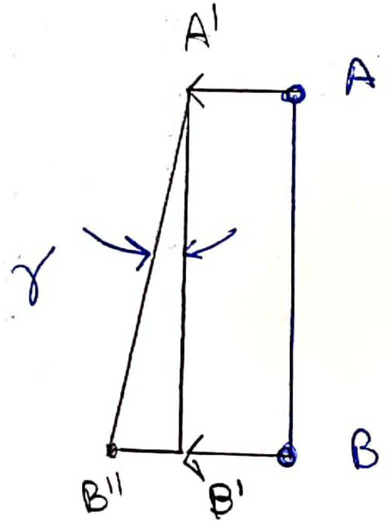
$$\overset{\frown}{B'B''} = \gamma \cdot dx.$$

$$\overset{\frown}{B'B''} = r \cdot d\phi(x)$$

$$\gamma \cdot dx = r \cdot d\phi(x).$$

$$\gamma = r \cdot \underbrace{\boxed{\frac{d\phi(x)}{dx}}}_{\text{CTE}} = r \cdot \underline{\chi(x)}$$

CURVATURA DE TORSIÓN



AB → GENERATRIZ INICIAL S/DEFORMAR

A'B'' → GENERATRIZ DEFORMADA

$0 \leq r \leq R$ $\quad \gamma \rightarrow$ VARIACIÓN LINEAL

$r = 0 \rightarrow \gamma = 0.$

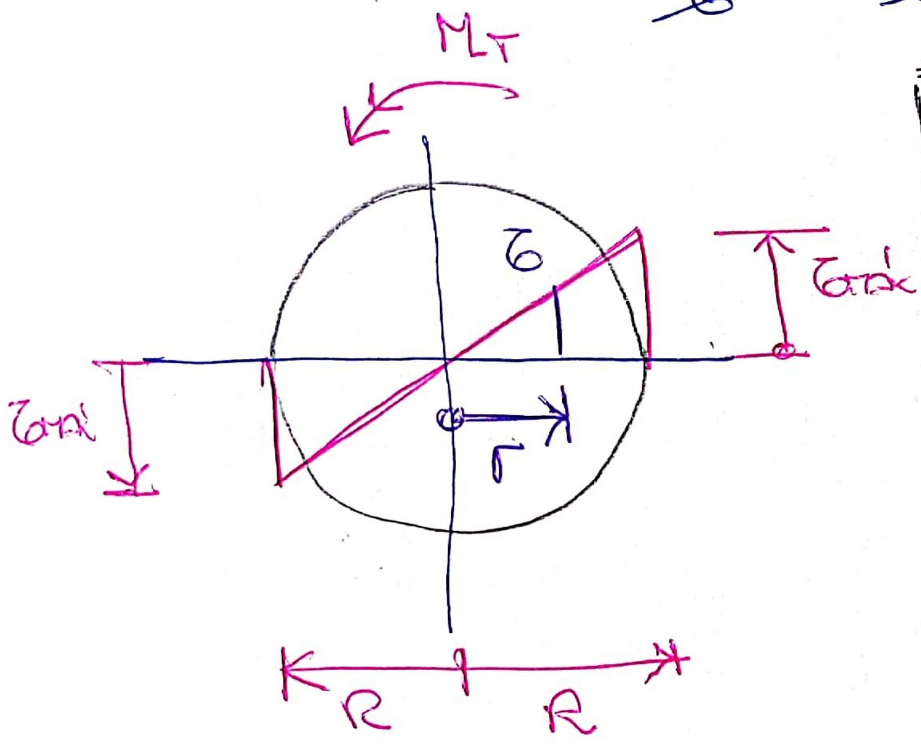
$r = R \rightarrow \gamma = \gamma_{máx} = R \cdot \dfrac{d\phi(x)}{dx}$

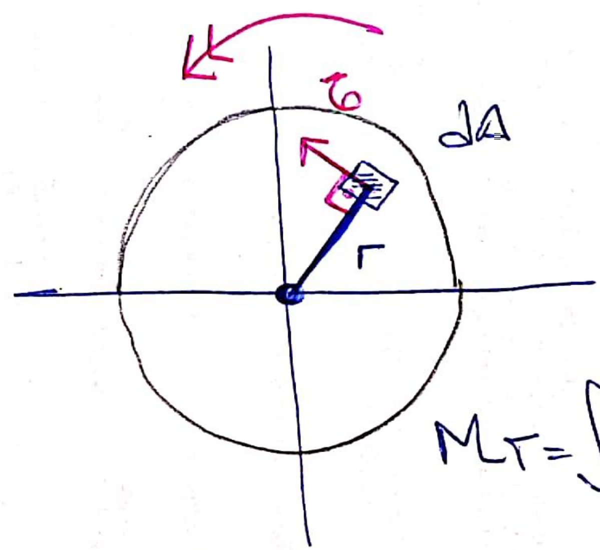
$$\left\{ \begin{aligned} \frac{\gamma}{r} &= \frac{d\phi(x)}{dx} \\ \frac{\gamma_{máx}}{R} &= \frac{d\phi(x)}{dx} \end{aligned} \right\} \longrightarrow \frac{\gamma}{r} = \frac{\gamma_{máx}}{R} \rightarrow \boxed{\gamma = \gamma_{máx} \cdot \frac{r}{R}}$$

$$\tau = G \cdot \gamma$$

$$\gamma = \frac{\tau}{G}$$

$$\frac{\tau}{\cancel{G}} = \frac{\tau_{máx}}{\cancel{G}} \cdot \frac{r}{R} \rightarrow \boxed{\tau = \tau_{máx} \cdot \frac{r}{R}}$$





$$dF = \tau \cdot dA$$

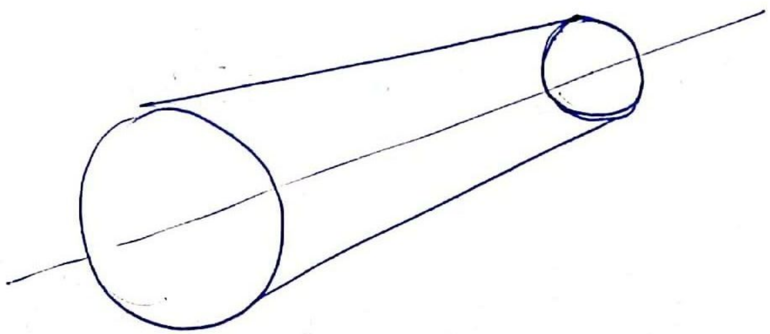
$$dM_T = dF \cdot r$$

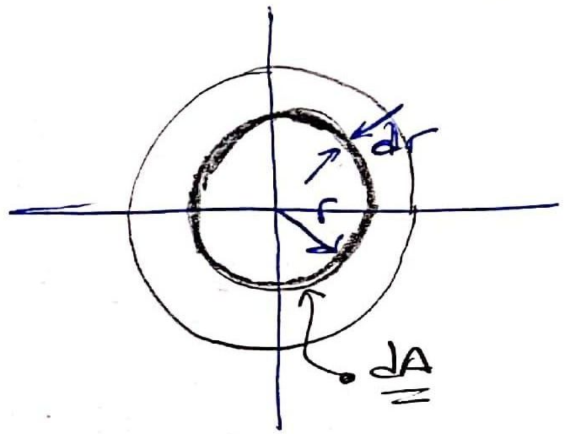
$$dM_T = \tau \cdot r \cdot dA$$

$$M_T = \int_A dM_T = \int_A \tau \cdot r \cdot dA$$

$$M_T = \int_A \frac{\tau_{máx}}{R} \cdot r^2 \cdot dA = \frac{\tau_{máx}}{R} \underbrace{\int r^2 dA}_{I_P}$$

$$M_T = \frac{\tau_{máx}}{R} \cdot I_p$$





$$\tau_{máx} = \frac{M_T(x)}{I_p} \cdot R$$

$$dA = 2\pi r\, dr$$

$$\boxed{\tau_{máx}(x, R) = \frac{M_T(x)}{I_p(x)} \cdot R(x)}$$

$$dI_p = r^2\, dA = 2\pi r^3\, dr$$

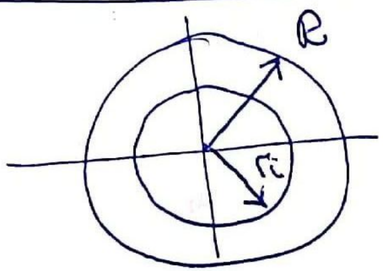
$$I_p = \int dI_p = \int_0^R 2\pi r^3\, dr$$

$$I_p = \cancel{2}\pi \frac{r^4}{\cancel{4}_2}\Big|_0^R = \frac{\pi R^4}{2} = \frac{\pi}{2}\left(\frac{D}{2}\right)^4$$

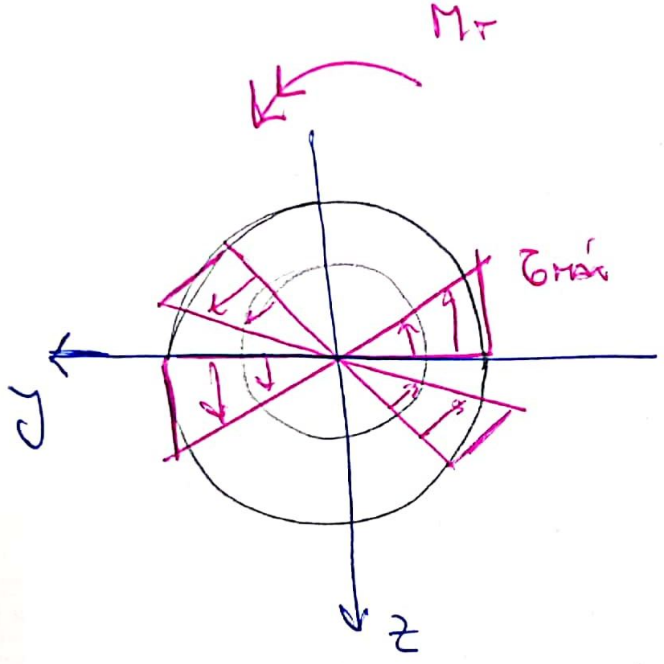
$$\tau_{máx} = \tau \cdot \frac{R}{r}$$

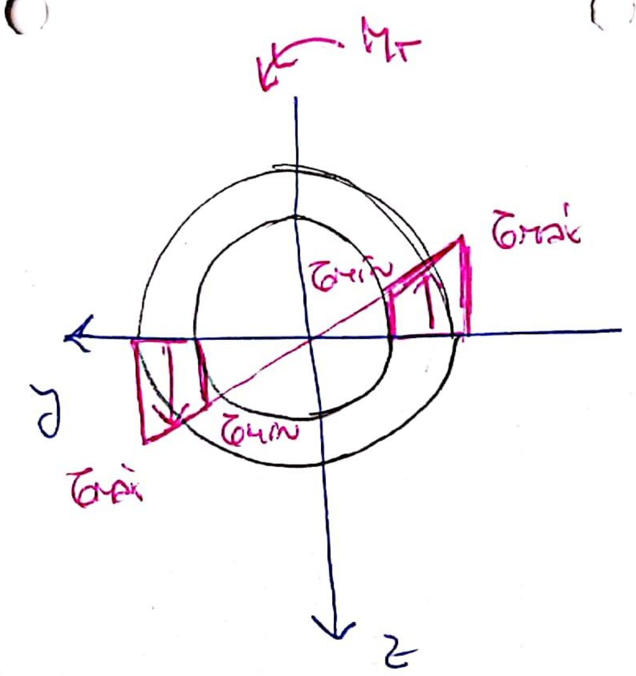
$$\boxed{\tau(x, r) = \frac{M_T(x)}{I_p(x)} \cdot r\,\cancel{(x)}}$$

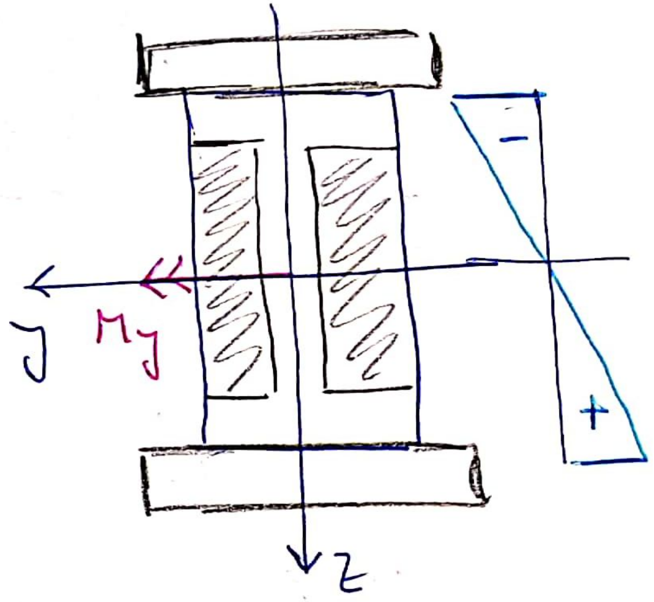
$$\boxed{I_p = \frac{\pi R^4}{2} = \frac{\pi D^4}{32}}$$



$$\boxed{I_p = \frac{\pi}{2}\left(R^4 - r_i^4\right) = \frac{\pi}{32}\left(D^4 - d_i^4\right)}$$







ÁNGULO DE TORSIÓN:

$$\gamma_{máx} = R \cdot \frac{d\phi(x)}{dx} = \frac{\tau_{máx}}{G}$$

$$d\phi(x) = \frac{\tau_{máx}}{G} \frac{1}{R} \cdot \cancel{dx}$$

$$d\phi(x) = \frac{M_T(x)}{G \cdot I_P} \cdot \frac{\cancel{R}}{\cancel{R}} \cdot dx$$

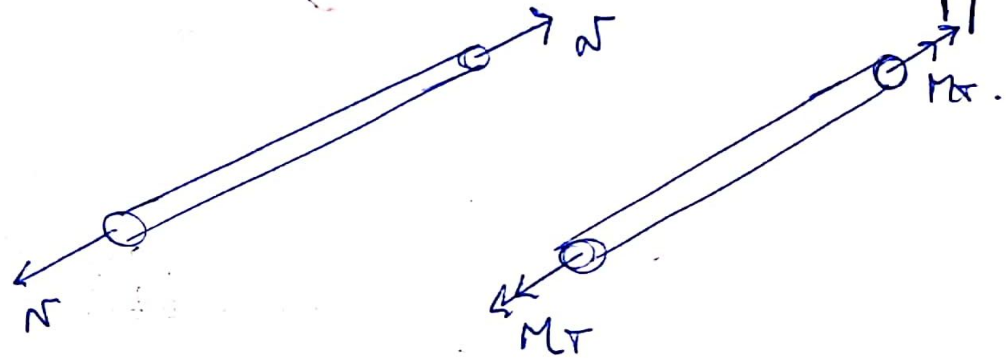
$$d\phi(x) = \frac{M_T(x)}{G\, I_P} \cdot dx.$$

$$\boxed{\phi(x) = \int d\phi(x) = \int \underbrace{\frac{M_T(x)}{G\, I_P}}_{\text{RIGIDEZ A LA TORSIÓN.}} \cdot dx}$$

Si $M_T(x) = cte$ $\quad G = cte \quad I_P = cte$

$$\boxed{\phi(x) = \frac{M_T}{G\, I_P} \cdot x} \qquad \boxed{\phi(x=L) = \frac{M_T L}{G\, I_P}}$$

$\phi(x)$ = ángulo de torsión.



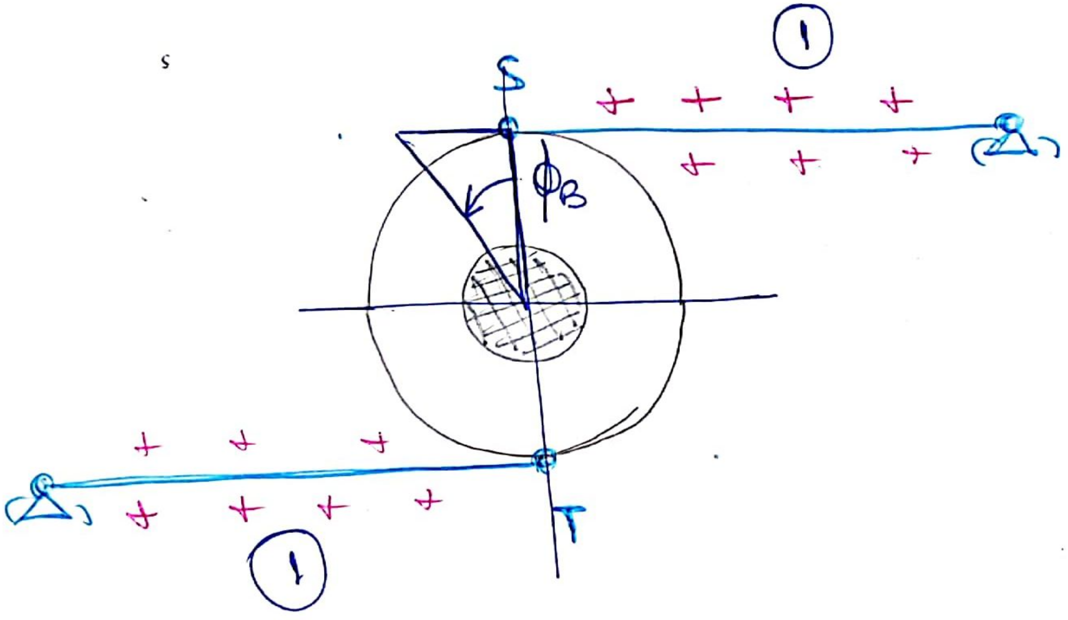
$$\boxed{\chi(x) = \frac{d\phi(x)}{dx}}$$ curvatura de torsión

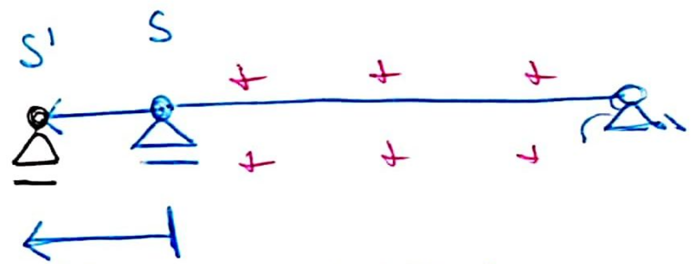
~~M_T = cte G = cte Ip = cte~~

$$\boxed{\chi(x) = \frac{M_T}{G \cdot I_p}}$$

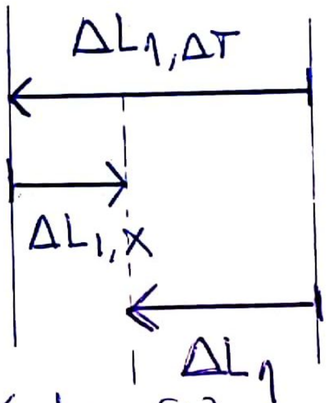
	Solicitación axil	Solicitación por torsión
ME	$N = N(x)$ $\sigma_x(x) = \dfrac{N(x)}{A(x)}$	$M_T = M_T(x)$. $\tau_{máx}(x) = \dfrac{M_T(x)}{I_p(x)} \cdot R$ $\tau(x,r) = \dfrac{M_T(x)}{I_p(x)} \cdot r$
MC	$\delta(x) = \Delta L(x) = \displaystyle\int \frac{N(x)}{E A(x)} dx$ $\varepsilon_x(x) = \dfrac{d\Delta L(x)}{dx}$	$\phi(x) = \displaystyle\int \frac{M_T(x)}{G\, I_p(x)} \cdot dx$. $\chi(x) = \dfrac{d\phi(x)}{dx}$ $\gamma = r \dfrac{d\phi(x)}{dx}$

EJERCICIO Nº 5:





$\Delta L_{1,\Delta T} = \alpha . \Delta T . L_1$



1) ECS. DE ~~EQU~~ COMPATIBILIDAD:

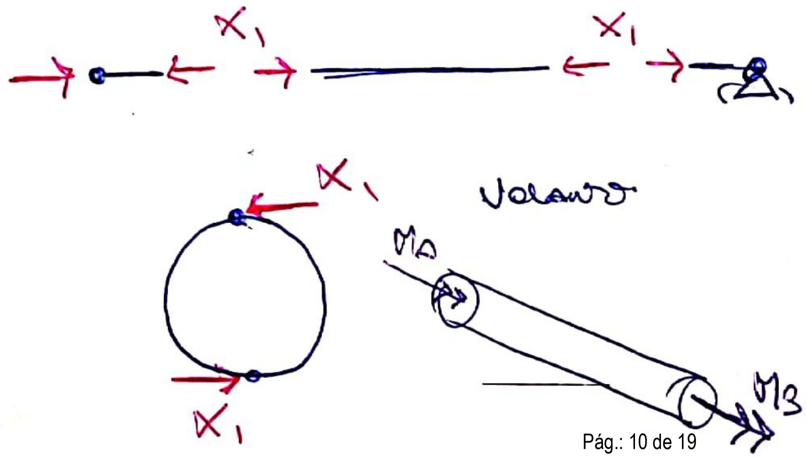
$$\Delta L_1 = \Delta L_{1,\Delta T} + \Delta L_{1,X} = \alpha \Delta T, L_1 + \frac{X_1 L_1}{E_1 A_1} \quad (1)$$

$$\Delta L_1 = \phi_B . \frac{DV}{2} \quad (2)$$

- Si $\phi_B > 0 \rightarrow \Delta L_1 > 0.$
- Si $\phi_B < 0 \rightarrow \Delta L_1 < 0.$

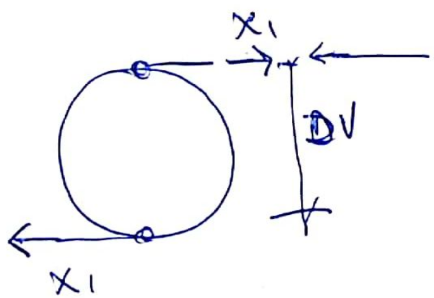
2) ECS. DE EQUILIBRIO

VISTO DESDE LA BARRA



$$\begin{cases} M_B = -X_1 \cdot DV & (3) \\ M_A + M_B = 0 & (4) \end{cases}$$

- Si $X_1 > 0 \rightarrow M_B < 0$.
- Si $X_1 < 0 \rightarrow M_B > 0$.



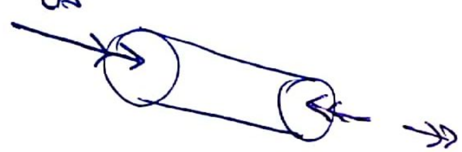
I) ECS. EQUILIBRIO

II) ECS. COMPATIBILIDAD

III) ECS. CONSTITUTIVAS

3) ECS. CONSTITUTIVAS

$$\phi_B = \int \frac{M_T(x)}{G \cdot I_P} \cdot dx. = -\frac{M_A L}{G \cdot I_P} \quad (5).$$



$$\phi_B \cdot \frac{DV}{2} = \alpha \cdot \Delta T_1 L_1 + \frac{X_1 L_1}{E_1 A_1}. \quad \text{DE (1) y (2)}$$

$$M_A - X_1 \cdot DV = 0 \rightarrow M_A = X_1 DV \quad \text{DE (3) y (4).}$$

$$\phi_B = -\frac{X_1 \cdot DV \cdot L}{G \cdot I_P}$$

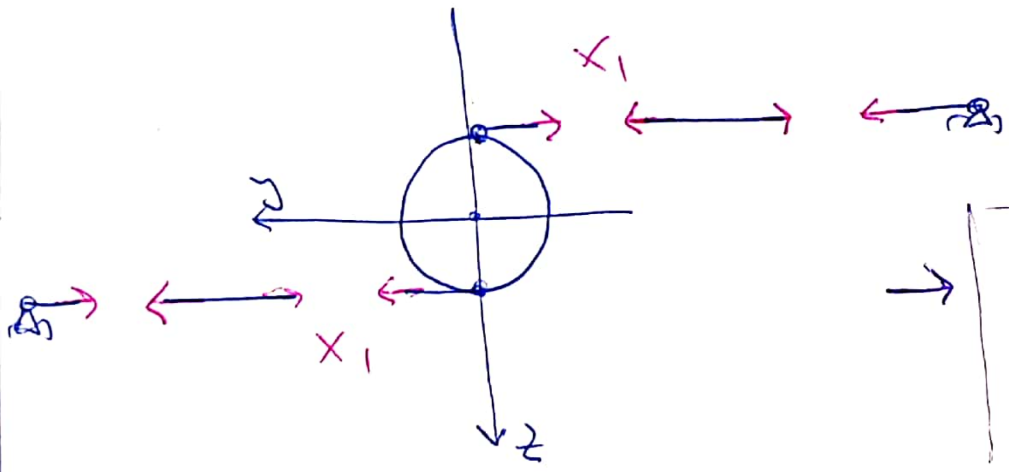
$$-\frac{X_1 \cdot DV^2 L}{2G I_P} = \alpha \cdot \Delta T_1 L_1 + \frac{X_1 L_1}{E_1 A_1}$$

$$-\alpha \cdot \Delta T_1 L_1 = \frac{X_1 L_1}{E_1 A_1} + \frac{X_1 DV^2 L}{2G I_P} =$$

$$= X_1 \left(\frac{L_1}{E_1 A_1} + \frac{DV^2 L}{2G I_P} \right)$$

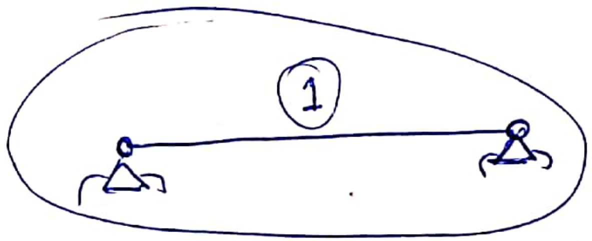
$$\boxed{X_1 = \frac{-\alpha \cdot \Delta T_1 \, L_1}{\frac{L_1}{E_1 A_1} + \frac{DV^2 L}{2 G I_p}}}$$

$\rightarrow$ Si $\Delta T_1 > 0 \rightarrow X_1 < 0 \rightarrow M_B > 0 \rightarrow$
$\rightarrow M_A < 0 \rightarrow \varphi_B > 0 \rightarrow \Delta L_1 > 0$



$\rightarrow$ Si $\Delta T_1 < 0 \rightarrow X_1 \rightarrow M_B \rightarrow$
$\rightarrow M_A \rightarrow \varphi_B \rightarrow \Delta L_1$

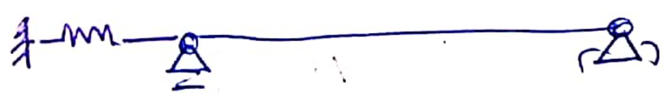
$$M_B = - \underbrace{\underbrace{X_1}_{>0} \cdot \underbrace{DV}_{>0}}_{>0}$$



$\Delta L_{1,\Delta T_1} = \alpha \cdot \Delta T_1 \, L_1$

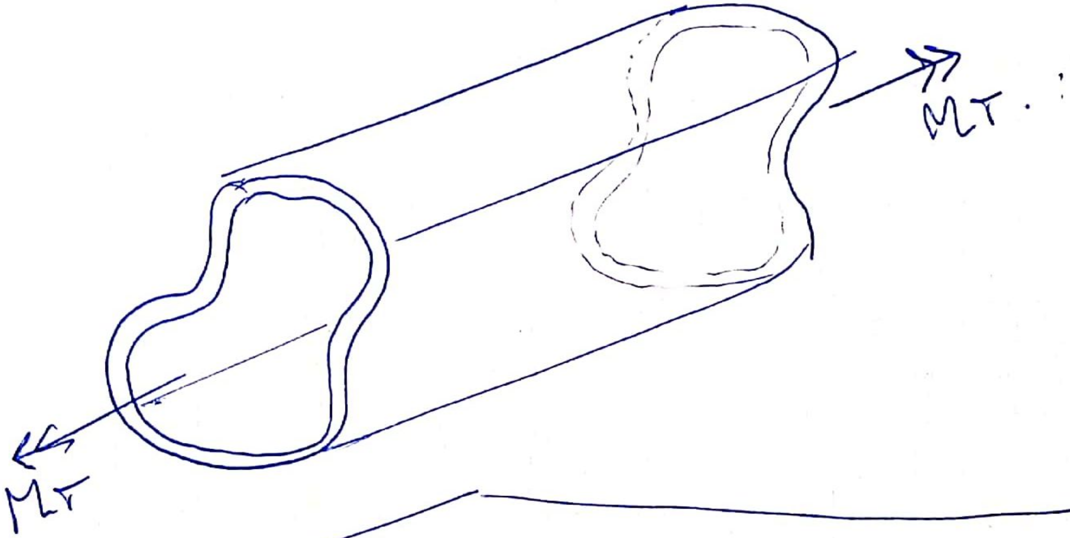
$\Delta L_{1,X_1} = \frac{X_1 L_1}{E_1 A_1}$

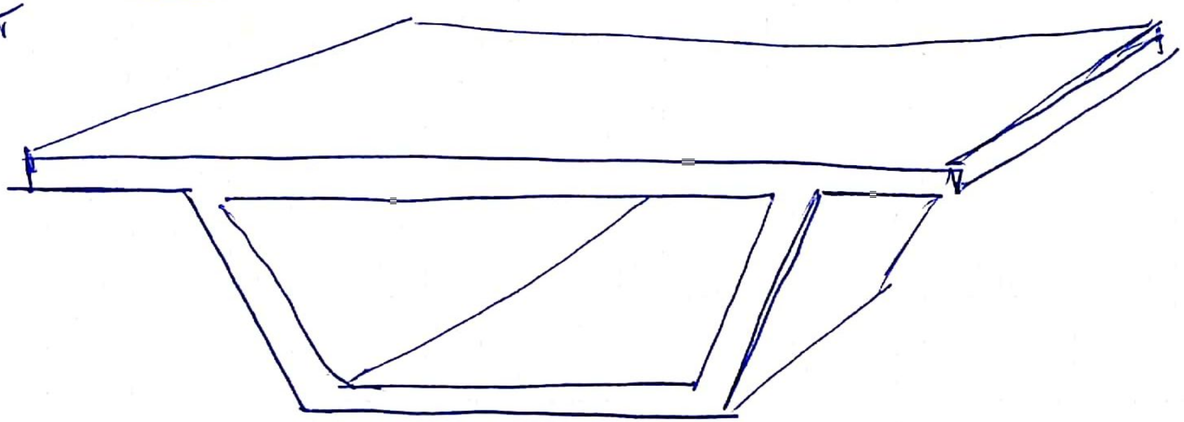
$\Delta L_{1,\Delta T_1} + \Delta L_{1,X_1} = 0 = \alpha \, \Delta T_1 L_1 + \frac{X_1 L_1}{E_1 A_1} \rightarrow$

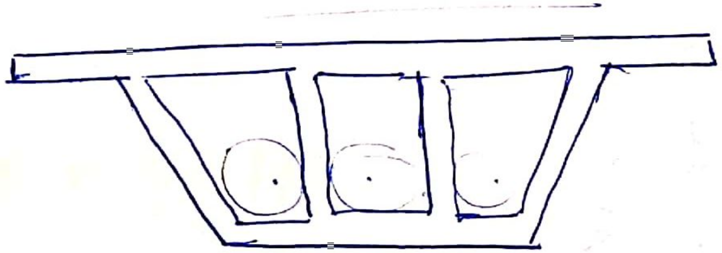


$$\boxed{X_1 = \frac{-\alpha \Delta T_1 \, L_1}{\frac{L_1}{E_1 A_1}}}$$

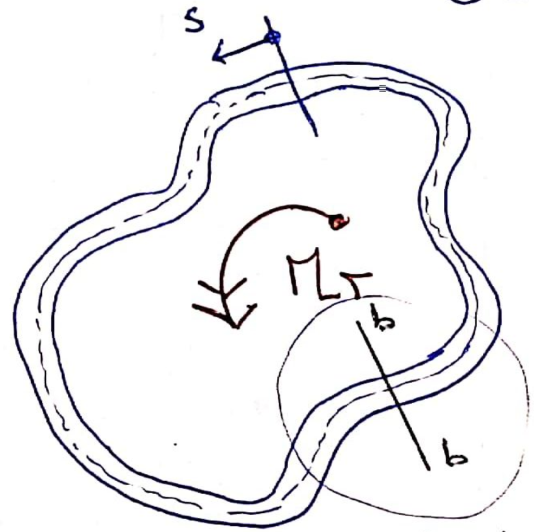
1) SECCIONES SIMPLEMENTE CONVEXAS Y DE PEQUEÑO ESPESOR:



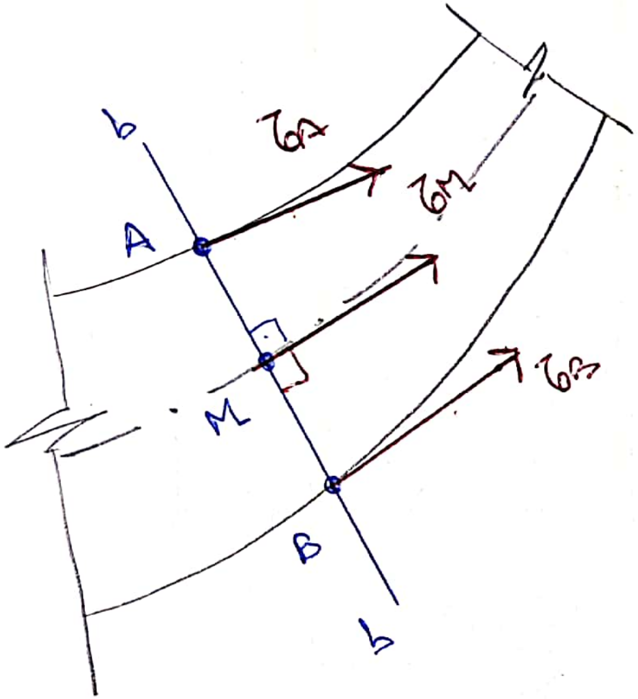


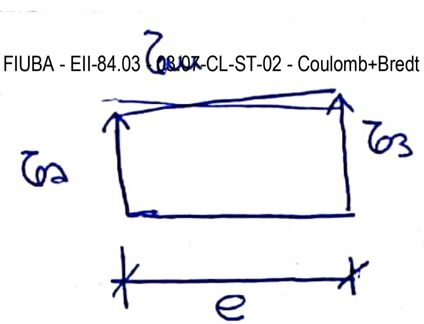


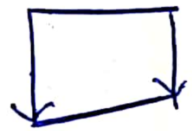
$e = e(s)$



b-b ⊥ tangente en la línea media.



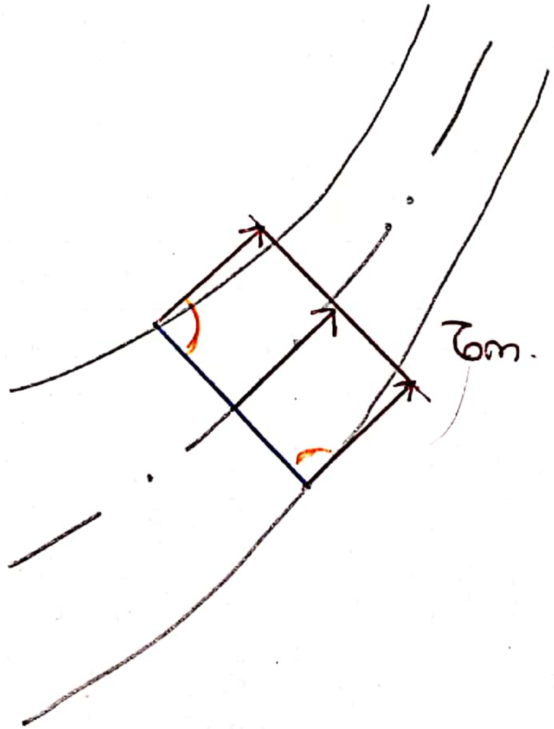




2 Suposiciones

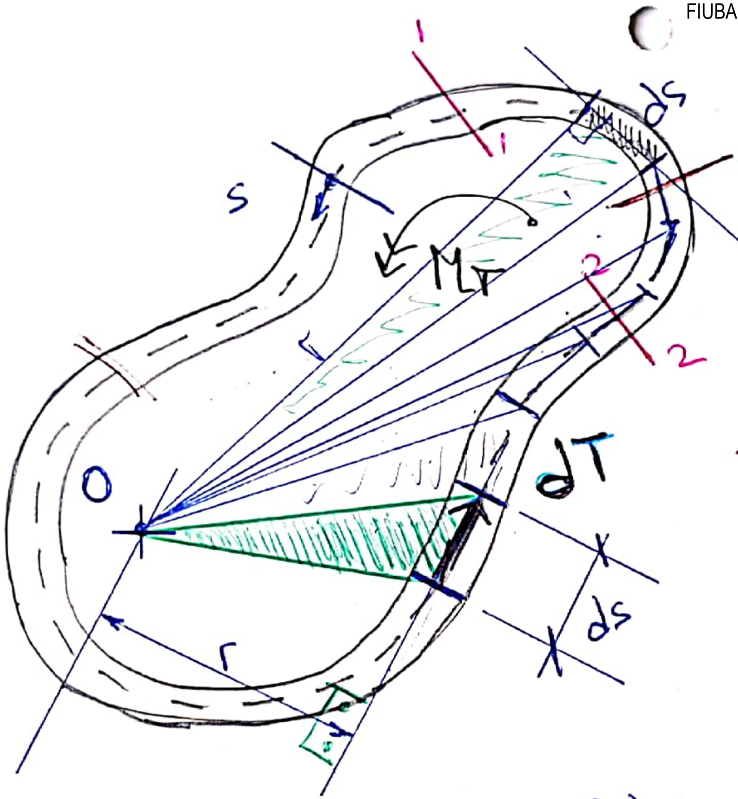
→ I) Dirección

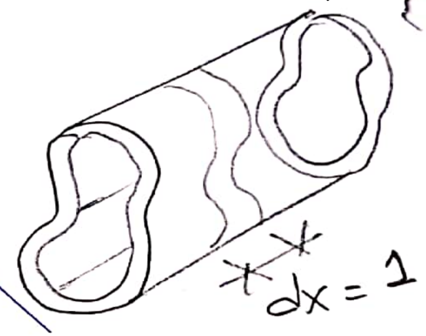
→ II) Intensidad



$$\gamma = \gamma_{máx} \cdot \frac{r}{R}$$

$$\tau_A < \tau_M < \tau_B.$$





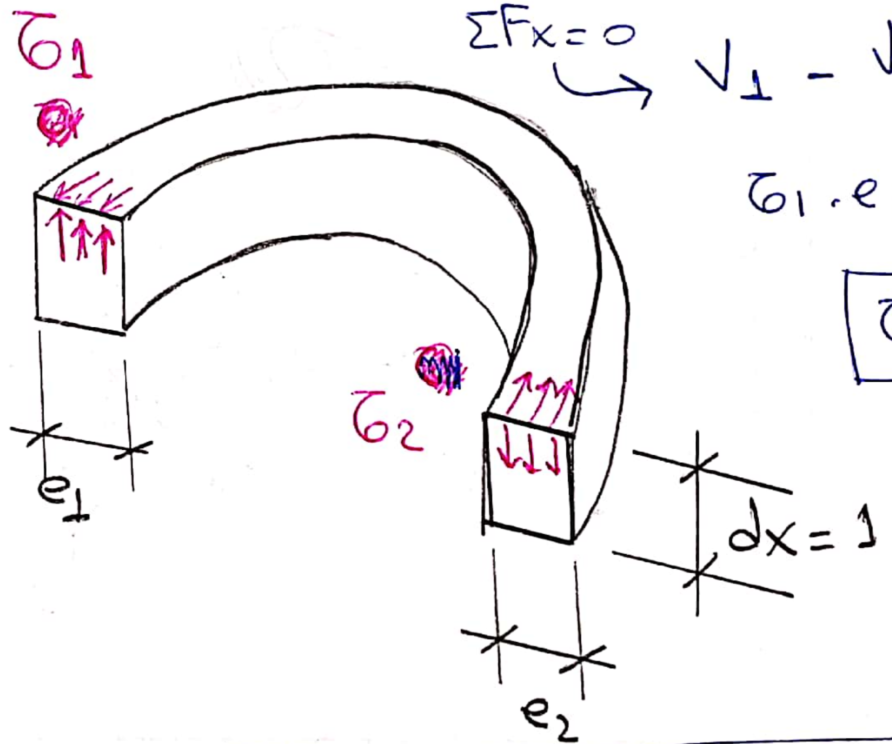
$V_1 = \tau_1 \cdot e_1 \cdot 1 \uparrow$

$V_2 = \tau_2 \cdot e_2 \cdot 1 \downarrow$

$\Sigma F_x = 0 \rightarrow V_1 - V_2 = 0.$

$\tau_1 \cdot e_1 \cdot 1 = \tau_2 \, e_2 \cdot 1$

$$\boxed{\tau_1 \cdot e_1 = \tau_2 \cdot e_2}$$



$dA = e \cdot ds \; ; \; e = e(s)$

$dT = \tau_m \cdot dA$

$dT = \tau_m \cdot e \cdot ds.$

$dM_T = r \cdot dT = \tau_m \cdot r \cdot e \, ds$

$$\boxed{M_T = \int dM_T = \int_S \tau_m \cdot r \cdot e \cdot ds}$$

$$\boxed{\tau_1 \cdot e_1 = \tau_2 \cdot e_2 = \dots = \tau_i \cdot e_i = \dots = \tau_n e}$$

$$M_T = M_x = \tau \cdot e \cdot \int_S r \cdot ds.$$

$$dA_\Delta = \frac{r \cdot ds}{2}$$

$$2 \cdot dA_\Delta = r \cdot ds$$

$\int_S r \cdot ds =$ EL ÁREA ENCERRADA BAJO LA LÍNEA MEDIA POR 2.

$$\int_S r \cdot ds = 2 \cdot \Omega$$

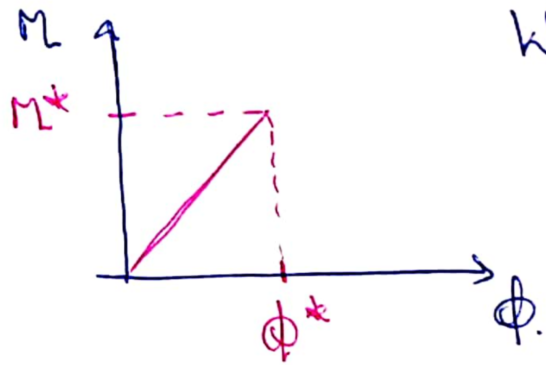
$$M_T = M_x = \tau \cdot e \cdot 2 \cdot \Omega$$

$$\tau = \frac{M_T}{2 \Omega\, e(s)} \rightarrow \boxed{\tau_{máx} = \frac{M_T}{2 \cdot \Omega \cdot e_{mín}}}$$

ÁNGULO DE TORSIÓN:

$$dW = M \cdot d\phi$$

$$W = \int dW = \int M \, d\phi.$$



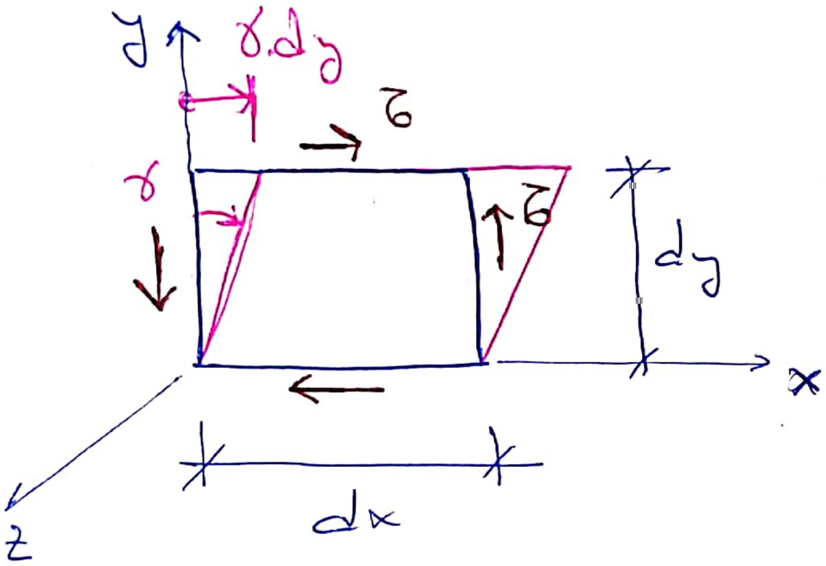
$$M = k \cdot \phi \rightarrow \phi = \frac{M}{k}$$

$$W = \int k \phi \, d\phi = \frac{1}{2} k \phi^2$$

$$W = \frac{1}{2} k \frac{M^2}{k^2} = \frac{1}{2} \frac{M^2}{k}$$

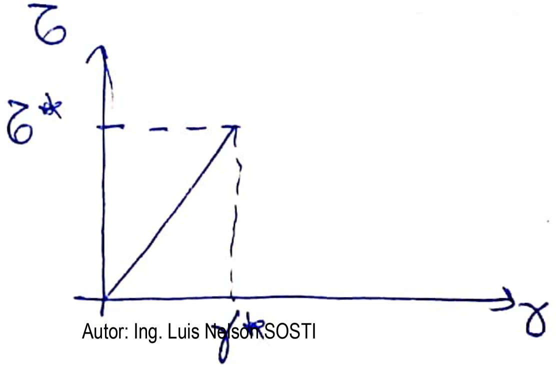
$$\boxed{W = \frac{1}{2} M^* \cdot \phi^*}$$

ENERGÍA INTERNA DE DEFORMACIÓN



$$dF = \tau \cdot dx \cdot dz$$

$$dU_i = \frac{1}{2}\left[\tau \left(dx\, dz\right]\right) . \gamma . dy .$$



$$dU_i = \frac{1}{2}\,\tau . \gamma . \underbrace{(dx\, dy\, dz)}_{dV} = \frac{1}{2}\,\tau . \gamma . dV .$$

$$\boxed{U_i = \int_V dU_i = \int_V \frac{1}{2}\,\tau\,\gamma\, dV}$$

$$\boxed{U_i = \int_V \frac{1}{2}\,\frac{\tau^2}{G}\, dV}$$

$$\frac{1}{2} M . \phi = \int_V \frac{1}{2}\,\frac{\tau^2}{G} . dV .$$

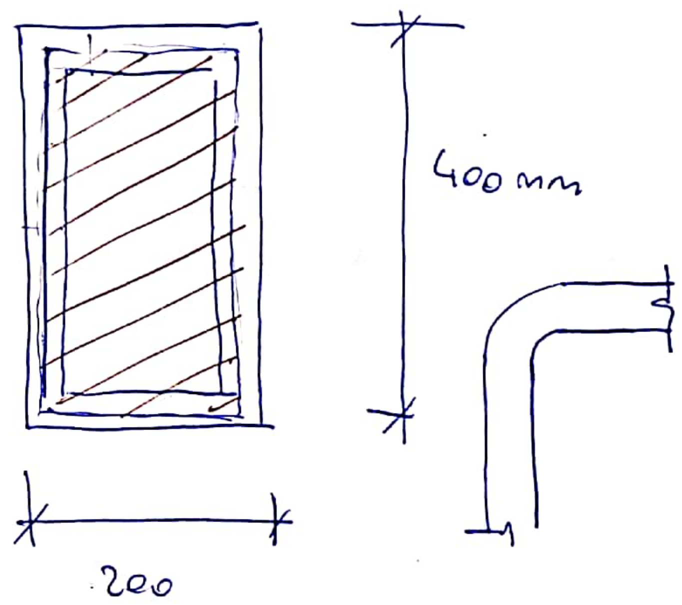
$$dV = . 1 . e(s) . ds$$

$$\tau = \frac{M_T}{2\Omega e} \rightarrow \tau^2 = \frac{M_T^2}{4\Omega^2 e^2}$$

$$\frac{1}{2} M \phi = \int_S \frac{1}{2}\,\frac{M_T^2}{4\Omega^2 e^{\cancel{2}}}\,\frac{1}{G} , 1 . \cancel{e} . ds .$$

$$\cancel{\frac{1}{2}} \cancel{M_T} . \phi = \cancel{\frac{1}{2}} . \frac{M_T^{\cancel{2}}}{4\Omega^2 G} \oint \frac{ds}{e}$$

$$\boxed{\phi = \frac{M_T(x)}{4\Omega^2 G} \int_S \frac{ds}{e(s)}}$$



- 1° Aproximación:

$$\boxed{\phi = \frac{M_T(x)}{4\Omega^2 G} \cdot \sum_{i=1}^{n} \frac{\Delta s}{e_{mi}}}$$

- si $\Delta s_i = cte.$

$$\boxed{\phi = \frac{M_T(x)}{4\Omega^2 G} \cdot \Delta s \cdot \sum_{i=1}^{n} \frac{1}{e_{mi}}}$$

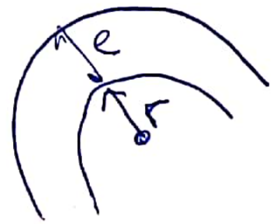
$\rho > e \rightarrow$ TODO ESTO ES VÁLIDO.

$\rho \cong e \rightarrow$ NO ES VÁLIDO.

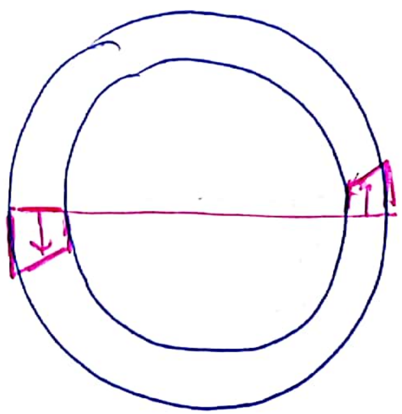
- si $e_{mi} = cte$

$$\boxed{\phi = \frac{M_T(x)}{4\Omega^2 G} \cdot \frac{S}{e}}$$

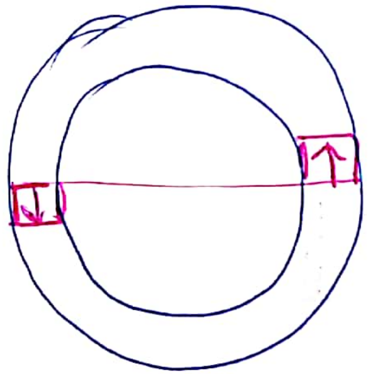
S = LONGITUD DE LA LÍNEA MEDIA



$\boxed{e \cong r}$



COULOMB



BREDT