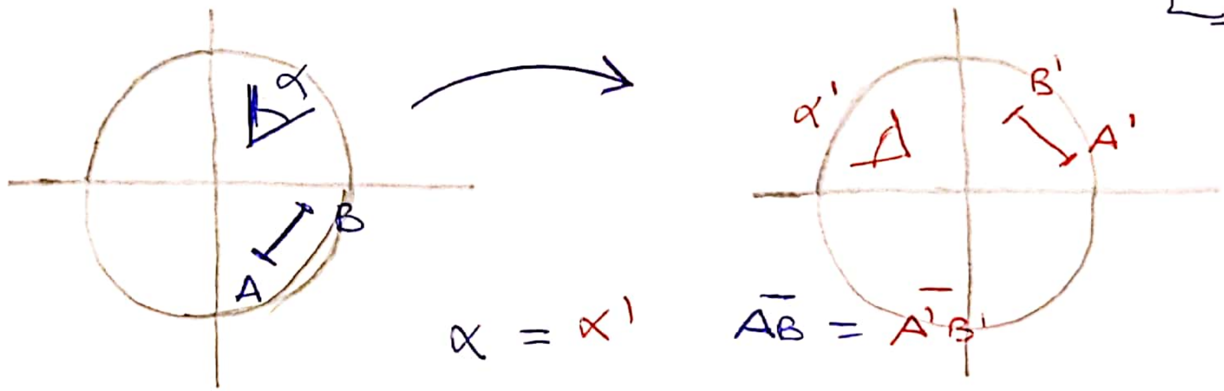
H1) → PLANAS y PARALELAS → NO SE ALABEAN.

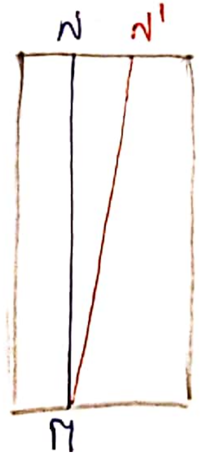
COROLARIOS DE LAS HIPÓTESIS DE COULOMB.

H2) → MANTIENEN SU FORMA → CÍRCULO → CÍRCULO.



$\alpha = \alpha'$ $\quad \overline{AB} = \overline{A'B'}$

H3) → GIRAN ALREDEDOR DEL EJE } → I) LAS GENERATRICES EN HÉLICES DE PASO MUY GRANDE.



II) LAS SECCIONES EXPERIMENTAN GIROS RELATIVOS.

07) - ECS. DE EQUIVALENCIA:

H1) → LAS SECCIONES SE MANTIENEN PLANAS → NO SE ALABEAN → LAS FIBRAS VARIAN SU LONGITUD ??

I) $\varepsilon = 0$ → $\Delta L = 0$

II) $\varepsilon = CTE$ → $\Delta L = CTE$

1º CASO

$\varepsilon_x = 0 \xrightarrow[\text{HOOKE}]{} \sigma_x = E \cdot \underbrace{\varepsilon_x}_{=0} \longrightarrow \sigma_x = 0 \longrightarrow$ 1º, 5º, 6º } SE SATISFACEN.

2º CASO

$\varepsilon_x = \text{CTE} \longrightarrow \sigma_x = E\,\varepsilon_x = \text{CTE} \longrightarrow$ 1º EC. EQ. $N = 0 = \int_A \sigma_x \cdot dA$

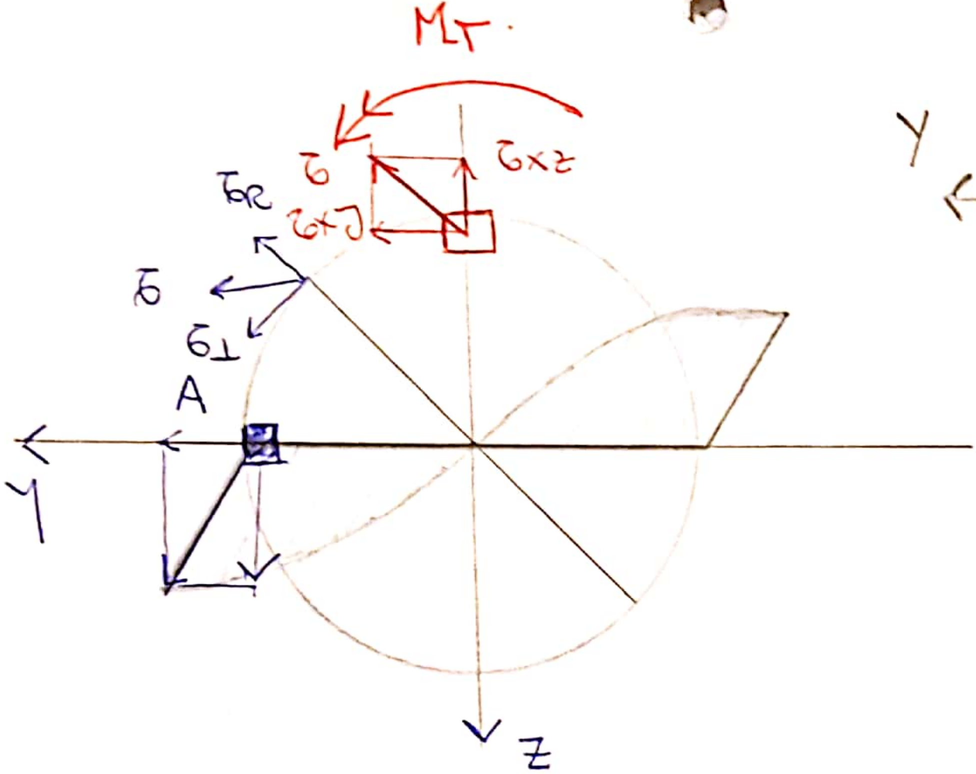
$0 = \sigma_x \int_A dA \longrightarrow 0 = \int_A dA$???

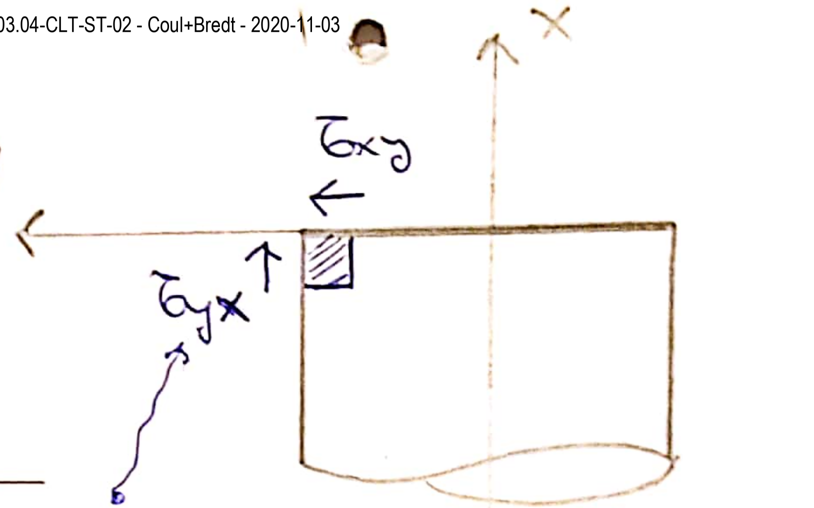
NO ES POSIBLE $\longrightarrow \boxed{\sigma_x = 0} \longrightarrow \left.\begin{array}{l} 1º \\ 5º \\ 6º \end{array}\right\}$ SE SATISFACEN

2º y 3º ECS. DE EQUIVALENCIA:

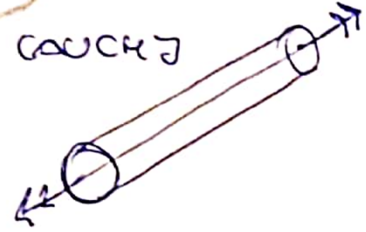
2º) $Q_y = 0 = \int_A \tau_{xy}\, dA$

3º) $Q_z = 0 = \int_A \tau_{xz}\, dA$



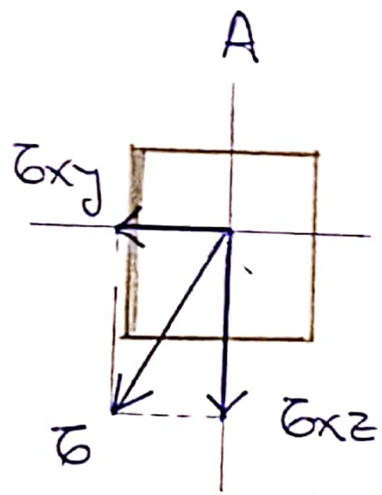


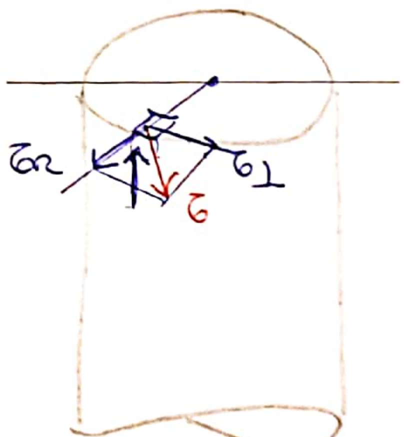
SURGE POR TEOREMA DE CAUCHY

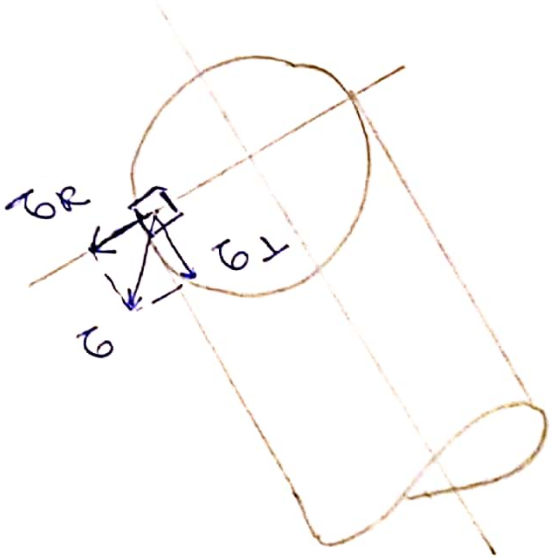


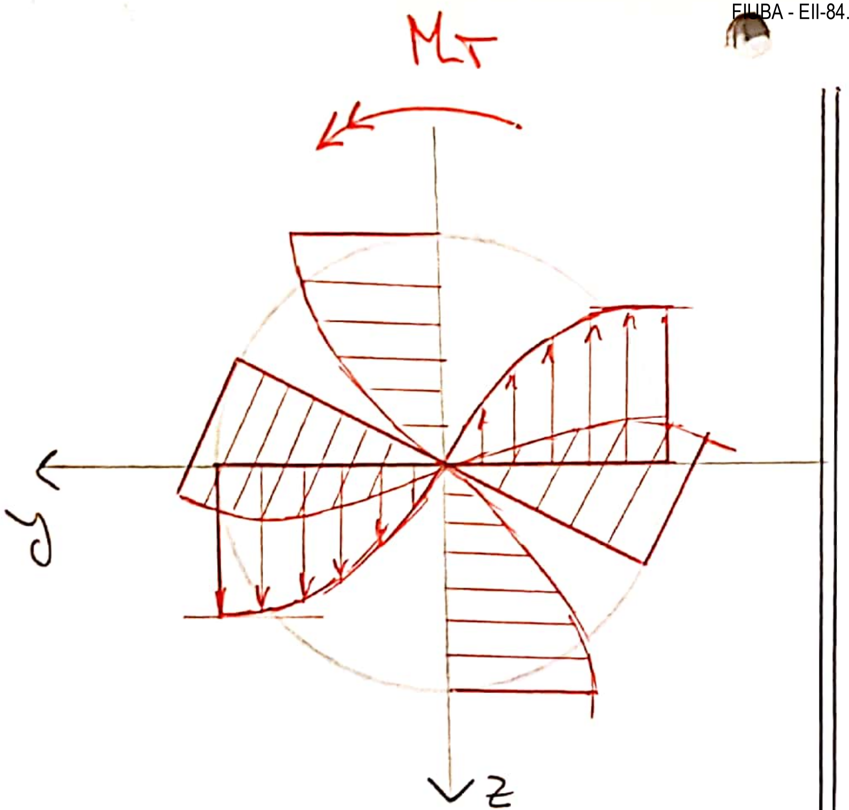
$$\rightarrow \boxed{\tau_{yx} = 0} \longrightarrow$$

→ LAS TENSIONES RADIALES SON <u>NULAS</u>







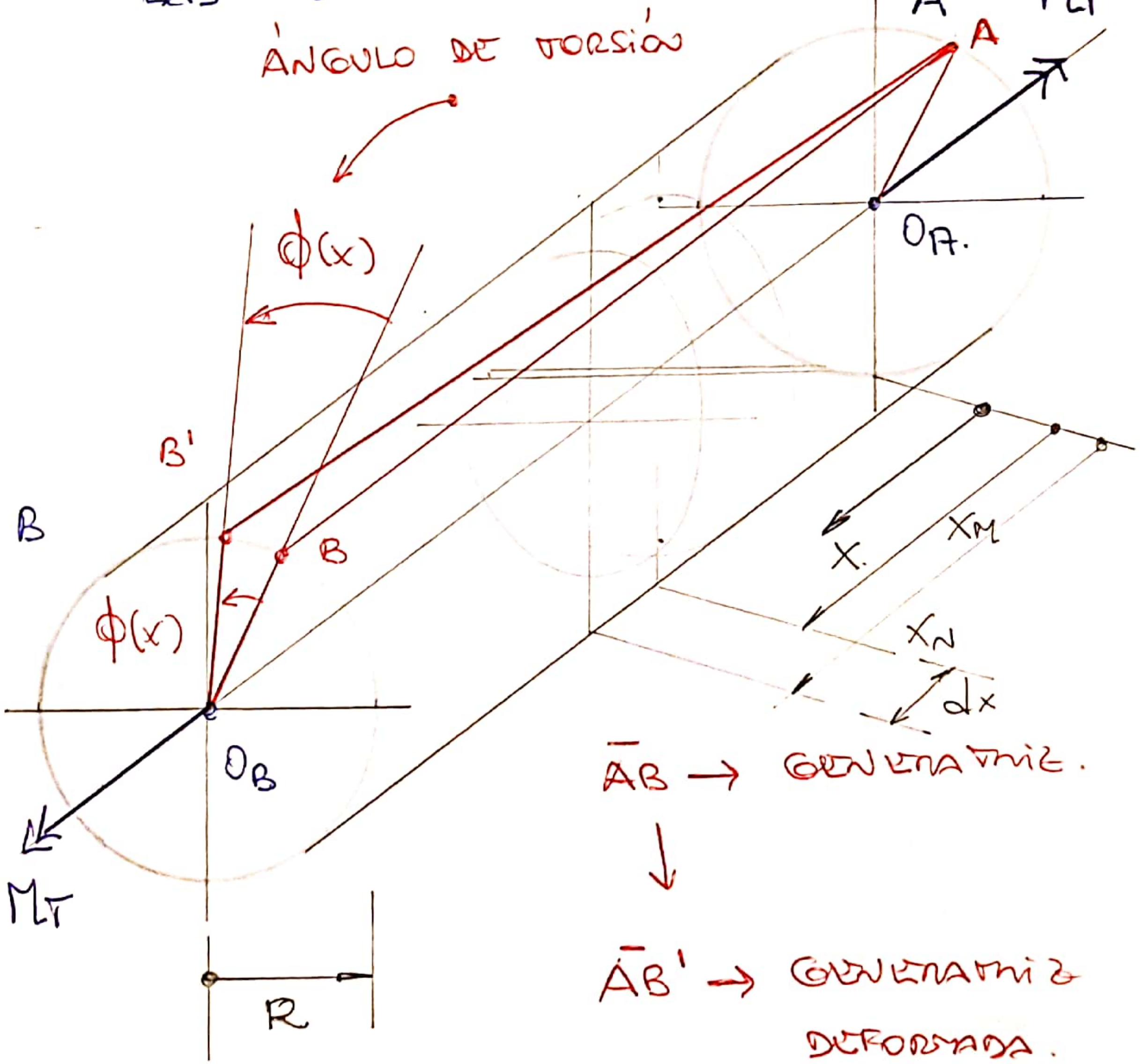


LA DISTRIBUCIÓN ES:

→ ANTISIMÉTRICA

→ DIRECCIÓN ⊥ AL DIÁMETRO (o RADIO

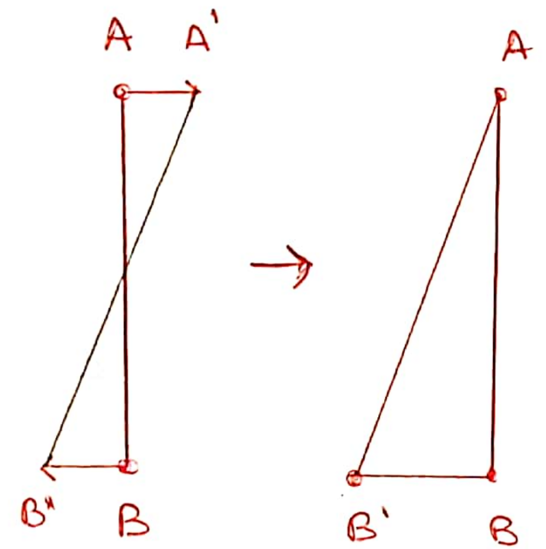
→ CONOCER LA LEY DE DISTRIBUCIÓN DE LAS TENSIONES TANGENCIALES.

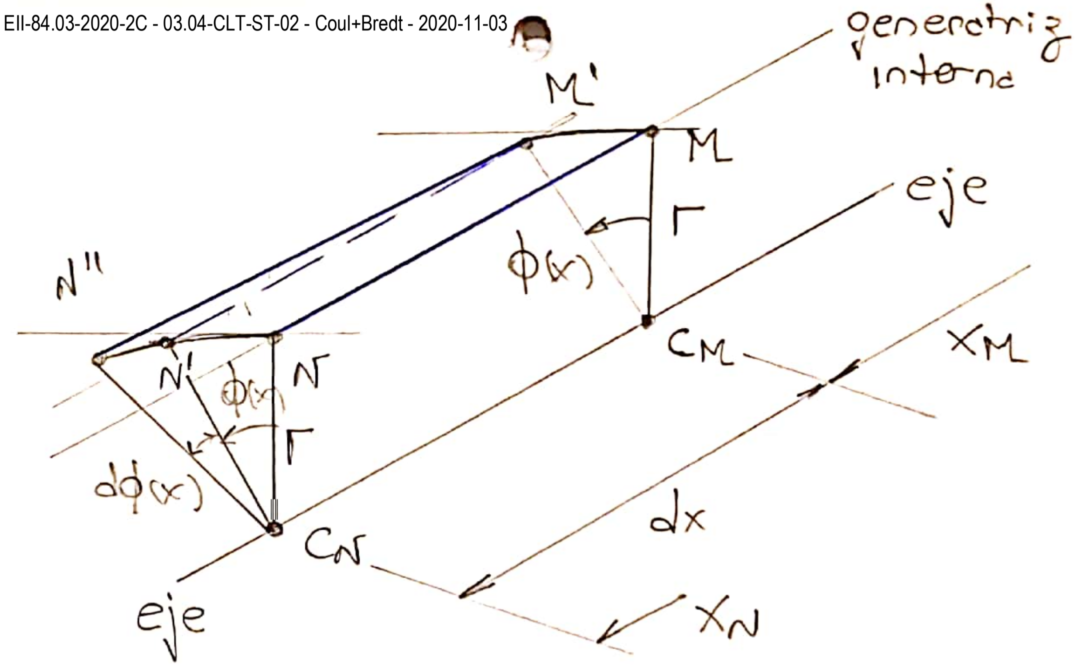


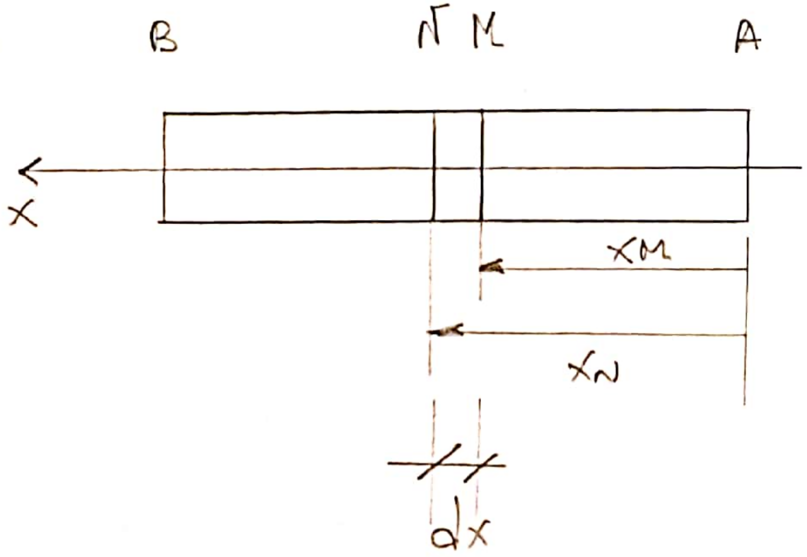
$\bar{AB}$ → GENERATRIZ.

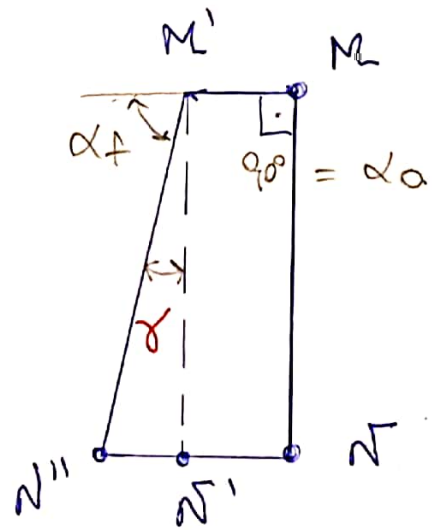
↓

$\bar{AB'}$ → GENERATRIZ DEFORMADA.









$\overline{MN}$: GENERATRIZ SIN DEFORMAR

$\overline{M'N''}$: GENERATRIZ DEFORMADA.

$$\alpha_0 - \alpha_f = 90° - \alpha_f = \gamma \text{ (DISTORSIÓN)}$$

$$\phi_M = \phi(x)$$

$$\phi_N = \phi(x) + d\phi(x).$$

$$\Delta\phi_{NM} = \phi_N - \phi_M = d\phi(x).$$

$$\left.\begin{aligned} \text{ARCO } \widehat{N'N''} &= r \cdot d\phi(x). \\ \widehat{N'N''} &= \gamma \cdot dx. \end{aligned}\right\}$$

$$\gamma \cdot dx = r \, d\phi(x).$$

$$\gamma = r \cdot \overset{\text{CTE}}{\boxed{\frac{d\phi(x)}{dx}}} = r \cdot \underbrace{\chi(x)}_{\text{CURVATURA DE TORSIÓN}}.$$

$\boxed{\chi(x)}$: CURVATURA DE TORSIÓN O ÁNGULO ESPECÍFICO DE TORSIÓN.

$$0 \leq r \leq R \rightarrow \boxed{\gamma \rightarrow \text{VARIACIÓN LINEAL}}$$

$$\left\{\begin{aligned} r = 0 &\longrightarrow \gamma = 0. \\ r = R &\longrightarrow \gamma = \gamma_{máx} = R \cdot \frac{d\phi(x)}{dx}. \end{aligned}\right.$$

$$\left.\begin{aligned} \frac{\gamma}{r} &= \frac{d\phi(x)}{dx} \\ \frac{\gamma_{máx}}{R} &= \frac{d\phi(x)}{dx}. \end{aligned}\right\} \rightarrow \frac{\gamma}{r} = \frac{\gamma_{máx}}{R}$$

$$\downarrow$$

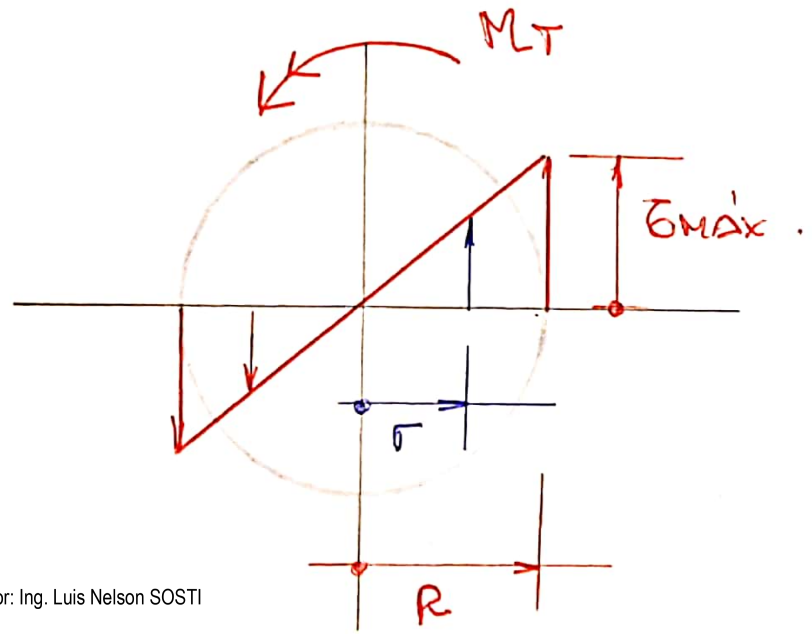
$$\boxed{\gamma = \gamma_{máx} \cdot \frac{r}{R}}$$

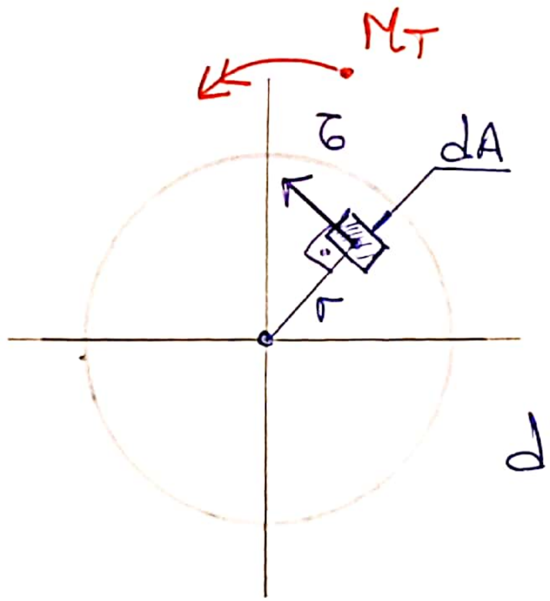
APLICAMOS LEY DE HOOKE

$$\tau = G \cdot \gamma \rightarrow \boxed{\gamma = \frac{\tau}{G}}$$

$$\frac{\tau}{\cancel{G}} = \frac{\tau_{máx}}{\cancel{G}} \cdot \frac{r}{R} \rightarrow$$

$$\rightarrow \boxed{\tau(r) = \tau_{máx} \cdot \frac{r}{R}}$$





$$\underbrace{dT}_{\text{DIFERENCIAL DE FUERZA.}} = \tau \cdot dA$$

$$dM_T = r \cdot dT$$

$$dM_T = r \cdot dT = \tau \cdot r \cdot dA \, .$$

$$M_T = \int_A dM_T = \int_A \tau \cdot r \cdot dA \, .$$

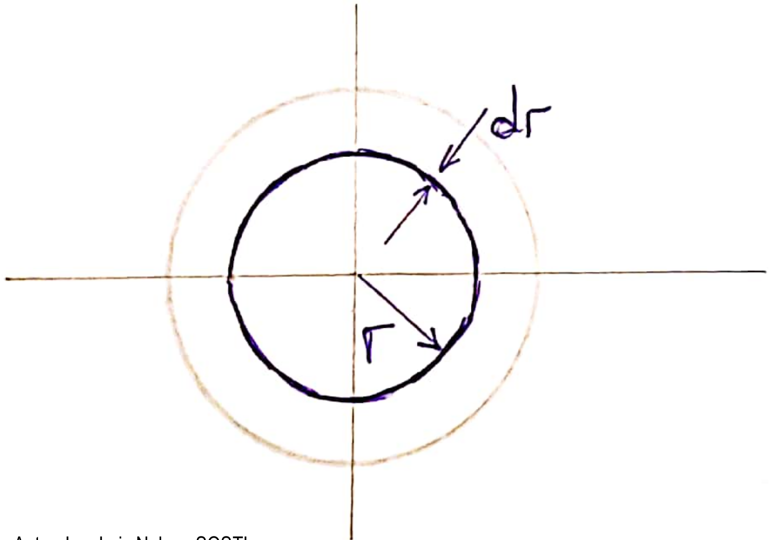
$$M_T = \int_A \frac{\tau_{máx}}{R} \cdot r^2 \, dA = \frac{\tau_{máx}}{R} \cdot \underbrace{\int_A r^2 \, dA}_{I_P.}$$

$$M_T = \frac{\tau_{máx}}{R} \cdot I_P. \longrightarrow$$

$$\tau_{máx} = \frac{M_T}{I_p} \cdot R.$$

$$\tau_{máx}(x, R) = \frac{M_T(x)}{I_p(x)} \, R(x).$$

$$\tau(x, r) = \frac{M_T(x)}{I_p(x)} \cdot r$$



$$dA = 2\pi r \, . \, dr$$

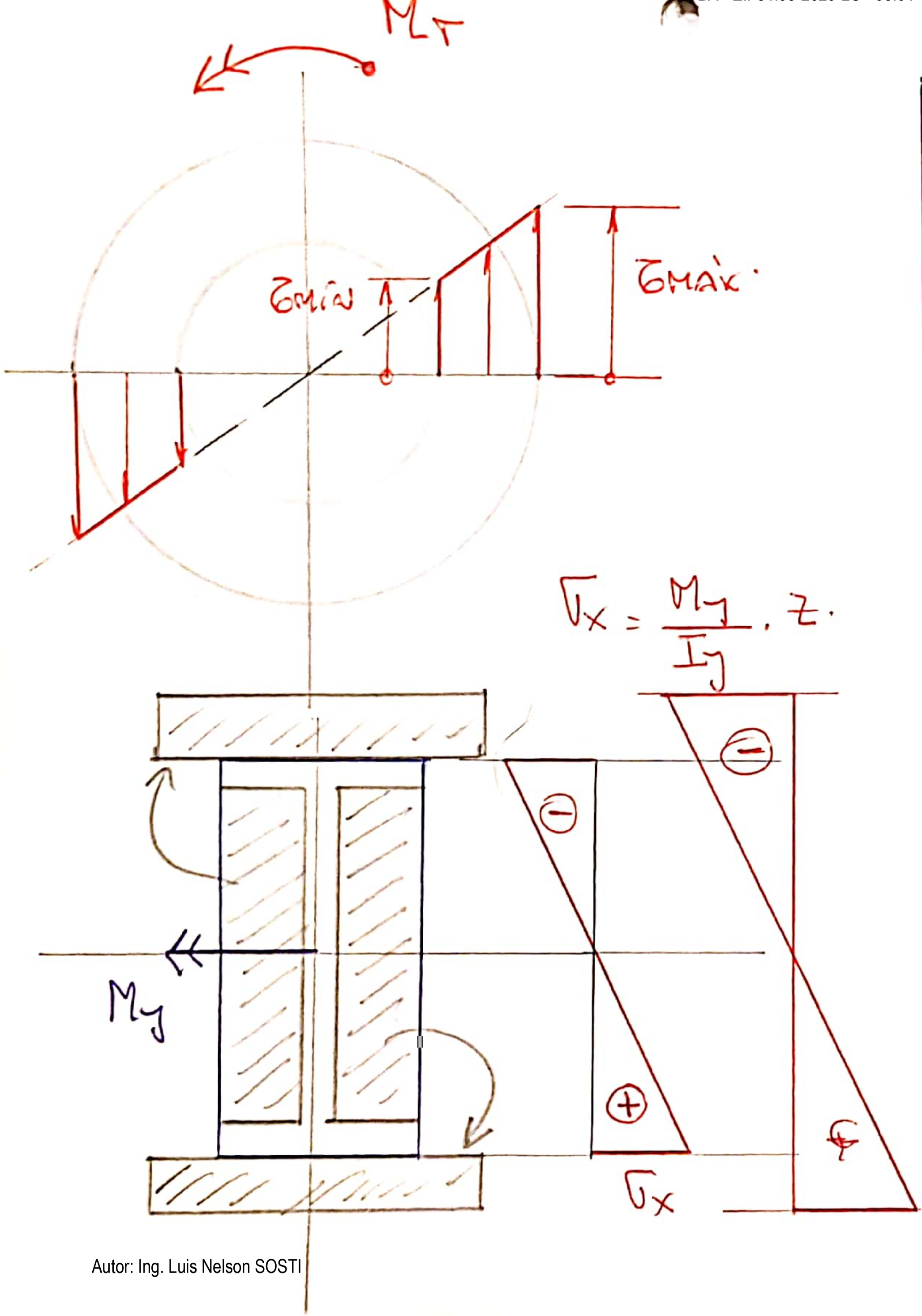
$$dI_p = r^2 \, . \, dA.$$

$$I_p = \int dI_p = \int_0^R r^2 dA = \int_0^R 2\pi r^3 \, dr$$

$$I_p = 2\pi \left. \frac{r^4}{4} \right|_0^R = \frac{\pi R^4}{2} = \frac{\pi D^4}{32}$$

$$\boxed{I_p = \frac{\pi R^4}{2} = \frac{\pi D^4}{32}}$$

$$\boxed{I_p = \frac{\pi}{2}\left[R^4 - r_i^4\right] = \frac{\pi}{32}\left[D^4 - d_i^4\right]}$$



ÁNGULO DE TORSIÓN:

$$\gamma_{máx} = R \cdot \frac{d\phi(x)}{dx} = \frac{\tau_{máx}}{G}$$

$$d\phi(x) = \frac{\tau_{máx}}{G} \frac{1}{R} dx.$$

$$\phi(x) = \int d\phi(x) = \int \frac{\tau_{máx}}{G} \cdot \frac{1}{R} dx$$

$$\phi(x) = \int \frac{M_T(x)}{G\, I_p} \cdot \frac{\cancel{R}}{\cancel{R}} dx.$$

$$\boxed{\phi(x) = \int \frac{M_T(x)}{G\, I_p} dx.}$$

11/20

- Si $\begin{cases} M_T(x) = cte. \\ G \cdot I_p = cte \end{cases}$

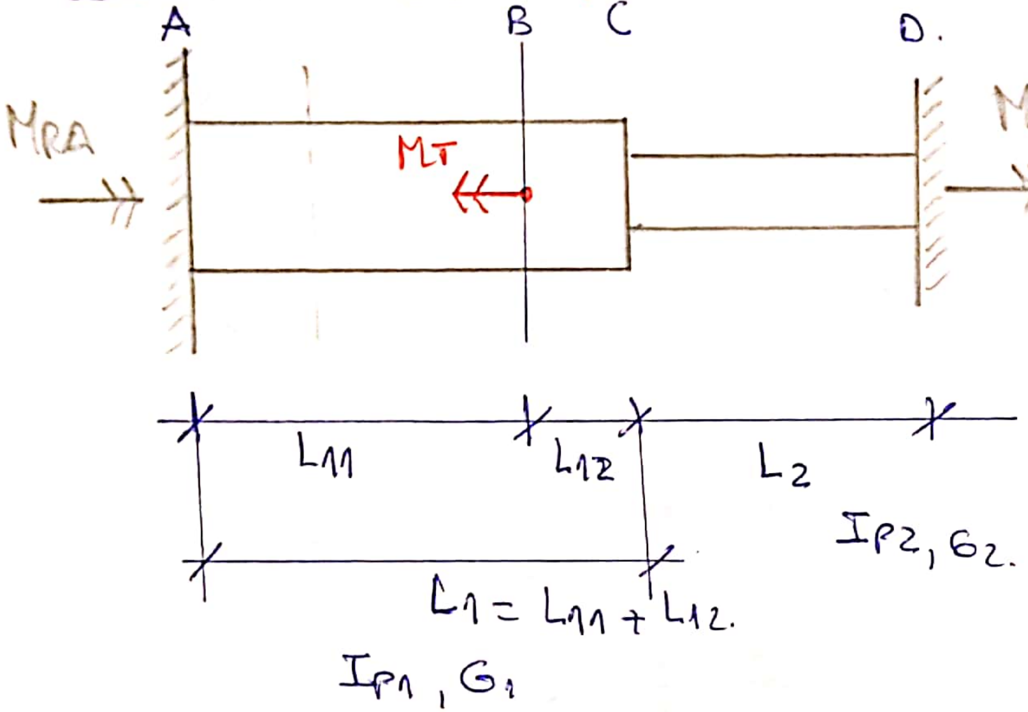
$$\phi(x) = \frac{M_T}{G I_p} \cdot x.$$

$$x = L \rightarrow \phi(x=L) = \frac{M_T L}{G I_p}.$$

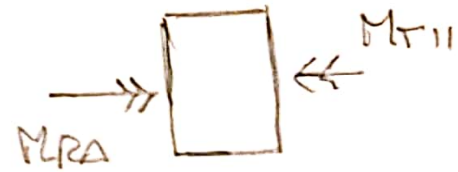
$$\boxed{\chi(x) = \frac{d\phi(x)}{dx} = \frac{M_T(x)}{G I_p}}$$

MAGNITUD	SA	ST
ESTÁTICAS	$N = N(x)$ $\sigma_x(x) = \dfrac{N(x)}{A(x)}$	$M_T = M_x = M_T(x).$ $\tau_{\max}(x) = \dfrac{M_T(x)}{I_p(x)} \cdot R$ $\tau(x,r) = \dfrac{M_T(x)}{I_p(x)} \cdot r$
CINEMÁTICAS	$\delta(x) = \Delta L(x) = \displaystyle\int \frac{N(x)}{E \cdot A(x)}\, dx$ $\varepsilon_x(x) = \dfrac{d\delta(x)}{dx}$	$\phi(x) = \displaystyle\int \frac{M_T(x)}{G \cdot I_p(x)}\, dx$ $\chi(x) = \dfrac{d\phi(x)}{dx}.$ $\gamma(x,r) = r \cdot \dfrac{d\phi(x)}{dx}$

EJEMPLO Nº 01: → HIPERESTÁTICO POR VÍNCULO EXTERNO.



$$\Delta\phi_{BA} = \frac{M_{T11} L_{11}}{G_1 I_{P1}}$$



$M_{T11} = -M_{RA}$.

$$\Delta\phi_{BA} = -\frac{M_{RA} L_{11}}{G_1 I_{P1}}.$$

$$\Delta\phi_{CB} = \frac{M_{T12} L_{12}}{G_1 I_{P1}}; \quad M_{T12} = -M_{RA} + M_T$$

$$\Delta\phi_{CB} = -\frac{(M_{RA} - M_T) L_{12}}{G_1 I_{P1}}$$

$$\Delta\phi_{DC} = \frac{M_{T2} L_2}{G_2 I_{P2}} = -\frac{(M_{RA} - M_T) L_2}{G_2 I_{P2}}$$

$\Sigma M_x = 0.$ → EC. DE EQUILIBRIO.

$$M_{RA} + M_{RD} - M_T = 0.$$

EC. DE COMPATIBILIDAD DE LOS DESPLAZAMIENTOS

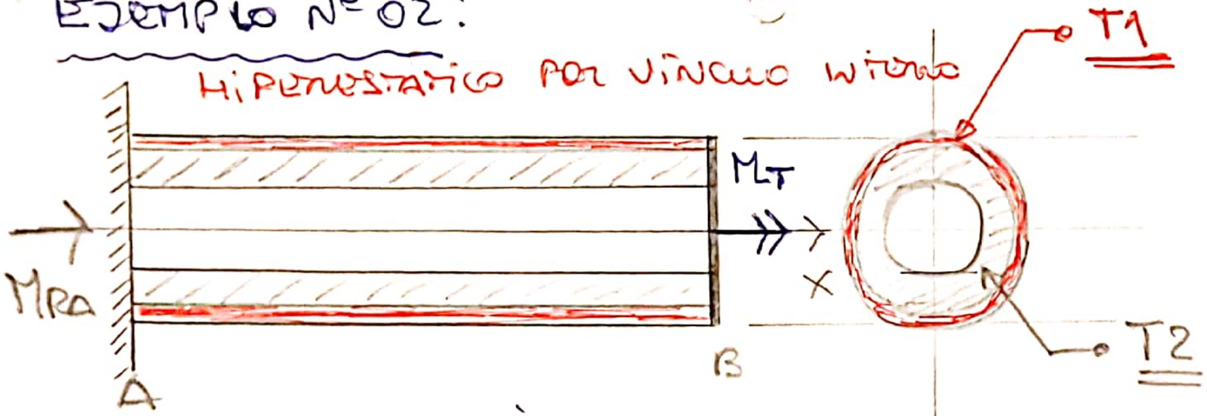
$$\Delta\phi_{DA} = 0 \rightarrow \phi_D - \phi_A = 0.$$

$$\Delta\phi_{BA} + \Delta\phi_{CB} + \Delta\phi_{DC} = 0.$$

$$-\frac{M_{RA} L_{11}}{G_1 I_{P1}} - \frac{(M_{RA} - M_T) L_{12}}{G_1 I_{P1}} - \frac{(M_{RA} - M_T) L_2}{G_2 I_{P2}} = 0$$

EJEMPLO Nº 02:

HIPERESTÁTICO POR VÍNCULO INTERNO



SOLICIT. INTERNA

$$\underbrace{M_T(x)}_{M_T} = M_{T1} + M_{T2}.$$

M_T ← DATO.

- 2 TUBOS CONCÉNTRICOS QUE GIRAN SOLIDARIAMENTE.

EC. DE EQUILIBRIO:

$\Sigma M_x = 0$ $\quad M_{RA} + M_T = 0.$

EC. COMPATIBILIDAD DE LOS DESPLAZAMIENTOS:

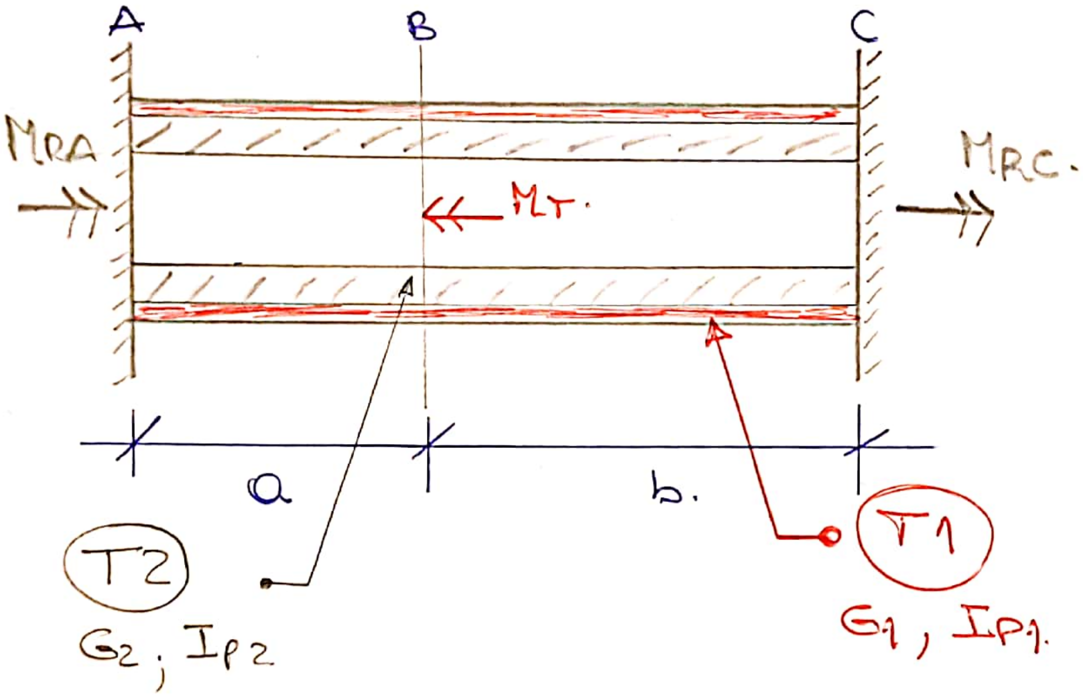
$$\phi_1(x) = \phi_2(x)$$

$$x = L \qquad \phi_1(x=L) = \phi_2(x=L).$$

$$\frac{M_{T1}\,L}{G_1\,I_{P1}} = \frac{M_{T2}\,L}{G_2\,I_{P2}}$$

EJEMPLO Nº 03:

HIPERESTÁTICO POR VÍNCULO EXTERNO E INTERNO.



- 2 TUBOS CONCÉNTRICOS QUE GIRAN SOLIDARIAMENTE.

EC. DE EQUILIBRIO GLOBAL.

$$\Sigma M_x = 0 \rightarrow M_{RA} + M_{RC} - M_T = 0$$

EC. DE COMPATIBILIDAD DE LOS DESPLAZAMIENTOS GLOBAL.

$$\Delta\phi_{CA} = 0.$$

$$\Delta\phi_{CA} = \Delta\phi_{BA} + \Delta\phi_{CB}.$$

$$\Delta\phi_{BA} = \frac{M_{Ta}\, a}{G\, I_P} = -\frac{M_{RA}\, a}{G\, I_P}.$$

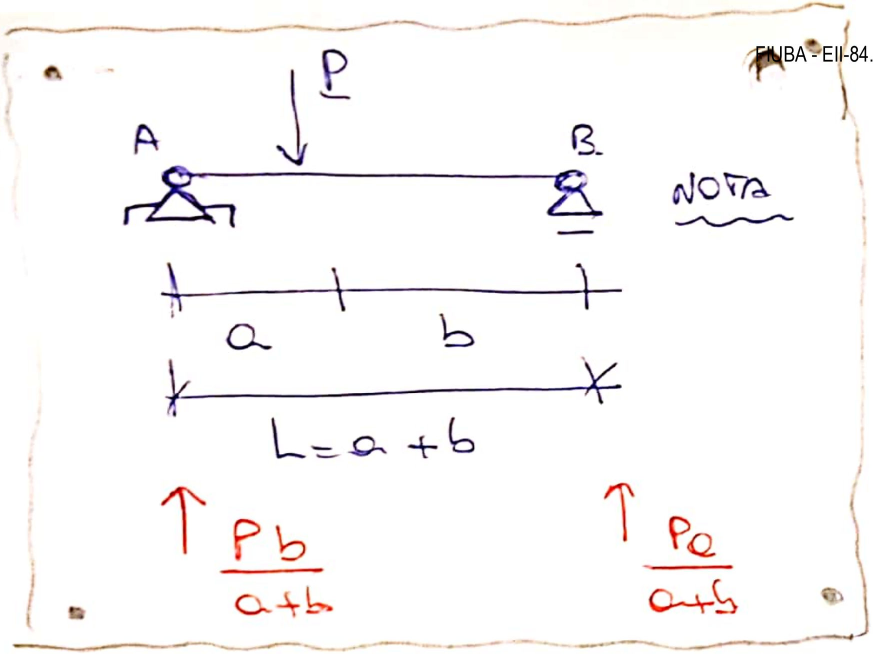
$$\Delta\phi_{CB} = \frac{M_{Tb}\, b}{G\, I_P} = -\frac{(M_{RA} - M_T)\, b}{G\, I_P}.$$

$$-\frac{M_{RA} \cdot a}{\cancel{G\, I_P}} - \frac{(M_{RA} - M_T)\, b}{\cancel{G\, I_P}} = 0$$

$$M_T\, b. = M_{RA}(a+b).$$

$$\boxed{M_{RA} = M_T \cdot \frac{b}{a+b}.}$$

$$\boxed{M_{RC} = M_T\, \frac{a}{a+b}}$$



EC. DE EQUILIBRIO PARTICULAR O INTERNO:

TRAMO 'a' $M_{Ta} = M^a_{T1} + M^a_{T2}$

$$M_{Ta} = -M_{RA}$$

EC. DE COMPATIBILIDAD EN EL TRAMO 'a'

$$\phi^a_1(x) = \phi^a_2(x)$$

$$x = a \rightarrow \phi^a_1(x=a) = \phi^a_2(x=a).$$

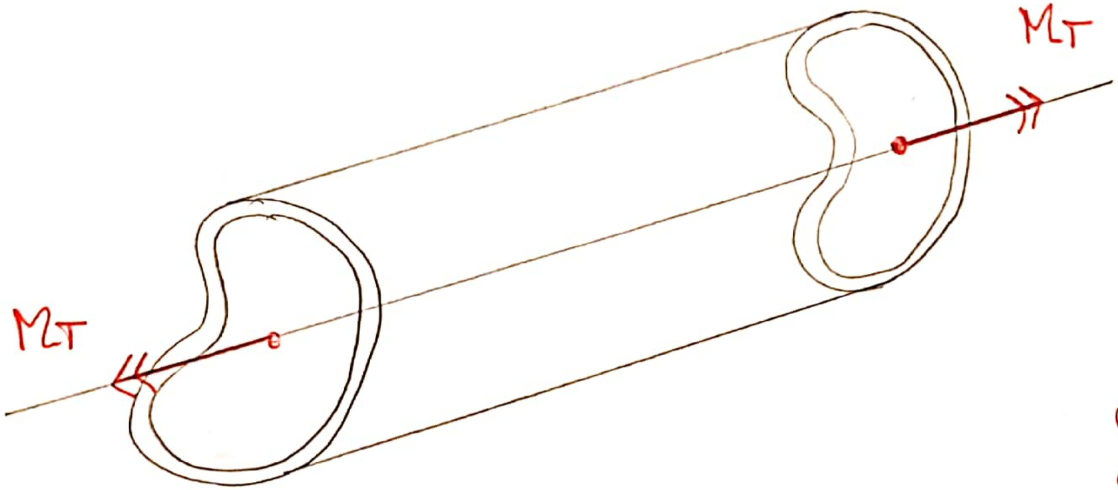
$$\frac{M^a_{T1}\, a}{G_1\, I_{P1}} = \frac{M^a_{T2}\, a}{G_2\, I_{P2}}.$$

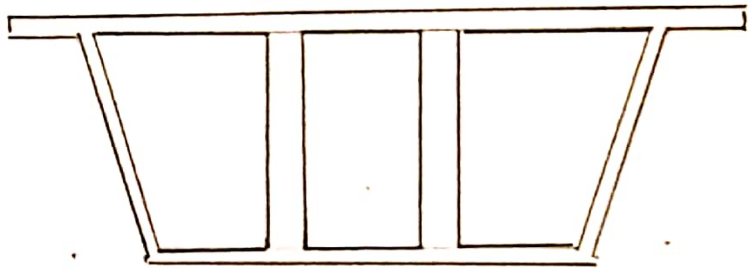
TRAMO 'b' $M_{Tb} = M^b_{T1} + M^b_{T2}.$

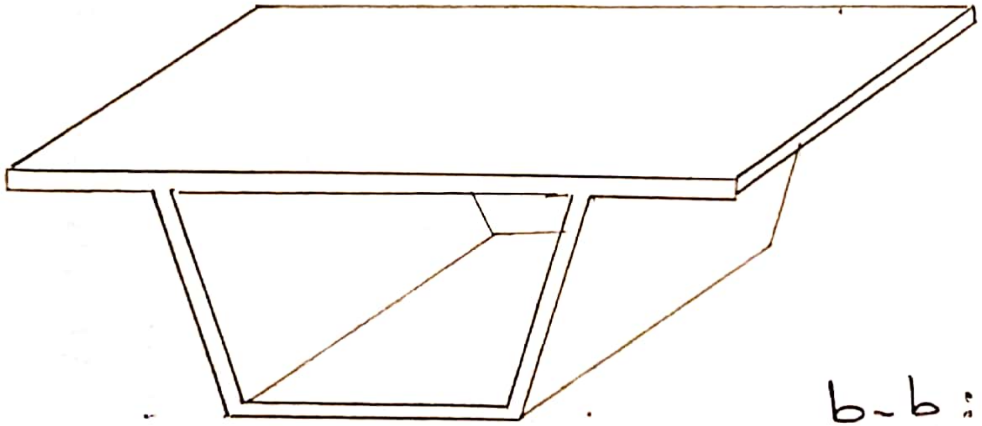
$$\phi^b_1(x) = \phi^b_2(x).$$

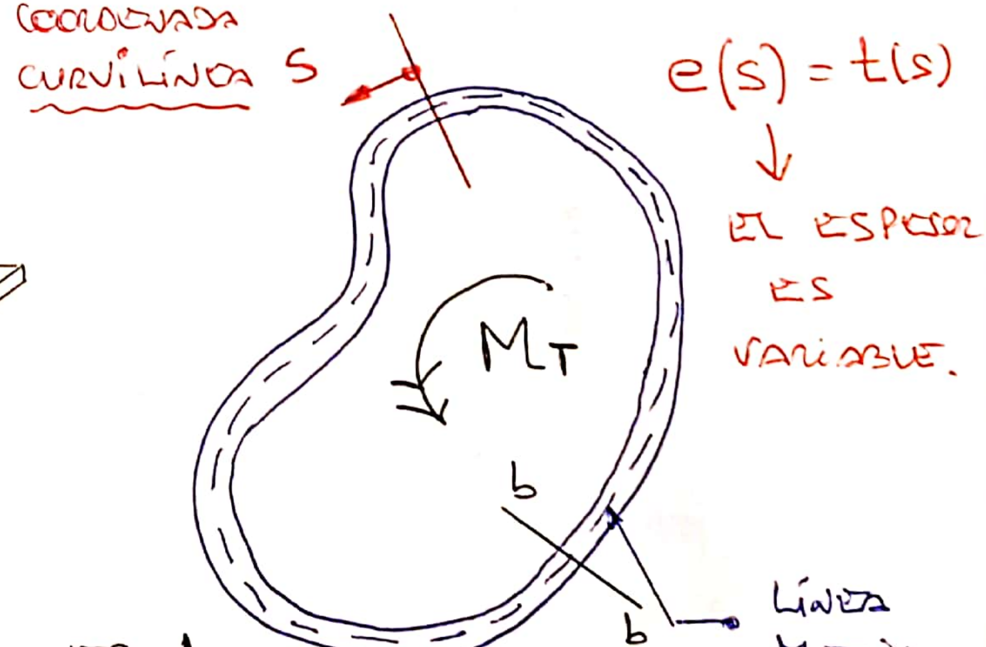
$$\frac{M^b_{T1}\, b}{G_1\, I_{P1}} = \frac{M^b_{T2}\, b}{G_2\, I_{P2}}.$$

SECCIONES TUBULARES SIMPLEMENTE CONEXAS DE PEQUEÑO ESPESOR

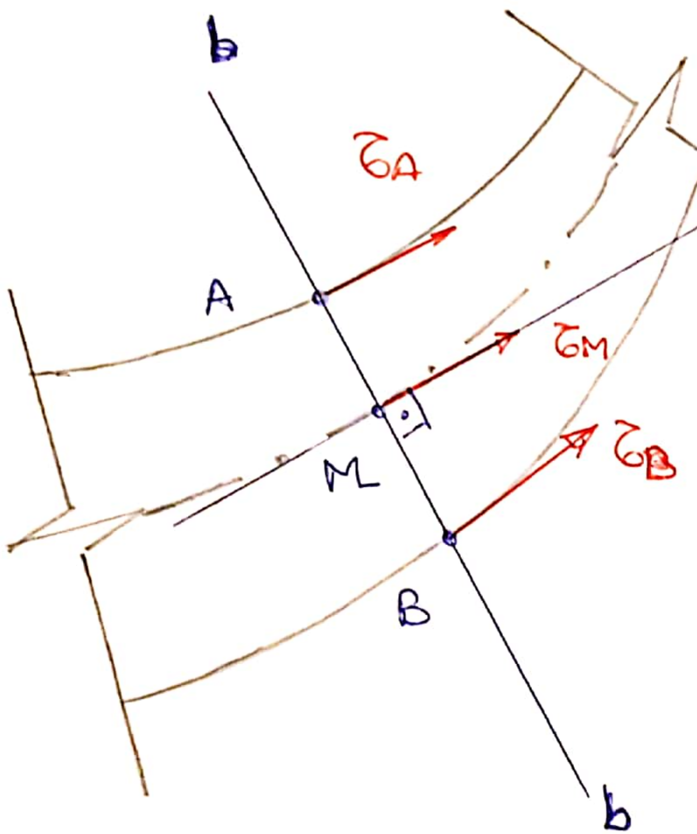






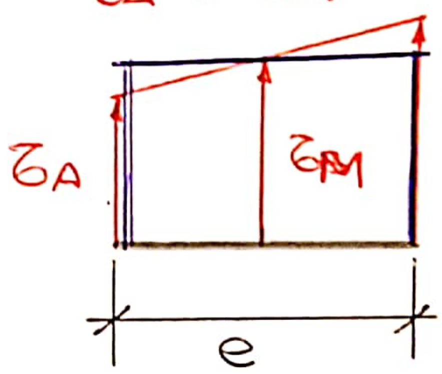


b-b: ES ⊥ A LA TANGENTE DE LA LÍNEA MEDIA DE ESE CORTE



TANGENTE A LA 'LM' EN EL CORTE b-b.

$\tau_A < \tau_M < \tau_B$.



$\tau_M = \tau_m = \tau_{ADOP}$

2 SUPOSICIONES:

BASADO EN QUE 'e' ES MUY CHICO. COMPARADO C/ LAS DIMENSIONES DE LA SECCIÓN:

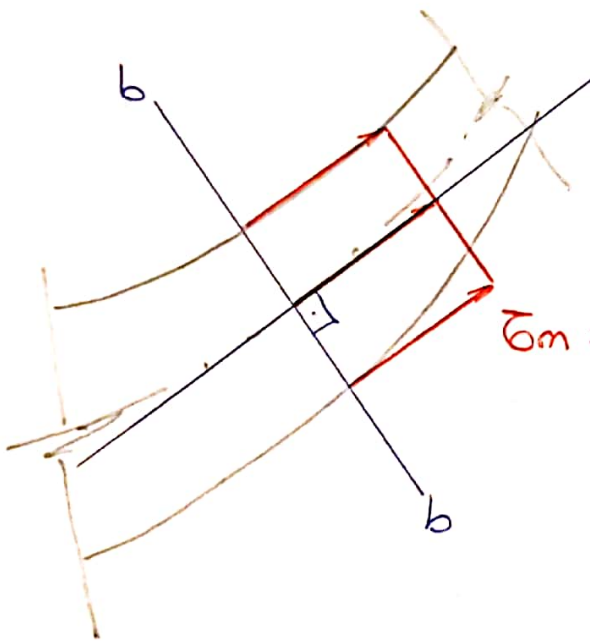
I) AL VALOR SE ADOPTA EL DE τ_M. COMO CONSTANTE EN EL ESPESOR.

II) LA DIRECCIÓN SERÁ LA DE LA TANGENTE EN LA LÍNEA MEDIA.

τ_A ES TANGENTE AL BORDE INTERNO EN 'A'

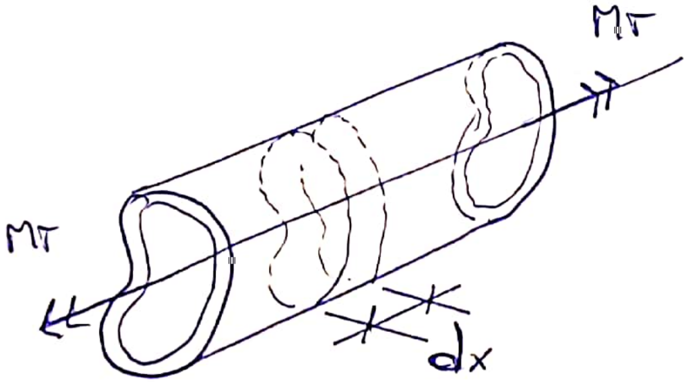
τ_B " " " EXTERNO EN 'B'.

τ_B ES " " LA 'LM' EN M.

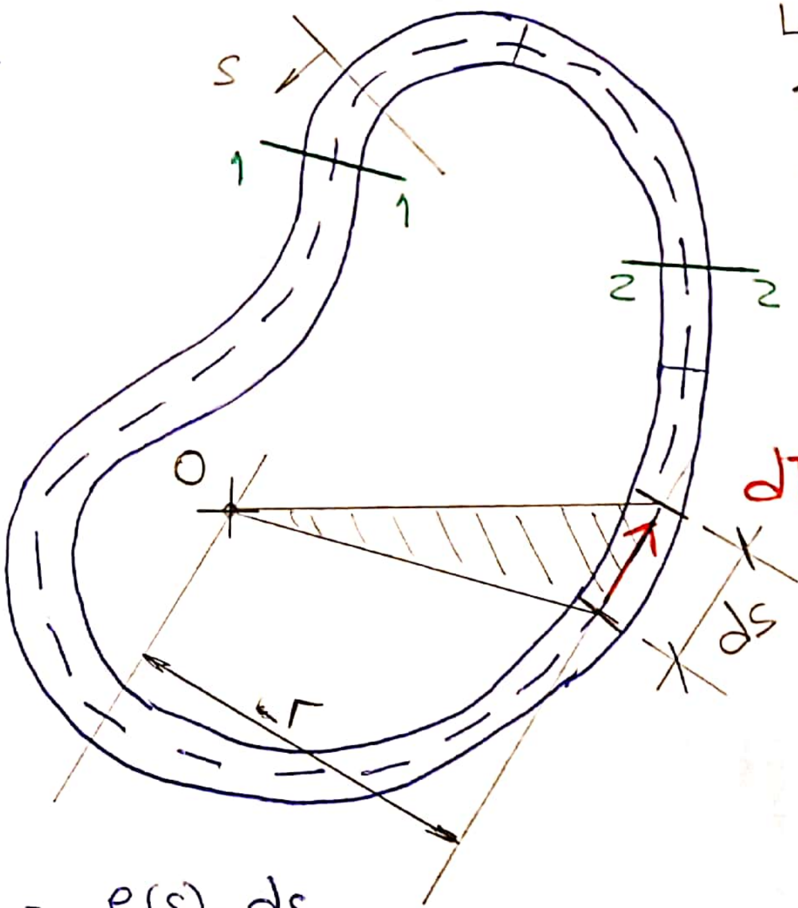


TANGENTE A LA 'LM' EN EL CORTE b-b.

$\tau_m = CTE.$



LOS CORTES 1-1 y 2-2 SON $\perp$ A LA LM

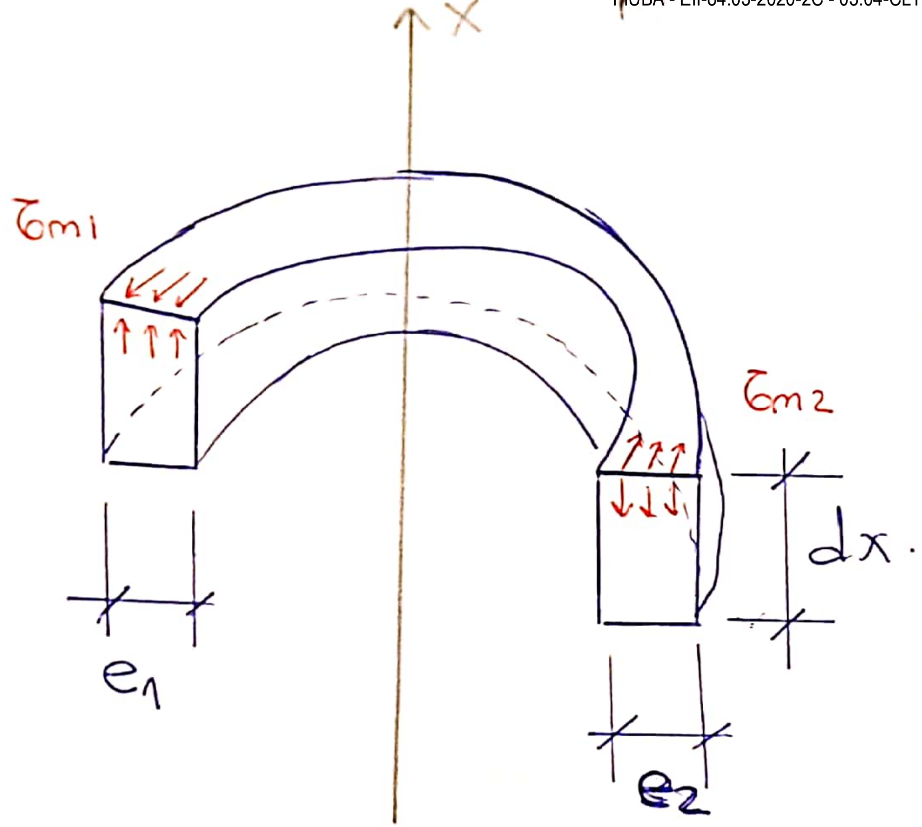


$$dA = e(s) \cdot ds$$

$$dT = \tau_m \cdot dA = \tau_m \cdot e \cdot ds.$$

$$dM_T = r\, dT = \tau_m \cdot r \cdot e \cdot ds$$

$$M_T = \int dM_T = \int_S \tau_m \cdot r \cdot e \cdot ds.$$



$$\boxed{\tau_{m1} \cdot e_1 = \tau_{m2} \cdot e_2}$$

$$\tau_{m1} \cdot e_1 = \tau_{m2} \cdot e_2 = \dots = \tau_{mi} \cdot e_i = \dots = \tau_{mn}\, e_n$$

$\tau_{mi} \cdot e_i =$ FLUJO DE LA TENSIONES TANGENCIALES

↓

FUERZA POR UNIDAD DE LONGITUD.

$\tau_m \cdot e = \text{CTE}$ → SALE FUERA DE LA INTEGRAL.

$$M_T = M_x(x) = \tau_m \cdot e \int_s r \cdot ds$$

$$V_1 = \tau_{m1} \cdot e_1 \cdot dx \uparrow$$

$$V_2 = \tau_{m2} \cdot e_2 \, dx \downarrow$$

$$V_1 - V_2 = 0.$$

$$\tau_{m1} \cdot e_1 \cdot \cancel{dx} - \tau_{m2} \cdot e_2 \, \cancel{dx} = 0$$

$$dA_\Delta = \frac{r \cdot ds}{2} \rightarrow 2\,dA_\Delta = r \cdot ds.$$

$$M_x = \tau_m \cdot e \int_s \underbrace{r \cdot ds}_{2\,dA_\Delta}.$$

EL ÁREA ENCERRADA
POR LA LÍNEA MEDIA
Y POR 2

$$\int_s dA_\Delta = \Omega$$

$$M_T = M_x(x) = \tau_m \cdot e \cdot 2\Omega.$$

$$\boxed{\tau_m(x) = \frac{M_T(x)}{2\,\Omega\,e(s)}}$$

$\tau_m(x,s)$.

$$\boxed{\tau_{m,\text{máx}}(x) = \frac{M_T(x)}{2 \cdot \Omega \cdot e_{\text{mín}}}}$$