Flexión Variable

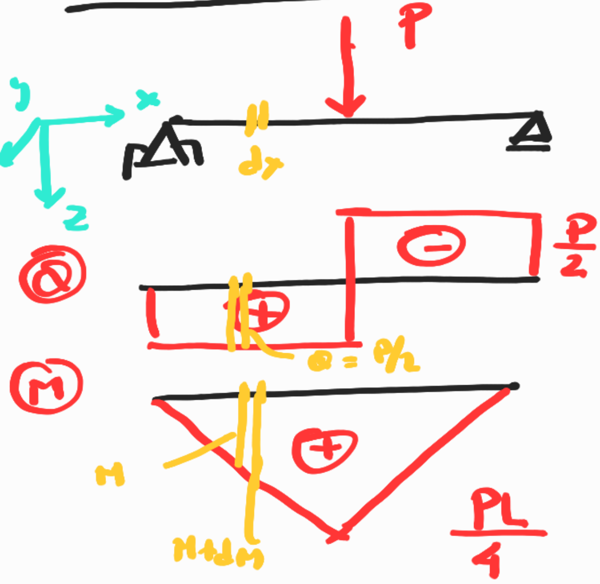
* CALCULAR σ, debido a M

↳ LM, LC, LE

Bernoulli Navier

$$\sigma = \frac{N}{A} + \frac{M_y}{J_y} z - \frac{M_z}{J_z} y$$

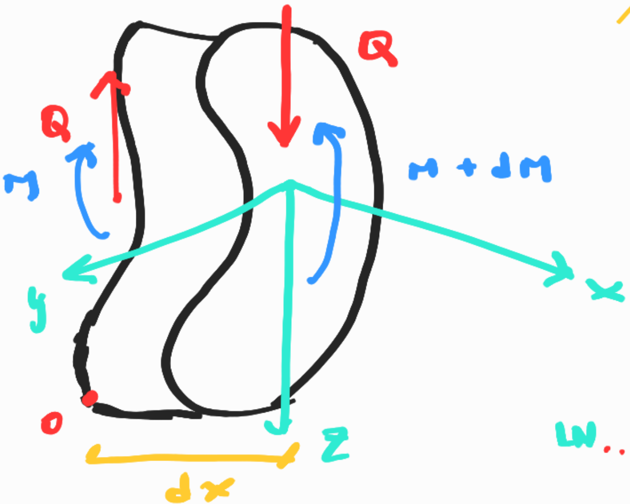
. EJES PRINCIPALES DE INERCIA



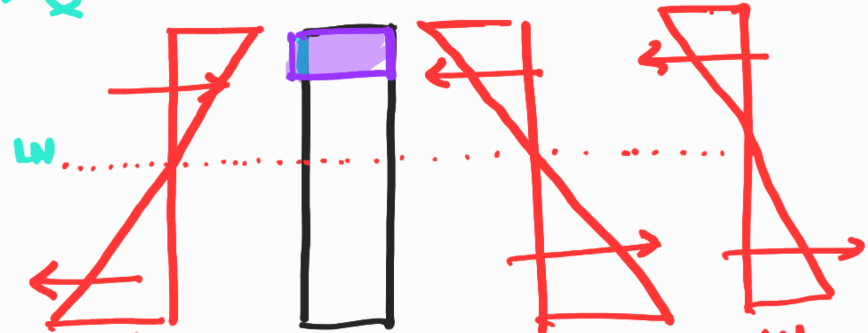
$$\Sigma_M = 0$$

$$\cancel{M} + dM - \cancel{M} - Q \cdot dx = 0$$

$$\boxed{Q_z = \frac{dM_y}{dx}}$$



Y, Z, son principales de inercia



$$\sigma = \frac{M_i}{J_y} z \qquad \sigma_i = \frac{M_i}{J_y} z + \frac{dM}{J_y} z$$

$$F_i = F_1$$

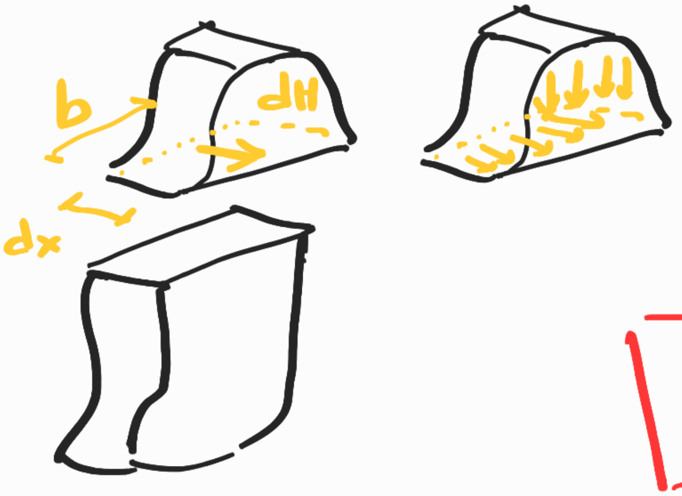
$$F_i + dF_i = F_2$$

$$dH = F_2 - F_1 = \int_{A^*} \sigma_i + d\sigma_i \, dA^* - \int_{A^*} \sigma_i \, dA^*$$

$$dH = \int_{A^*} \frac{\cancel{M} + dM}{J_y} z \, dA^* - \cancel{\int \frac{M}{J_y} z \, dA^*} \rightarrow S_y^{A^*}$$

$$dH = \int \frac{dM}{J_y} z \, dA^* = \frac{dM}{J_y} \int_{A^*} z \cdot dA^*$$

$$\boxed{dH = \frac{dM}{J_y} S_y^{A^*}}$$



$$\tau_{resbalamento} = \frac{dH}{dx \cdot b}$$

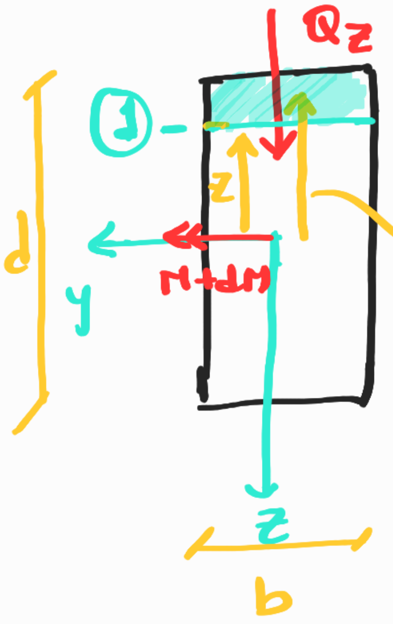
$$\tau = \frac{dM}{dx} \cdot \frac{S_y^{A^*}}{J_y \, b}$$

$\frac{dM}{dx} \to Q_z$

$$\boxed{\tau_{resbalamento} = \frac{Q_z \, S_y^{A^*}}{J_y \, b}}$$

$$\frac{dH}{dx} = q = \frac{Q_z \, S_y^{A^*}}{J_y}$$

↳ FLUJO DE CORTE

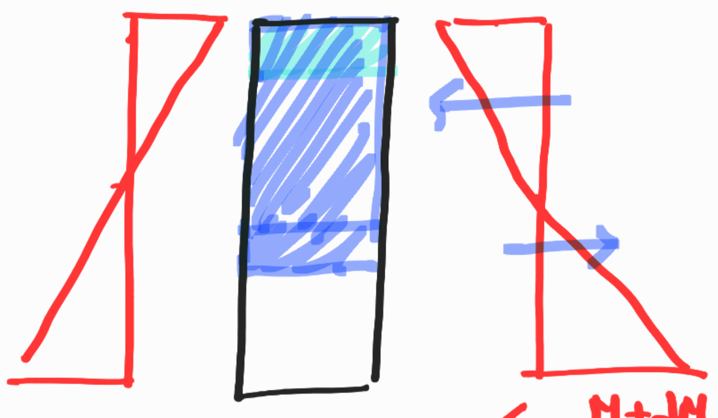


$\frac{d}{2} - z$

$$\frac{z + \left(\frac{d}{2} - z\right)}{2}$$

$$z + \frac{d}{4} - \frac{z}{2}$$

$$\frac{1}{2}\left(z + \frac{d}{2}\right)$$



$\sigma = \frac{M_y z}{J_y}$ $\sigma = \frac{M+dM}{J_y} \cdot z$

(1) $$\tau = \frac{Q_z \, S_{A^*}}{J_y \, b} = \frac{Q_z \, \overbrace{b\left(\frac{d}{2} - z\right)}^{A^*} \frac{1}{2}\left(\frac{d}{2} + z\right)}{\frac{b d^3}{12} \cdot b}$$

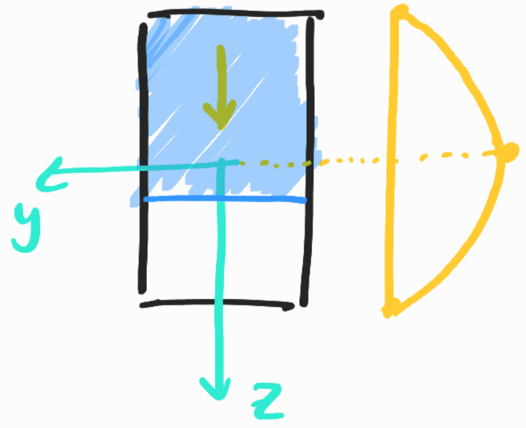
$$\tau = \frac{Q_z \left(\left(\frac{d}{2}\right)^2 - z^2\right) \cancel{\frac{b}{2}}}{\frac{b d^3}{\cancel{12}\,6} \cancel{b}} = \frac{6\, Q_z}{b d^3}\left[\left(\frac{d}{2}\right)^2 - z^2\right]$$

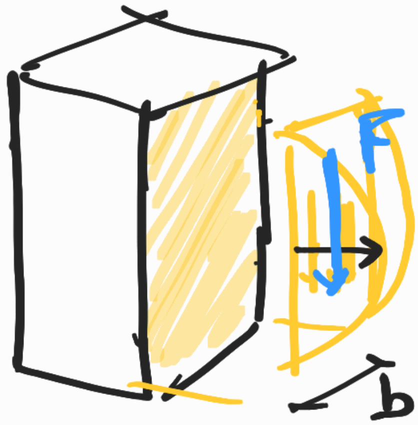
$$z = -\frac{d}{2} \longrightarrow \tau = 0$$

$$\frac{3\,\cancel{6}\, Q_z \, \cancel{d^2}}{\ldots} = \frac{3\, Q_z}{\ldots}$$

$z = 0 \longrightarrow \frac{\cdots}{b\, d^3 / 12} \quad \frac{\cdots}{2} \frac{\cdots}{b \cdot d}$

$z = \frac{d}{2} \longrightarrow \tau = 0$





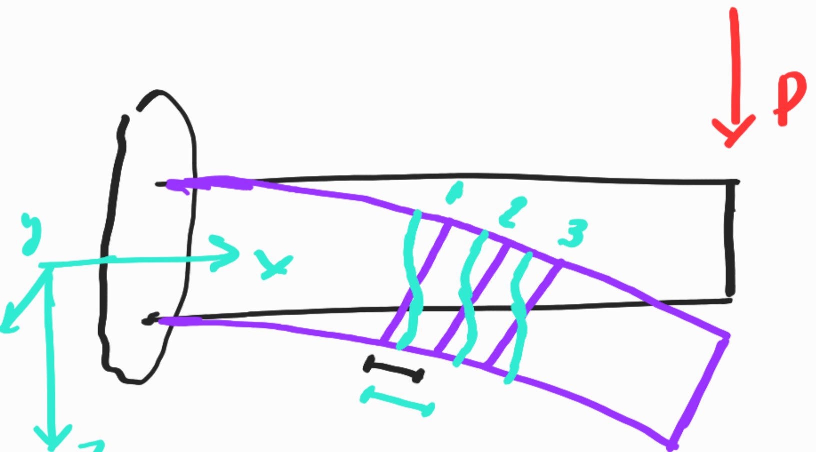
$Q_z = \int \tau_{xz}\, dA$

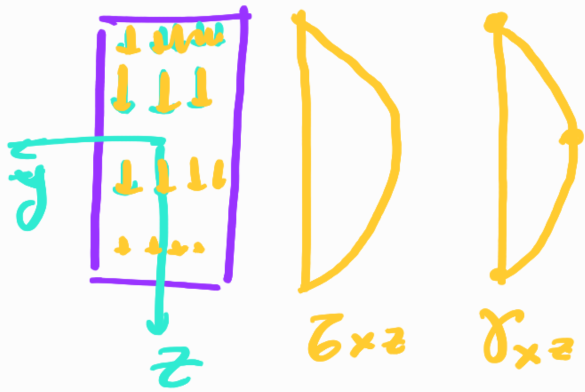
$\frac{3}{2} \frac{Q_z}{b \cdot d}$

Área $= \frac{2}{3} f \cdot L$

$F = \cancel{\frac{2}{3}} \cdot \cancel{\frac{3}{2}} \frac{Q_z}{\cancel{b \cdot d}} \cancel{d \cdot b} = Q_z$ ✓

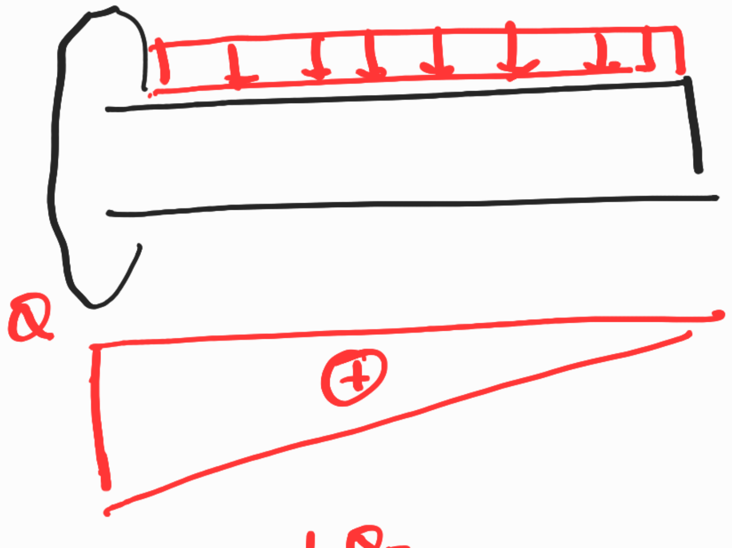
$\tau = G \gamma$

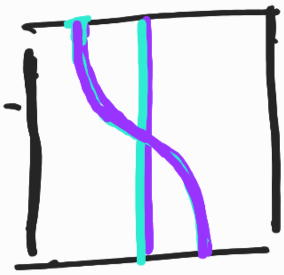




$\varepsilon_x = 0$

ALABEO ES IGUAL

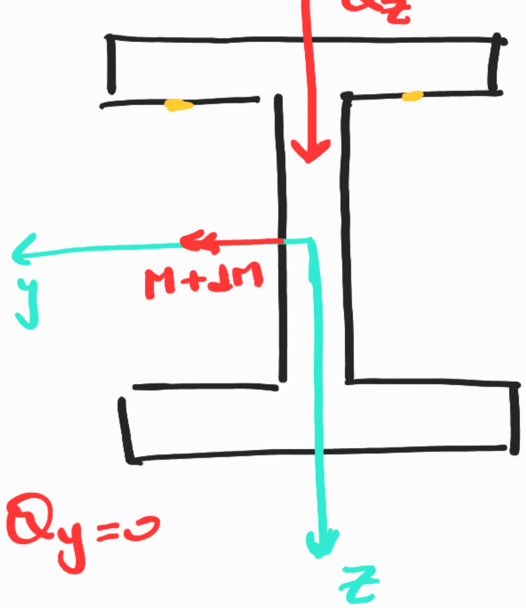


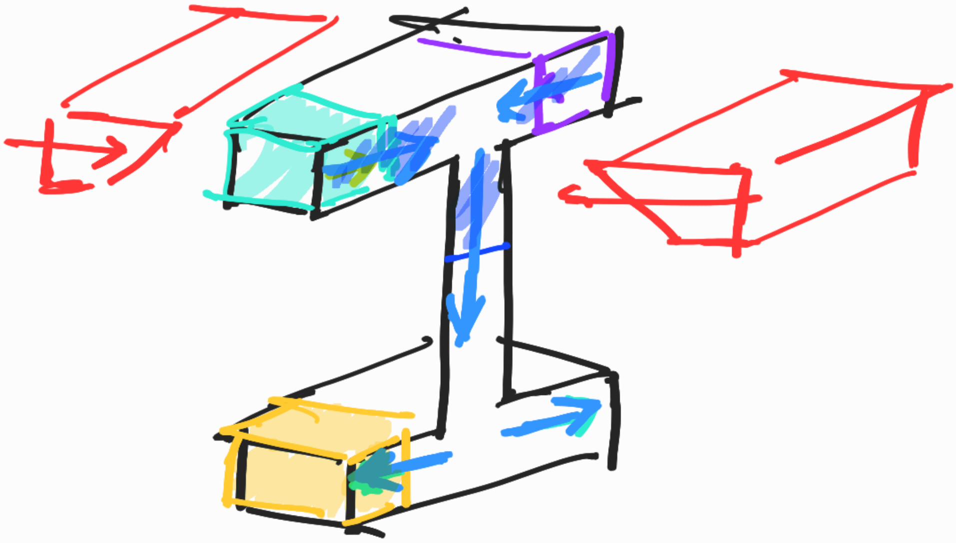


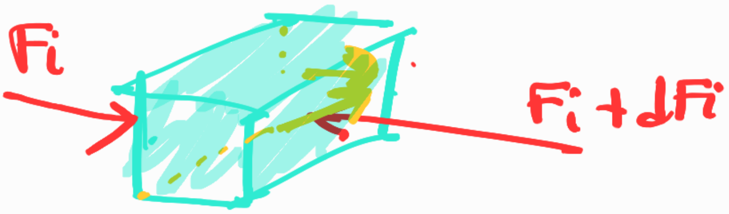
ALABEO NO ES CONSTANTE

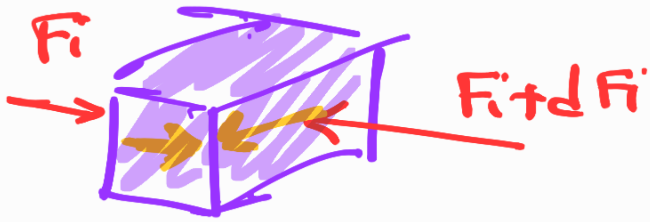
$\Downarrow$

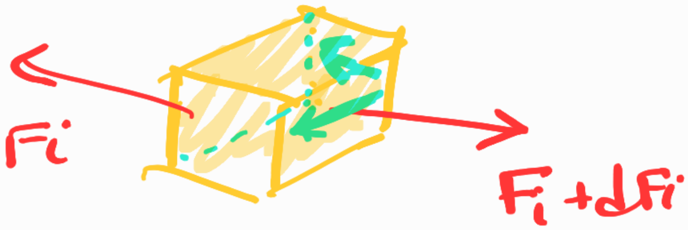
γ son despreciables

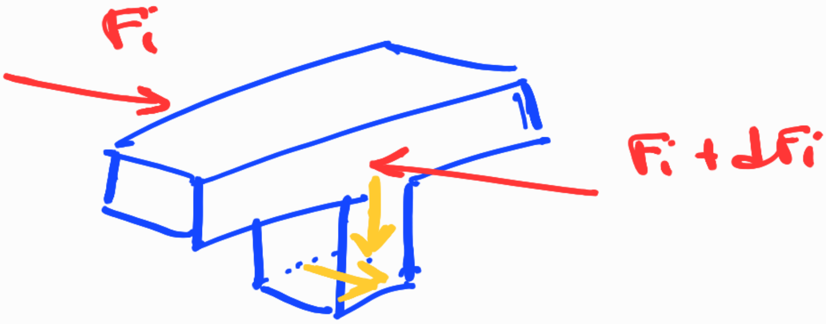








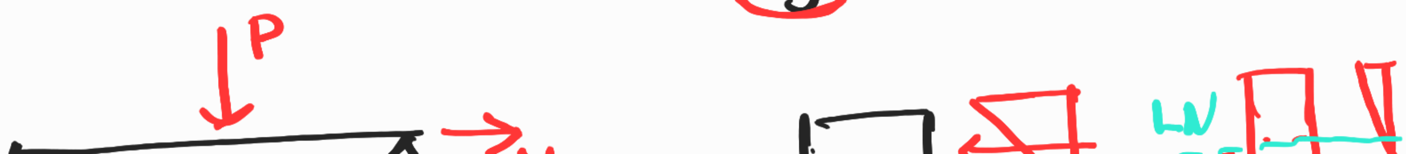


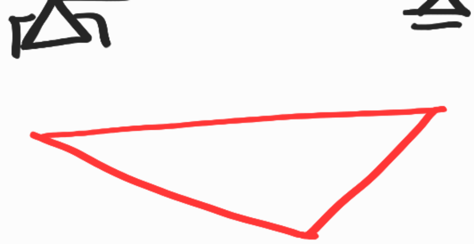


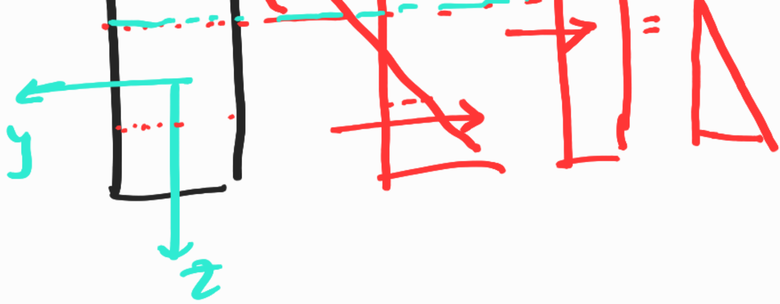
$$\tau = \frac{Q\ S}{J\ b}$$

→ PRINCIPALES DE INERCIA

$$Q_z \longrightarrow \quad \tau = \frac{Q_z \cdot S_y}{J_y \cdot b} \qquad \boxed{y \to LN}$$







1) Q

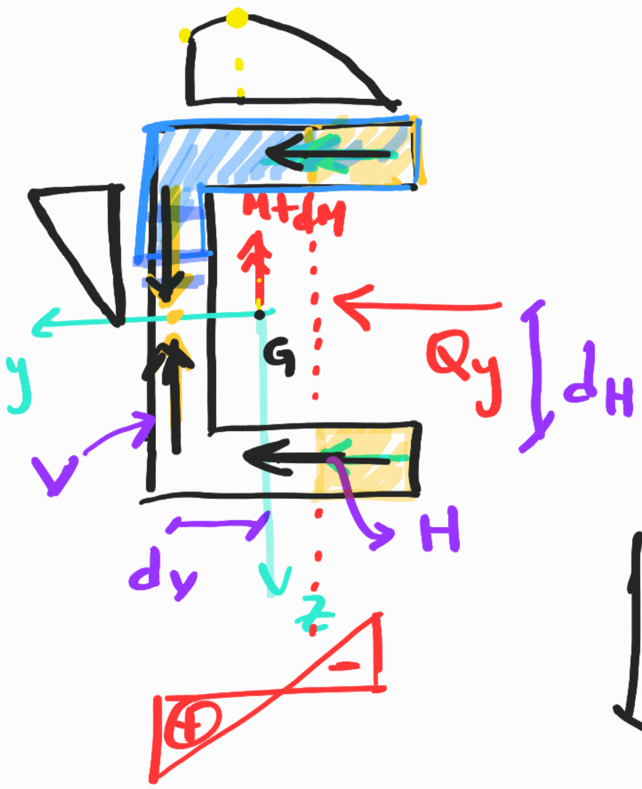
2) dM

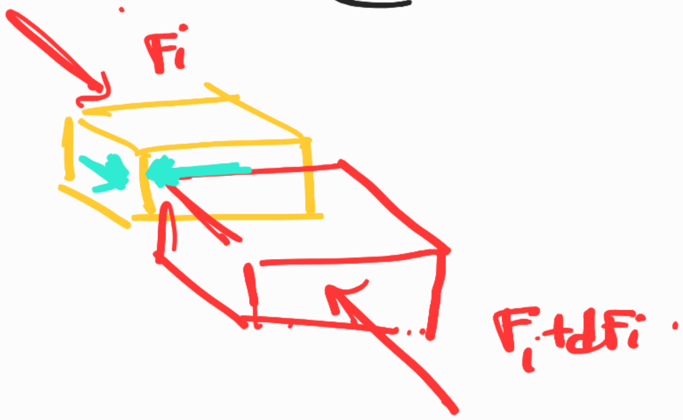
3) $d\sigma \longrightarrow$ debido a $dM \Longrightarrow d\sigma$ ↓ LN del dM

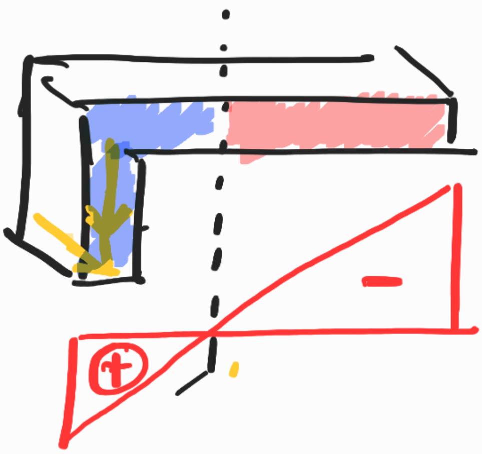
4) $\Longrightarrow$ Aparecen $\tau_{\text{resbalamiento}}$

Centro de corte

$$\tau = \frac{Q_y \cdot S_z^*}{J_z \cdot b} \qquad Q_y = \frac{dM_z}{dx}$$







Ecuaciones Equivalencia

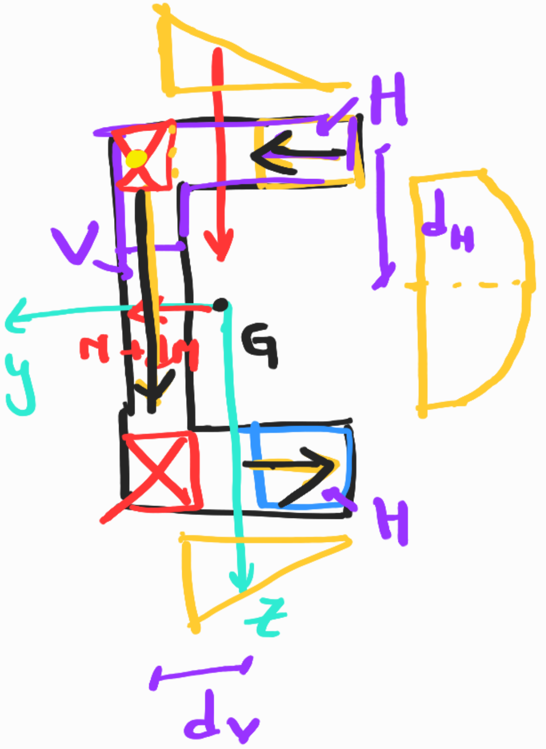
$Q_z = 0 \longrightarrow Q_z = V - V = 0$

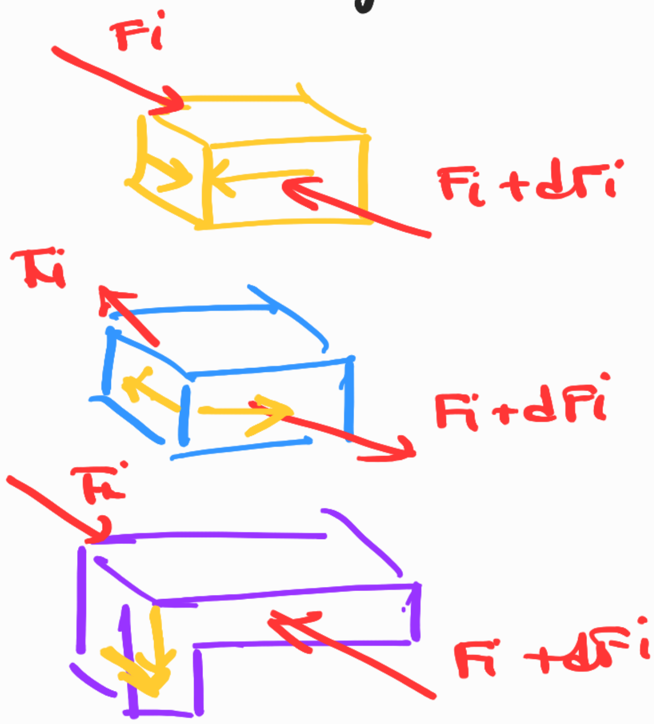
$Q_y = Q_y \longrightarrow Q_y \approx 2H$

$$M_T = \int (\tau_{xz} \cdot y + \tau_{xy} \cdot z)\, dA$$

$$M_T = V \cdot d_V - V \cdot d_V + H \cdot d_H - H \cdot d_H = 0$$

$$\tau = \frac{Q_z \cdot S_y}{J_y \cdot b}$$

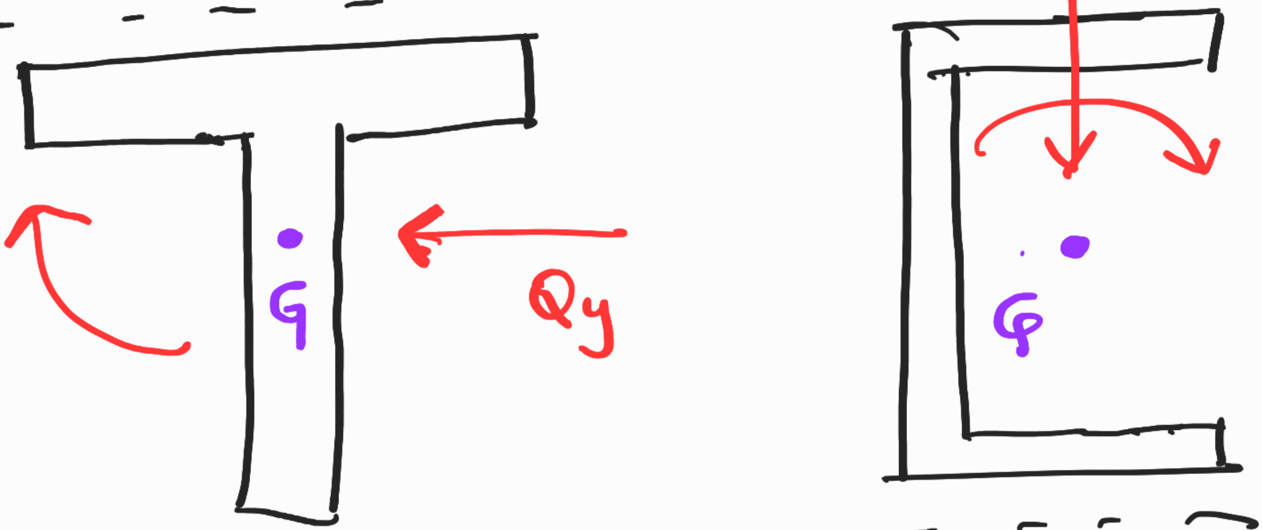


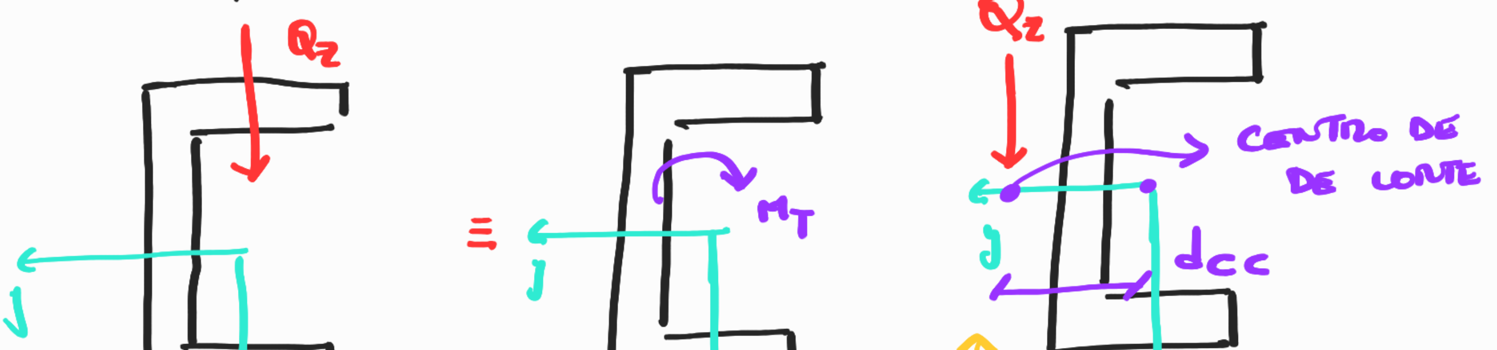


$$Q_z = V$$

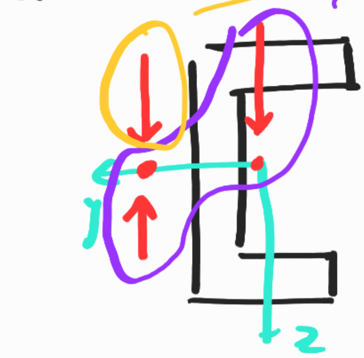
$$Q_y = 0 = H - H = 0$$

$$M_t = H \cdot d_H + H \cdot d_H + V \cdot d_v \neq 0$$

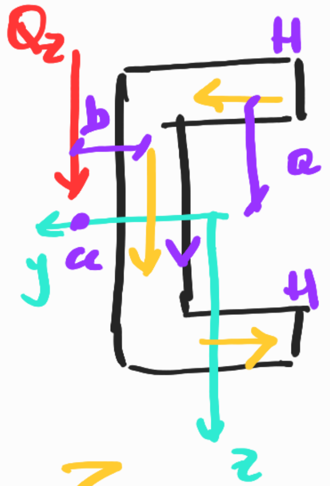




CAMBIAR CR



$$\boxed{M_t = d_{cc} \cdot Q_z}$$



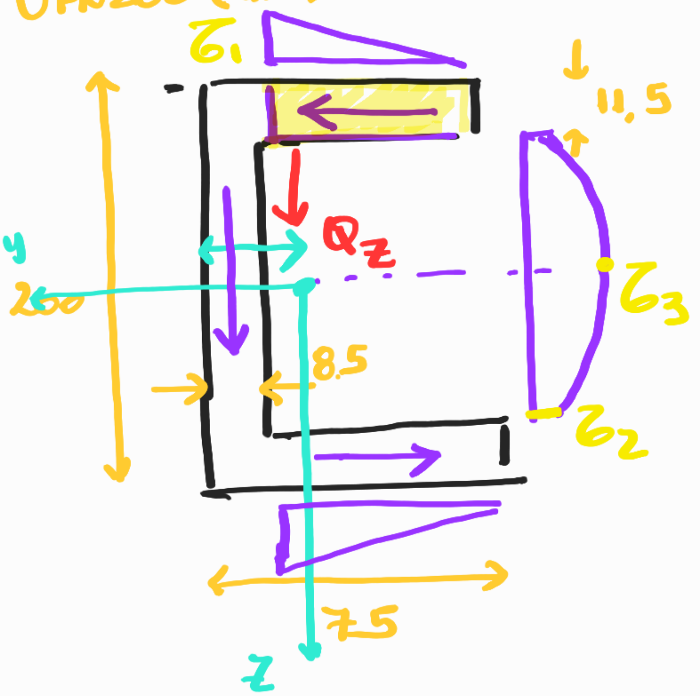
$$M_T = H \cdot a + H a - V \cdot b =$$

$$M_{T_{cc}}^{cc} = 2Ha - Vb = 0$$

$$\Rightarrow b \checkmark$$

$\tau_{\text{Jourawski}} = \tau_{\text{Jourawski}}$ con Q por G

UPN200 (mm)



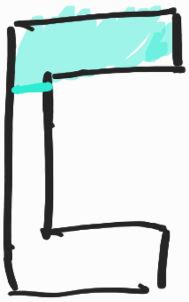
$$\tau_1 = \frac{Q_z \cdot S_y}{J_y \cdot b} \qquad J_y = 1910\,cm^4$$

$$\tau_1 = \frac{Q_z\,(1{,}15cm \cdot (7.5cm - 0{,}85cm)) \cdot \left(10cm - \frac{1{,}15cm}{2}\right)}{1910\,cm^4 \cdot 1{,}15\,cm}$$

$$\tau_1 = 0.0328\,Q_z$$

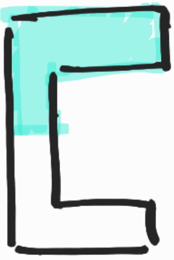
$$H = \tau_1 \cdot \frac{(7{,}5cm - 0{,}85cm) \cdot 1{,}15cm}{2}$$

$$\boxed{H = 0{,}1255\,Q_z}$$

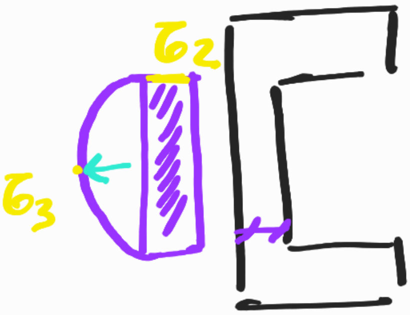


$$\tau_2 = \frac{Q_z\left(7{,}5cm \cdot 1{,}15cm\left(10cm - \frac{1{,}15cm}{2}\right)\right)}{1910\,cm^4 \cdot 0{,}85\,cm}$$

$$\tau_2 = 0{,}0501\,Q_z$$

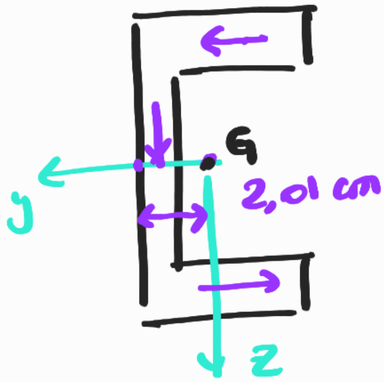


$$\tau_3 = \frac{Q_z\, S}{1910\,cm^4 \cdot 0{,}85\,cm} = \frac{Q_z\; 114\,cm^3}{1910\,cm^4 \cdot 0{,}85\,cm} = 0.0702\,Q_z$$



$$V = \tau_2\left(20\,cm - 2\cdot 1{,}15\,cm\right)\cdot 0{,}85\,cm\; +$$
$$\frac{2}{3}\left(\tau_3 - \tau_2\right)\left(20\,cm - 2\cdot 1{,}15\,cm\right)\cdot 0{,}85\,cm$$

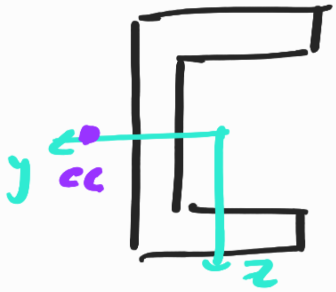
$$V = 0.955\,Q_z$$



$$M_T = H\left(20\,cm - 1{,}15\,cm\right) + V\left(2{,}01\,cm - \frac{0{,}85\,cm}{2}\right)$$

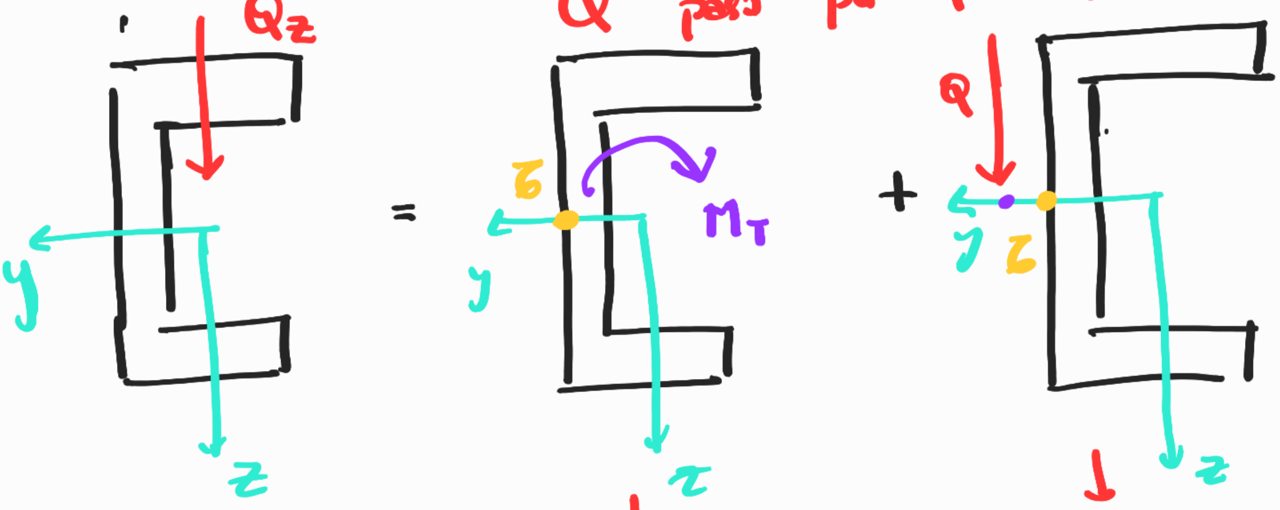
$$M_T = 3{,}88\,cm\; Q_z$$

$$d_{cc} = \frac{M_t}{Q_z} = 3{,}88\,cm$$



$$M_{T_{cc}} = 0$$

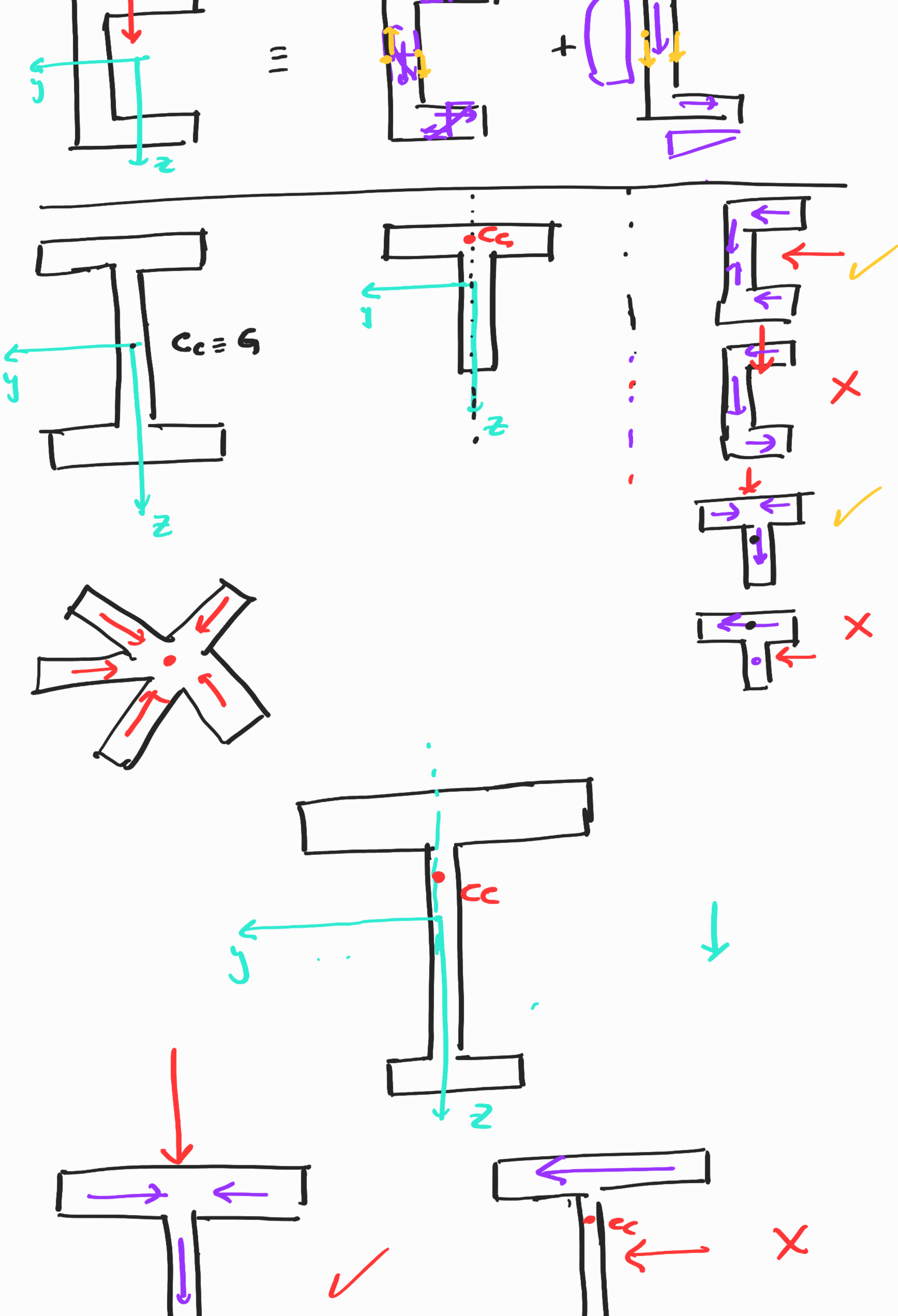
Q pasa por G $\Rightarrow M_T$



τ Saint Venant

τ Jourawski

τ — suma $\tau_{Saint} + \tau_{Jourawski}$



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Cc
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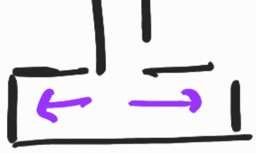

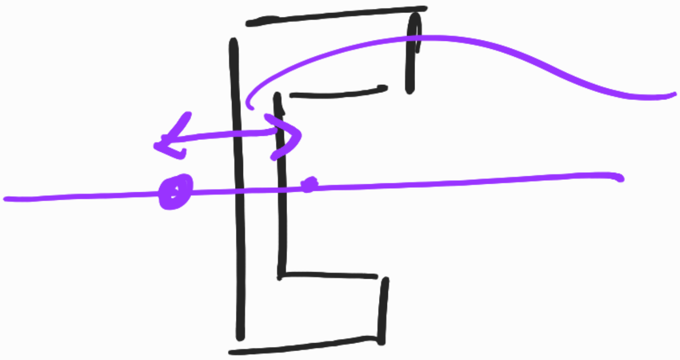

ESTA TABULADO