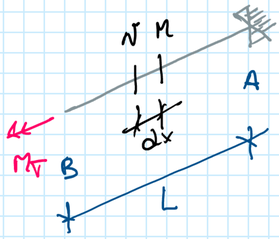
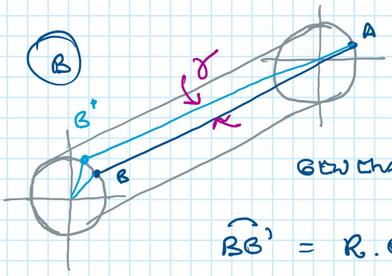
12.01 - ARAn - ST - INTRODUCCIÓN:

martes, 14 de diciembre de 2021 10:56

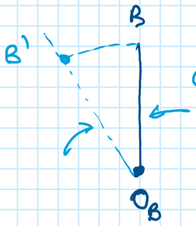




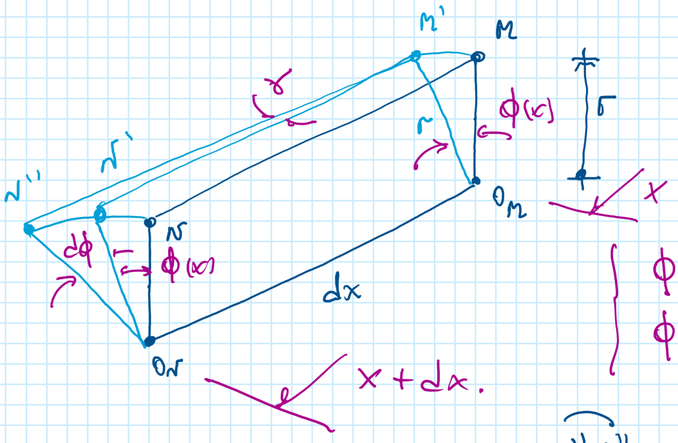
Geometría $\overline{AB} \rightarrow \overline{AB'}$

$$\widehat{BB'} = R.\phi(x) = L.\gamma$$

$$\gamma_{máx} = \frac{R}{L}.\phi(x) = \frac{R}{L}\phi_L$$



$\phi(x) = \phi(L).$



$\overline{MN}$: Geometría inicial.

$\downarrow$

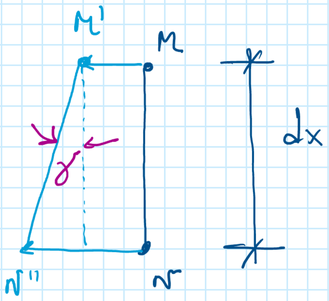
$\overline{M'N''}$: " inicial deformada

$$\begin{cases} \phi_M = \phi(x) \\ \phi_N = \phi(x) + d\phi(x) \end{cases}$$

$$\widehat{N'N''} = \gamma.dx = r\,d\phi(x)$$

$$\boxed{\gamma = r.\underbrace{\frac{d\phi(x)}{dx}}_{\chi_t(x)}}$$

$\chi_t(x)$: curvatura por torsión.



P/ una sección cualquiera $d\phi(x) = cte$.

$$\gamma = \gamma(r)$$

Si $r = 0 \rightarrow \gamma = 0.$

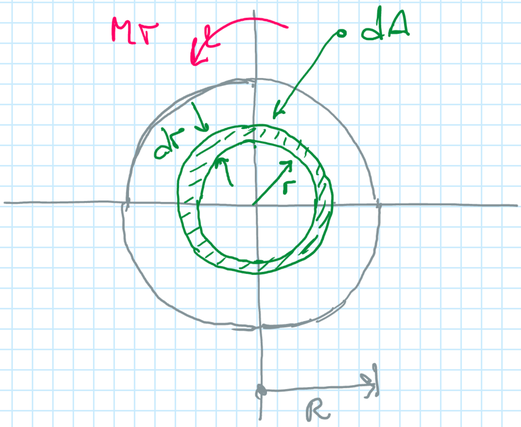
Si $r = r \rightarrow \gamma = r\,\frac{d\phi(x)}{dx}$

Si $r = R \Rightarrow \gamma_{máx} = R.\frac{d\phi(x)}{dx}$

$$\frac{d\phi(x)}{dx} = \frac{\gamma}{r} = \frac{\gamma_{máx}}{R} \rightarrow \boxed{\gamma = \gamma_{máx}.\frac{r}{R}}$$

Lineal

Estas deducciones fueron realizadas en forma independiente del tipo de material con el cual fue realizado el ensayo, y también en forma independiente del tipo de comportamiento del mismo así como de la relación entre tensiones y deformaciones.



$$dA = 2\pi r.dr$$

$$dT = \tau.dA = \tau.2\pi r\,dr$$

$$dM_T = r.dT = \tau.2\pi r^2 dr$$

$$M_T = \int dM_T = \int_0^R 2\pi r^2 \tau\,dr$$

$$\boxed{M_T = 2\pi\int_0^R \tau.r^2 dr}$$

{ Ec. de equivalencia entre tensiones y esf. interna }

12.01 - ARAn - ST - INTRODUCCIÓN:

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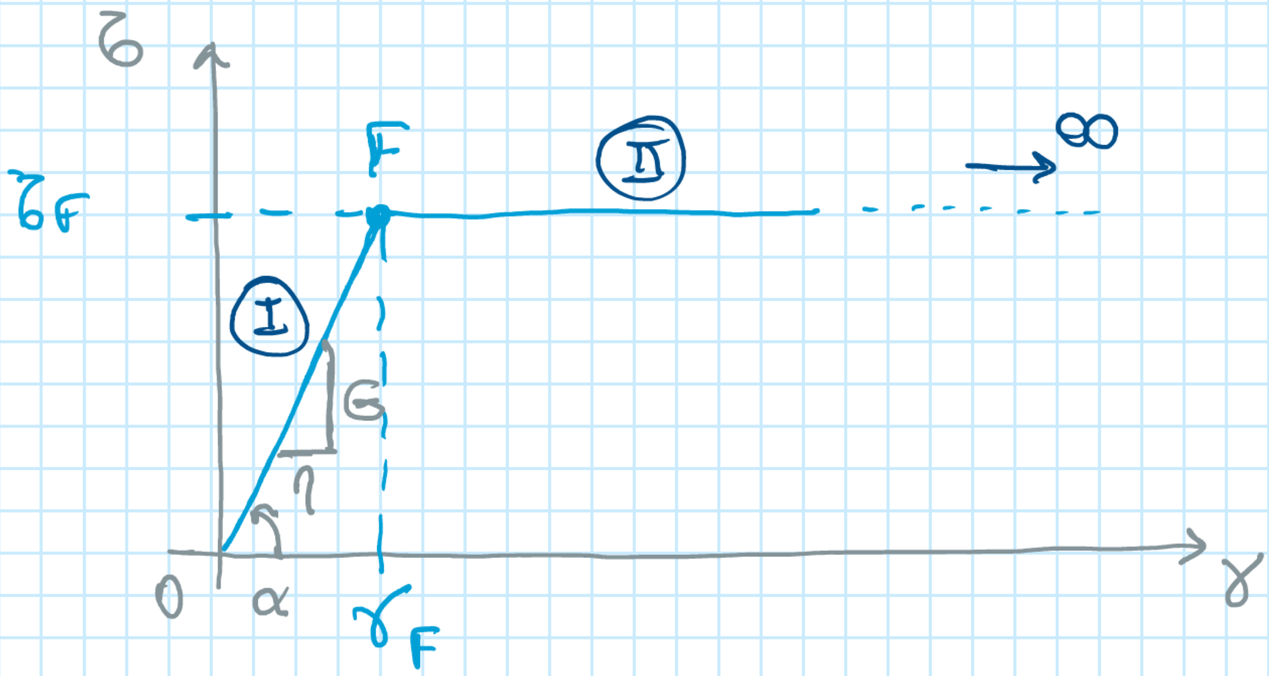
$$M_T(x) = 2\pi \int_0^R \tau \cdot r^2 \, dr$$



$\tan \alpha = G$

I Tramo OF

$\tau = f(\gamma) = G \cdot \gamma$

$\tau_F = G \cdot \gamma_F$.

II Tramo F-∞

$\tau = f(\gamma) = cte.$

$\tau = f(\gamma) = \tau_F = cte.$

$\gamma = \gamma_{máx} \cdot \frac{r}{R}$ ← Ecuación lineal

$\tau = G \cdot \gamma \rightarrow \gamma = \frac{\tau}{G}$

$$\frac{\tau}{\cancel{G}} = \frac{\tau_{máx}}{\cancel{G}} \cdot \frac{r}{R} \rightarrow \boxed{\tau = \tau_{máx} \cdot \frac{r}{R}}$$

- Si estamos dentro del período elástico: $\tau \leq \tau_F$

$$M_T = 2\pi \int_0^R \tau \cdot r^2 \, dr = 2\pi \int_0^R \frac{\tau_{máx}}{R} r^3 \, dr$$

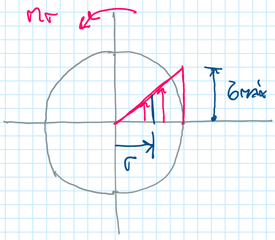
$$M_T = 2\pi \frac{\tau_{máx}}{R} \cdot \frac{r^4}{4}\Big|_0^R = \cancel{2}^{1}\pi \frac{\tau_{máx}}{\cancel{R}} \cdot \frac{R^{\cancel{4}^{3}}}{\cancel{4}_{2}}$$

$$M_T = \frac{1}{2}\pi R^3 \tau_{máx} = \frac{\tau_{máx}}{R} \cdot \underbrace{\frac{\pi R^4}{2}}_{I_p} = \frac{\tau_{máx}}{R} \cdot \frac{\pi D^4}{32}$$

$$M_T = \frac{\tau_{máx}}{R} \cdot I_p \rightarrow \boxed{\tau_{máx} = \tau_R = \frac{M_T}{I_p} \cdot R}$$

12.02 - COMPORTAMIENTO ELASTO-PLÁSTICO:

martes, 14 de diciembre de 2021 11:28

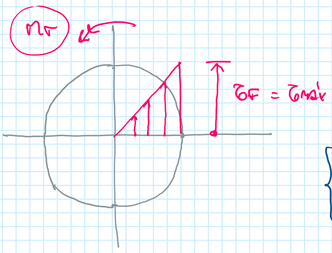


$$\begin{cases} \tau_{máx} = \dfrac{M_T}{R} \cdot I_p \\ \tau(x,r) = \dfrac{M_T}{r} \cdot I_p \end{cases}$$

$$\tau_{máx} \leq \tau_F$$

Si $\tau_{máx} = \dfrac{\tau_F}{CS} = \tau_{adm} \rightarrow M_{T,adm}$

$$\boxed{M_{T,adm} = \tau_{adm} \cdot \frac{I_p}{R}}$$



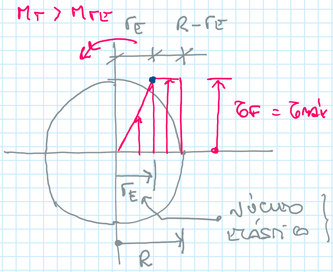
$$\tau_{máx} = \tau_F = \frac{M_{T,E}}{I_p} \cdot R$$

$$M_{T,E} = \tau_F \cdot \frac{I_p}{R}$$

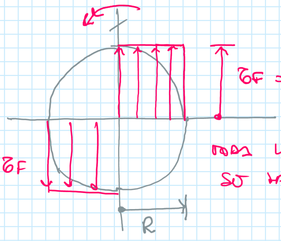
$$\begin{cases} \text{Si } M_T(x) \leq M_{T,E} \rightarrow \text{elástico} \\ \text{Si } M_T(x) > M_{T,E} \rightarrow \text{anelástico.} \end{cases}$$

$$M_{T,E} = M_{T,F} = \frac{\tau_F}{\cancel{R}} \cdot \frac{\pi R^{\cancel{4}\,3}}{2} \rightarrow \boxed{M_{T,E} = M_{T,F} = \frac{1}{2}\pi R^3 \tau_F} \leftarrow$$

$M_T > M_{TE}$



$M_T = M_{T,P}$



toda la sección se ha plastificado.

sección parcialmente plastificada:

$$\begin{cases} 0 \leq r \leq r_E \rightarrow \tau_r = \dfrac{r}{r_E}\tau_F \\ r_E \leq r \leq R \rightarrow \tau_r = \tau_F \end{cases}$$

$$M_T = 2\pi \int_0^R r^2 \cdot \tau_r \cdot dr.$$

$$M_T = 2\pi \int_0^{r_E} r^2 \frac{r}{r_E} \cdot \tau_F \, dr + 2\pi \int_{r_E}^R r^2 \tau_F \, dr$$

$$M_T = 2\pi \frac{\tau_F}{r_E} \cdot \frac{r^4}{4}\Big|_0^{r_E} + 2\pi \tau_F \frac{r^3}{3}\Big|_{r_E}^R$$

$$M_T = \cancel{2}^{\,1}\pi \frac{\tau_F}{r_E} \cdot \frac{r_E^4}{\cancel{4}_{\,2}} + \frac{2}{3}\pi\tau_F\left[R^3 - r_E^3\right] \leftarrow$$

$$M_T = \frac{\tau_F}{r_E} \cdot \frac{\pi r_E^4}{2} + \frac{2}{3}\pi\tau_F\left(R^3 - r_E^3\right) = \boxed{\frac{\tau_F}{r_E} I_{p,NE} + \frac{2}{3}\pi\tau_F\left(R^3 - r_E^3\right)} \quad (1).$$

momento de inercia polar del núcleo elástico $I_{p,NE}$

$$M_T = \pi \frac{\tau_F}{\cancel{r_E}} \cdot \frac{r_E^{\cancel{3}\,3}}{2} + \frac{2}{3}\pi\tau_F\left(R^3 - r_E^3\right) =$$

$$M_T = \frac{1}{2}\pi r_E^3 \tau_F + \frac{2}{3}\pi\tau_F R^3 - \frac{2}{3}\pi r_E^3 \tau_F =$$

$$= \underbrace{\frac{2}{3}\pi R^3 \tau_F} - \frac{1}{6}\pi r_E^3 \tau_F$$

$$\boxed{M_T = \frac{2}{3}\pi R^3 \tau_F \left[1 - \frac{1}{4}\frac{r_E^3}{R^3}\right]} \leftarrow \quad (2)$$

$$\boxed{M_{T,E} = M_{T,F} = \frac{1}{2}\pi R^3 \tau_F} \leftarrow \qquad \frac{\frac{2}{3}\pi R^2 \cdot \tau_F}{\underbrace{\frac{1}{2}\pi R^3 \tau_F}_{M_{TE}}} = \frac{4}{3} \rightarrow \frac{2}{3}\pi R^3 \tau_F = \frac{4}{3} M_{TE}$$

$$\boxed{M_T = \frac{4}{3} M_{TE}\left[1 - \underbrace{\frac{1}{4}\frac{r_E^3}{R^3}}\right]} \leftarrow \quad (3)$$

En la plastificación total $\rightarrow r_E = 0$

$$\boxed{M_{T,P} = \frac{4}{3} M_{TE} = \frac{2}{3}\pi R^3 \tau_F}$$ momento torsor plástico

12.03 - VARIABLES CINEMÁTICAS:

martes, 14 de diciembre de 2021 12:18

$$\gamma = \frac{r}{L} \cdot \phi$$

Si $r = R$ y en $r = R \rightarrow \sigma = \sigma_F \wedge r < R \rightarrow \sigma < \sigma_F \rightarrow$

$\rightarrow \boxed{M_T = M_{TE}}$

$\gamma = \gamma_F \wedge \phi = \phi_E$ = ángulo de torsión en el comienzo de la plastificación p/ la sección.

$$\boxed{\gamma_F = \gamma_R = \frac{R}{L}\phi_E}$$

$\rightarrow$ Si $\boxed{M_T > M_{TE}}$ $\quad \phi(x) = \int \frac{M_T(x)}{G \cdot I_P} \cdot dx$. NO la puedo aplicar

En $r = r_E \rightarrow \gamma = \gamma_F$.

$$\gamma_F = \frac{r_E}{L} \cdot \phi$$

$$\frac{r_E}{\cancel{L}} \cdot \phi = \frac{R}{\cancel{L}} \cdot \phi_E$$

$$\boxed{\frac{r_E}{R} = \frac{\phi_E}{\phi}}$$

$$M_T = \frac{2}{3}\pi R^3 \sigma_F \left(1 - \frac{r_E^3}{R^3}\right) = \frac{2}{3}\pi R^3 \sigma_F \left(1 - \frac{\phi_E^3}{\phi^3}\right) \quad (4a)$$

$$M_T = \frac{4}{3} M_{TE} \left(1 - \frac{\phi_E^3}{\phi^3}\right) \quad (4b)$$

$$\phi = x \cdot L \qquad \frac{d\phi}{dx} = x \rightarrow d\phi = x\,dx.$$

$$\boxed{M_T = \frac{2}{3}\pi R^3 \sigma_F \left(1 - \frac{x_E^3}{x^3}\right) = \frac{4}{3} M_{TE} \left(1 - \frac{x_E^3}{x^3}\right)} \quad (5)$$

Gráficos $\quad M_T = f(\phi) \rightarrow M_T = f(x)$

diagrama momento curvatura.

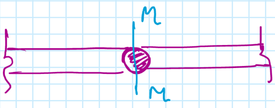
$$M_{T,P} = \frac{2}{3}\pi R^3 \sigma_F$$

Si $M_T = M_{T,P} \rightarrow \cancel{\frac{2}{3}\pi R^3 \sigma_F} = \cancel{\frac{2}{3}\pi R^3 \sigma_F}\left(1 - \frac{\phi_E^3}{\phi^3}\right)$

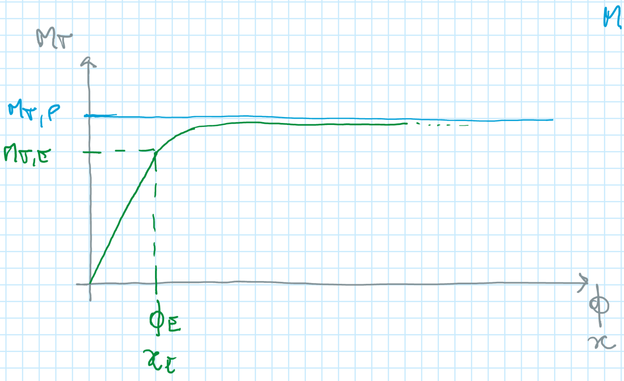
$\cancel{1} = \cancel{1} - \frac{\phi_E^3}{\phi^3} \rightarrow 0 = \frac{\phi_E^3}{\phi^3} \leftarrow \phi_E \Rightarrow M_T = M_{T,E}$

$\phi_P = \phi \quad \phi \rightarrow M_T = M_T$

Si $M_T \rightarrow M_{T,P} \rightarrow \phi_P \rightarrow \infty$



$M_M = 0$



Si $\phi \leq \phi_E \rightarrow \phi = \frac{M_T}{G \cdot I_P} \cdot L \rightarrow \underbrace{M_T = \frac{G \cdot I_P}{L} \cdot \phi}_{} = G I_P x$

Si $\phi > \phi_E \rightarrow M_T = \frac{4}{3} M_{T,E}\left[1 - \frac{\phi_E^3}{\phi^3}\right] = \frac{4}{3} M_{T,E}\left[1 - \frac{x_E^3}{x^3}\right]$

12.04 - EJERCICIO:

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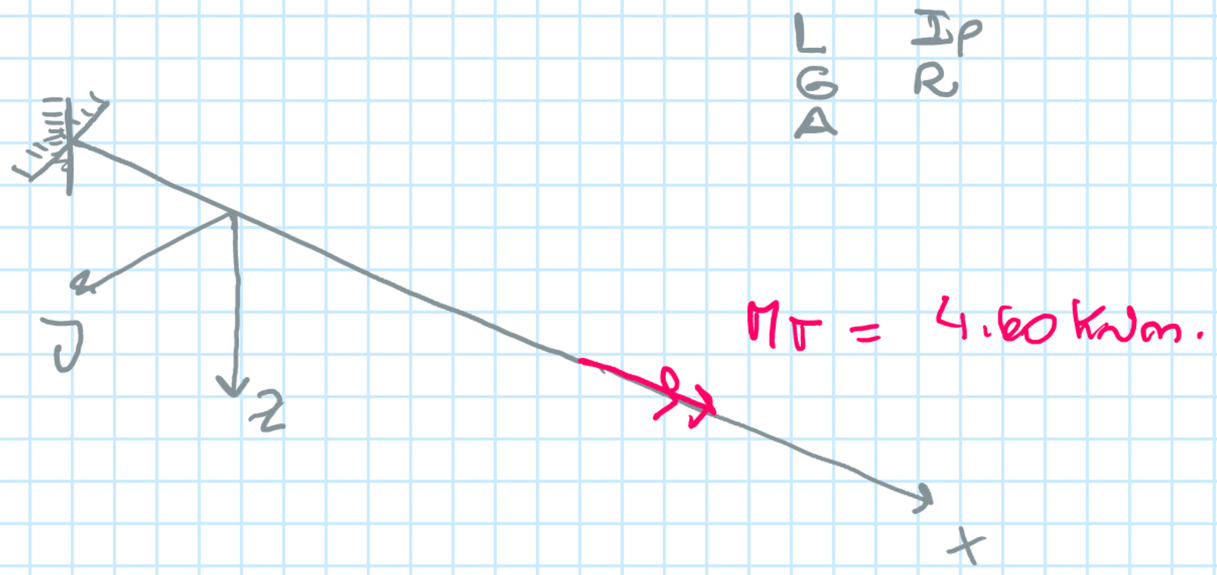


$L = 1{,}20\ m$

$D = 50\ mm = 5\ cm$

COMPORTAMIENTO DE MATERIAL

ELASTO PLÁSTICO IDEAL

$\tau_F = 15\ kN/cm^2$

$G = 7700\ kN/cm^2$

$$M_{T,E} = \frac{\tau_F \cdot I_P}{R} \quad ; \quad I_P = \frac{\pi D^4}{32} = \frac{\pi \cdot 5^4}{32} = 61{,}36\ cm^4$$

$$M_{T,E} = \frac{15\ kN}{cm^2} \cdot \frac{61{,}36\ cm^4}{2{,}5\ cm} = 3{,}68\ kNm.$$

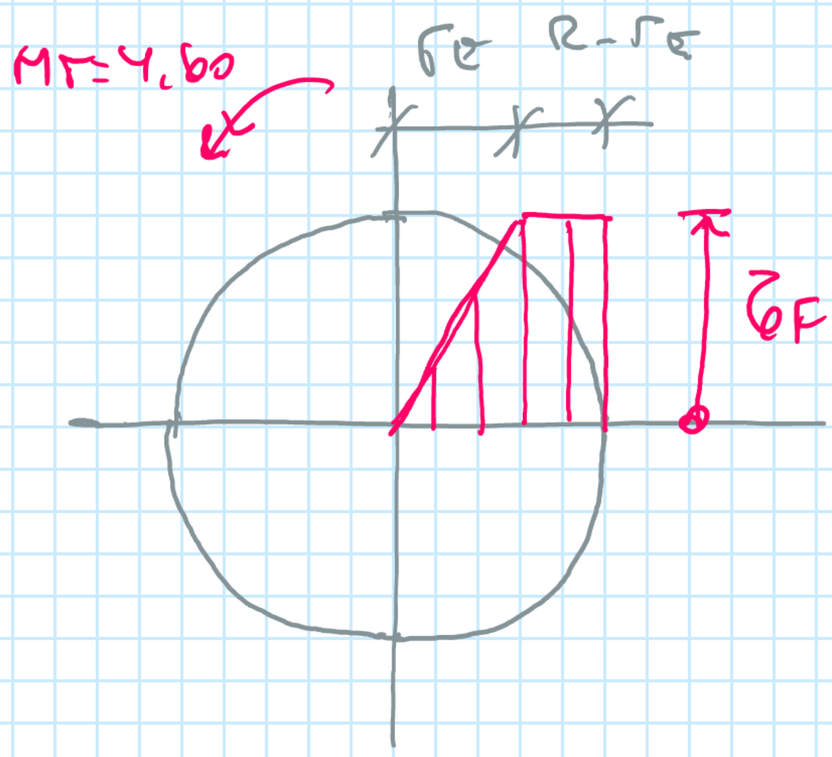
$$M_{T,P} = \frac{4}{3} M_{T,E} = \frac{4}{3}\ 3{,}68 = 4{,}91\ kNm$$

$$\left.\begin{array}{ccccc} M_{TE} & < & M_T & < & M_{TP} \\ 3{,}68 & < & 4{,}60 & < & 4{,}91 \end{array}\right\} \longrightarrow \text{PLASTIFICADA PARCIALMENTE.}$$

RADIO DEL NÚCLEO ELÁSTICO

$$M_T = \frac{4}{3} M_{T,E} \left(1 - \frac{r_E^3}{R^3}\right)$$

$$r_E = \sqrt[3]{\left(1 - \frac{3}{4}\frac{M_T}{M_{TE}}\right) R^3} \longrightarrow \boxed{r_E = 1{,}578\ cm = 15{,}78\ mm}$$



ÁNGULO DE TORSIÓN:

$$\left.\begin{array}{l} \gamma_F \cdot L = R \cdot \phi_E \\ \gamma_F L = r_E\, \phi \end{array}\right\} \longrightarrow \frac{R\,\phi_E}{\cancel{L}} = \frac{r_E\,\phi}{\cancel{L}}$$

$$\phi = \frac{R \cdot \phi_E}{r_E} = \frac{25\ mm \cdot 0{,}09351\ rad}{15{,}78\ mm}$$

$$\boxed{\phi_E = \frac{M_{TE}\, L}{G\, I_P}} = \frac{368\ \cancel{kNcm}\ 120\ \cancel{cm}}{7700\ \frac{\cancel{kN}}{\cancel{cm^2}} \cdot 61{,}36\ \cancel{cm^4}} = 0{,}09351\ rad.$$

$$\boxed{\phi = 0{,}1481\ rad}$$

$$\gamma_E = \frac{\phi_E}{L} = 7{,}7925 \cdot 10^{-4}$$

$$\gamma = \frac{\phi}{L} = 1{,}234 \cdot 10^{-3}\ \frac{rad}{cm}.$$

12.04 - EJERCICIO:

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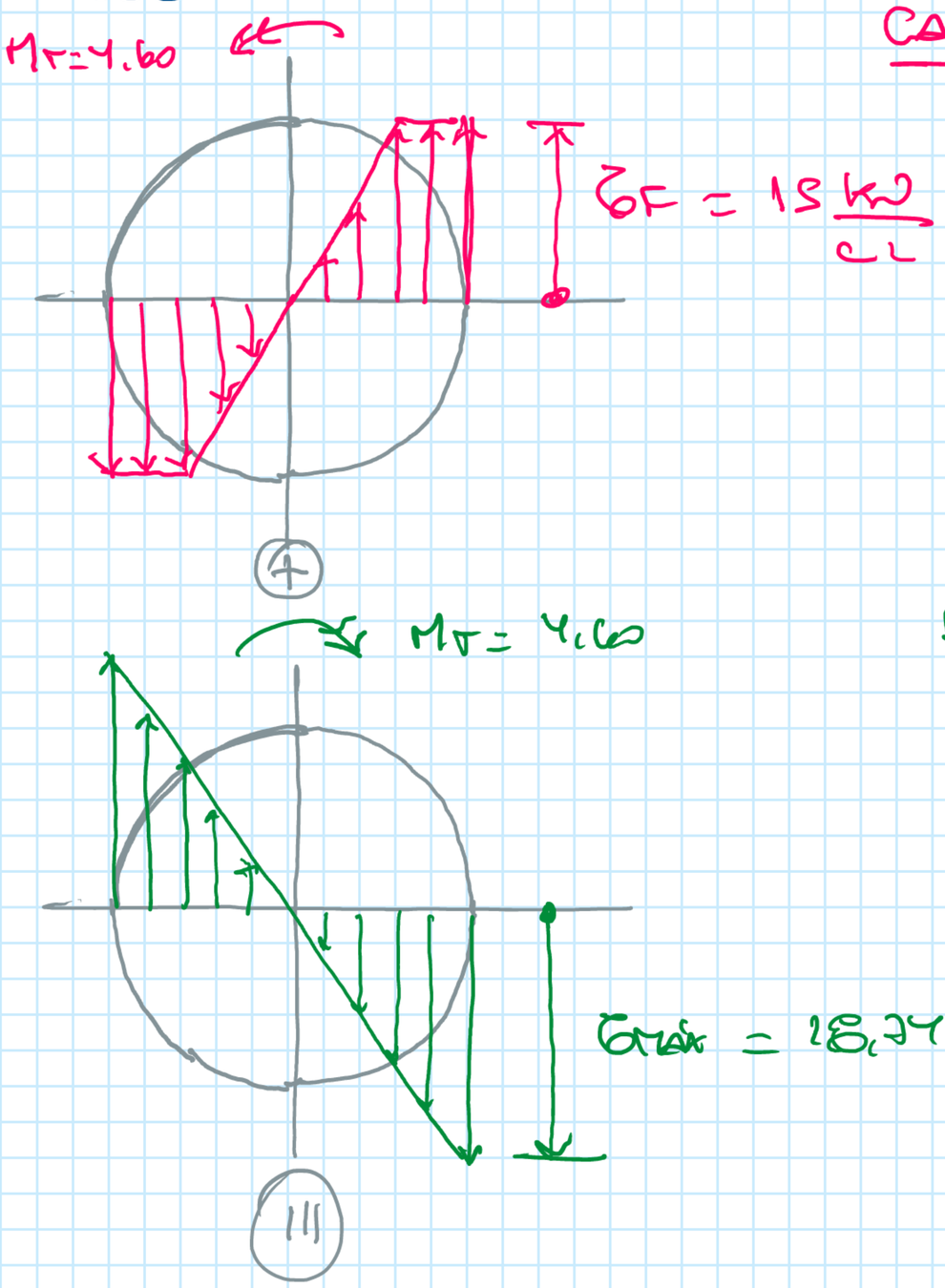
DESCARGA: → ELÁSTICA.

CARGA.

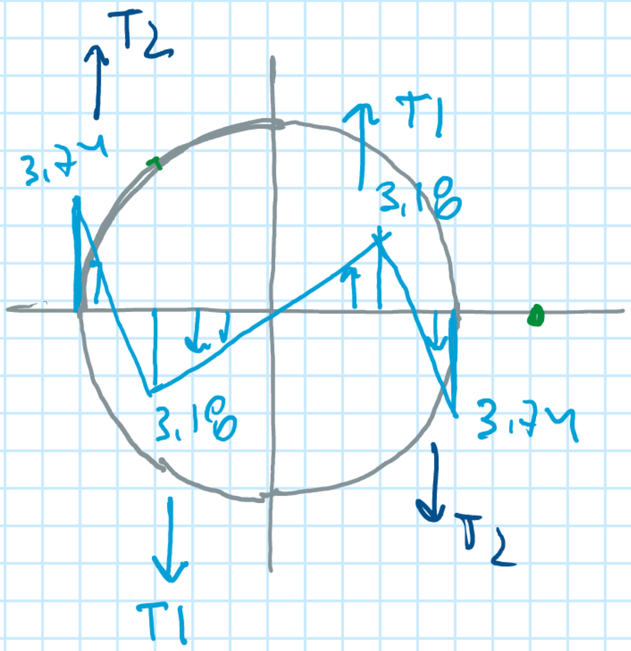


DESCARGA: → ELÁSTICA

$$\tau = \frac{M_T}{I_P} \cdot R = \frac{460\ kNcm}{61,36\ cm^4} \cdot 2,5\ cm$$

$$\tau_{máx} = 18,24\ kN/cm^2.$$

$$\tau = \frac{M_T \cdot r}{I_P} = \frac{460 \cdot 1,578}{61,36} = 11,82\ \frac{kN}{cm^2}$$



TENSIONES RESIDUALES.

↳ AUTOEQUILIBRADO.

$$M_{T1} + M_{T2} = 0$$

$$T_1 z_1 + T_2 z_2 = 0$$

$$\phi_{descarga} = \frac{M_{T,desc} \cdot L}{G \cdot I_P} \leftarrow \text{crítico}$$

$$\phi_{residual} = \phi_{carga} - \phi_{descarga}.$$

→ X descarga X residual