SOLICITACIÓN POR FLEXIÓN EN RÉGIMEN ANELÁSTICO:

I) → FLEXIÓN SIMPLE.

II) → FLEXIÓN COMPUESTA.

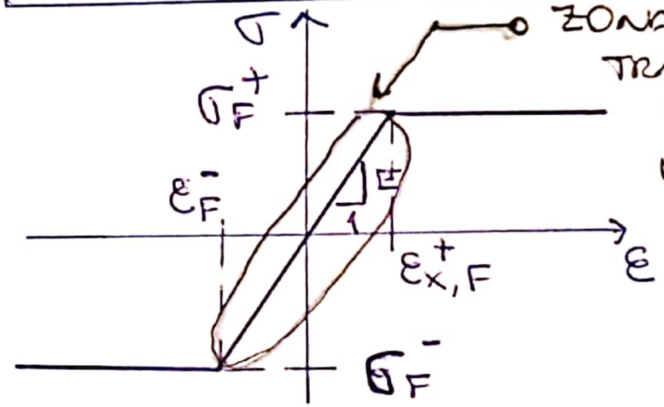
1) PARTIENDO DE LA HBN:

$$\begin{cases} \varepsilon_{x,z} = \dfrac{z}{z_{máx}} \cdot \varepsilon_{x,máx} \\ \varepsilon_{x,z} = \dfrac{z}{1} \cdot \bar{\varepsilon}_x \end{cases}$$

2) ECS DE EQUIVALENCIA:

$$\begin{cases} N = 0 = \int_A \sigma_x \cdot dA \\ M_y = \int_A \overbrace{\sigma_x \cdot z \cdot dA}^{dF} \end{cases}$$

3) PERÍODO ELÁSTICO



4) EXPRESIÓN DE FLEXIÓN:

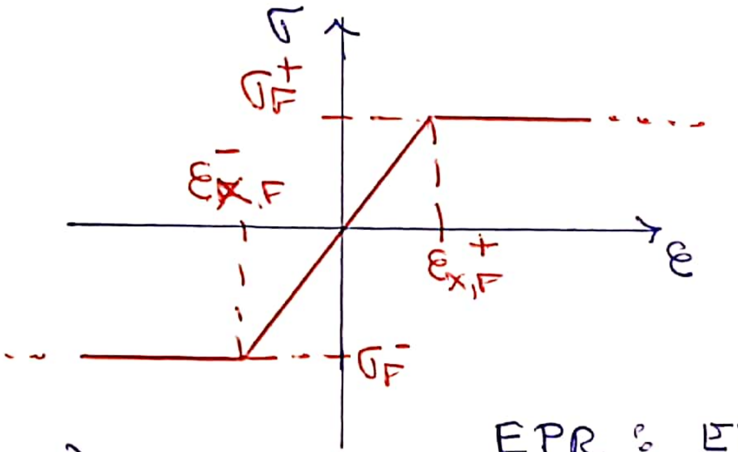
$$\sigma_x = \frac{M_y(x)}{I_y} \cdot z$$

↳ FLEXIÓN SIMPLE RECTA.

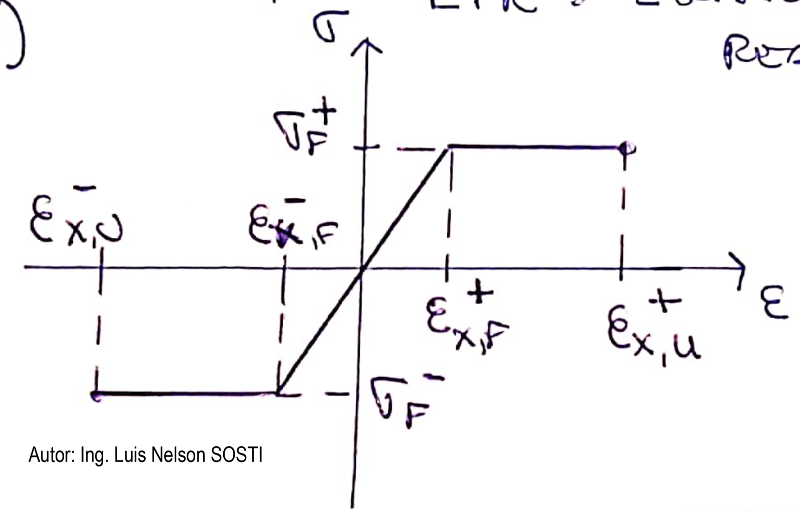
FLEXIÓN SIMPLE RECTA - ANELÁSTICO

I → TIPOS DE COMPORTAMIENTO DEL MATERIAL:

1) → EPI: ELASTO-PLÁSTICO IDEAL.

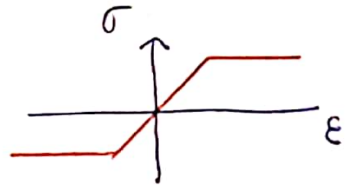


2) EPR: ELASTO-PLÁSTICO REAL.

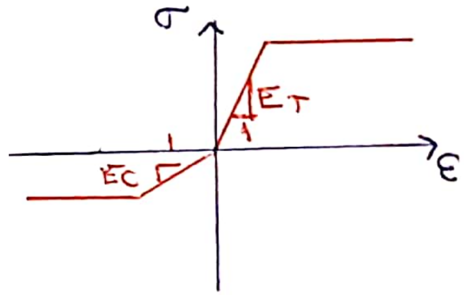


3) - VARIOS:

a) → $E_T = E_C$.



b) → $E_T \neq E_C$.



c) → $\begin{cases} \sigma_F^+ = \sigma_F^- \\ \sigma_F^+ \neq \sigma_F^- \end{cases}$

d) → $\begin{cases} \varepsilon_F^+ = \varepsilon_F^- \\ \varepsilon_F^+ \neq \varepsilon_F^- \end{cases}$

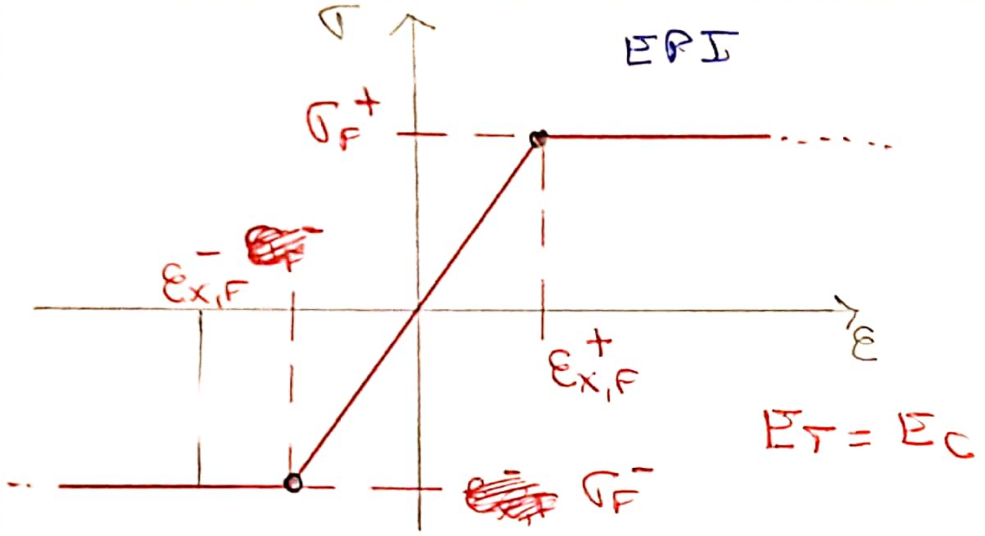
II → TIPOS DE SECCIONES:

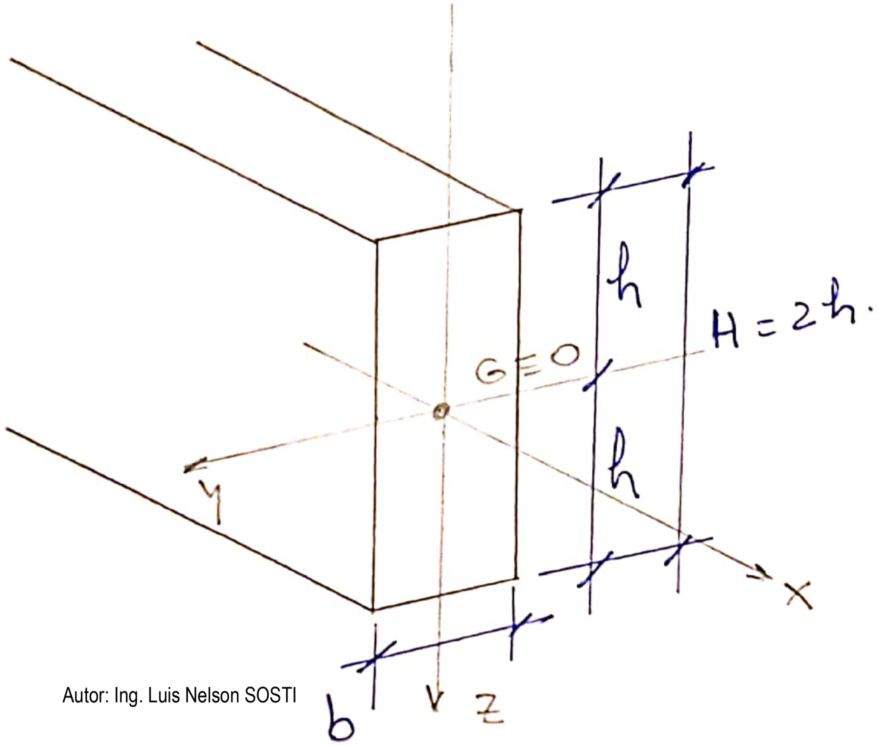
1) → DE DOBLE SIMETRÍA.

2) → " SIMPLE SIMETRÍA.

3) → CUALESQUIERA S/ EJES DE SIMETRÍA.

SECCIÓN RECTANGULAR - EPI. - DIAGRAMA BILINEAL:





PERÍODO ELÁSTICO

- Si $\sigma_{máx} \leq \sigma_F \rightarrow \sigma_{máx} = \dfrac{M_y}{I_y} \cdot h.$

- $\sigma_{ADM} = \dfrac{\sigma_F}{CS} \rightarrow \sigma_{ADM} = \dfrac{M_{y,ADM}}{I_y} \cdot h.$

$$M_{y,ADM} = \frac{I_y}{h} \cdot \sigma_{ADM} = \frac{b(2h)^3}{12\,h}\,\sigma_{ADM}$$

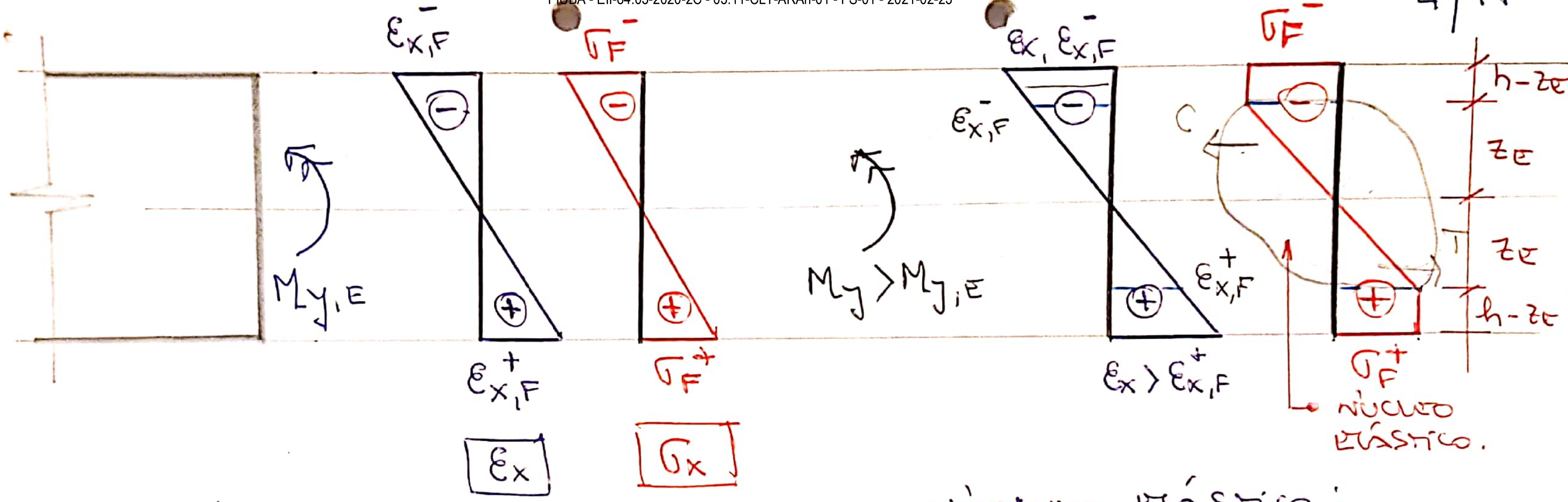
$$\boxed{M_{y,ADM} = \frac{2}{3}\,b h^2 \sigma_{ADM}} \quad (1)$$

- $\sigma_{máx} = \sigma_F \rightarrow \sigma_F = \dfrac{M_{y,E}}{I_y} \cdot h.$

$$M_{y,E} = \frac{I_y}{h} \cdot \sigma_F \rightarrow \boxed{M_{y,E} = \frac{2}{3}\,b h^2 \sigma_F} \quad (2)$$

$M_{y,E}$: MOMENTO FLECTOR ELÁSTICO.

$M_{y,E} = M_{y,F}$



- EN RÉGIMEN ANELÁSTICO SIGUE VALIENDO $\boxed{HBN}$

$$\sigma_{x,F} = \frac{M_{y,E}}{I_y}\, h = \frac{M_{y,E}}{I_y/h} = \frac{M_{y,E}}{S_y}$$

S_y: MÓDULO RESISTENTE ELÁSTICO.

$$\boxed{M_{y,E} = \sigma_{x,F} \cdot S_y} \quad (3).$$

- NÚCLEO ELÁSTICO:

$$-z_E \leq z \leq z_E$$

- ZONA PLÁSTICA:

$$|(h - z_E)|$$

I $\quad z_E \leq z \leq h$

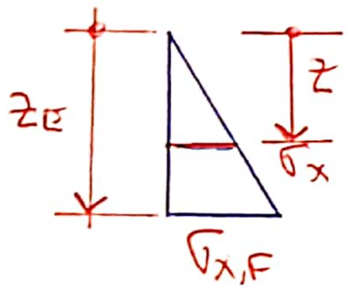
II $\quad -h \leq z \leq -z_E$.

ECS. DE EQUIVALENCIA:

$$M_y = \int_A \sigma_x . z . dA = \int_{-h}^{h} \sigma_x . z . \underbrace{b\, dz}_{dA} = 2 . \int_0^h \sigma_x . z . b\, dz$$

$$M_y = 2 \left[\int_0^{z_E} \sigma_x . z . b\, dz + \int_{z_E}^{h} \sigma_x . z . b\, dz \right]$$

$$0 \leq z \leq z_E \rightarrow \frac{\sigma_x}{z} = \frac{\sigma_{x,F}}{z_E} \rightarrow \sigma_x = \sigma_{x,F} . \frac{z}{z_E}$$



$$z_E \leq z \leq h \rightarrow \sigma_x = \sigma_{x,F} .$$

$$M_y = 2 \left[\int_0^{z_E} \sigma_{x,F} \frac{z^2}{z_E} b\, dz + \int_{z_E}^{h} \sigma_{x,F} . z\, b\, dz \right] .$$

$$M_y = 2 \left[\frac{\sigma_{x,F} . b}{z_E} \int_0^{z_E} z^2\, dz + \sigma_{x,F} . b \int_{z_E}^{h} z\, dz \right]$$

$$M_y = 2\left[\frac{\sigma_{x,F} \cdot b}{z_e} \cdot \frac{z^3}{3}\Big|_0^{z_e} + \sigma_{x,F} \cdot b \; \frac{z^2}{2}\Big|_{z_e}^{h}\right] = 2\left[\frac{\sigma_{x,F}\, b}{\cancel{z_e}}\,\frac{z_e^{\cancel{3}\,2}}{3} + \sigma_{x,F}\, b\left(\frac{h^2}{2} - \frac{z_e^2}{2}\right)\right]$$

$$M_y = \frac{2}{3}\, b \cdot z_e^2 \cdot \sigma_{x,F} + b \cdot (h^2 - z_e^2)\, \sigma_{x,F} \,.$$

$$M_y = \frac{2}{3}\, b\, z_e^2 \cdot \sigma_{x,F} + b h^2 \sigma_{x,F} - b z_e^2 \sigma_{x,F}$$

$$\boxed{M_y = b h^2 \sigma_{x,F} - \frac{1}{3}\, b z_e^2 \sigma_{x,F}} \quad (4)$$

$$\boxed{M_y = b h^2 \sigma_{x,F}\left[1 - \frac{1}{3}\,\frac{z_e^2}{h^2}\right]} \quad (5).$$

$$(2) \quad M_{y,e} = \frac{2}{3}\, b h^2 \sigma_{x,F}$$

$$\frac{M_y}{M_{y,e}} = \frac{3}{2} \rightarrow \boxed{M_y = \frac{3}{2} M_{y,e}}$$

$$\boxed{M_y = \frac{3}{2} M_{y,e}\left[1 - \frac{1}{3}\,\frac{z_e^2}{h^2}\right]} \quad (6)$$

SECCIONES "PARCIALMENTE PLASTIFICADAS".

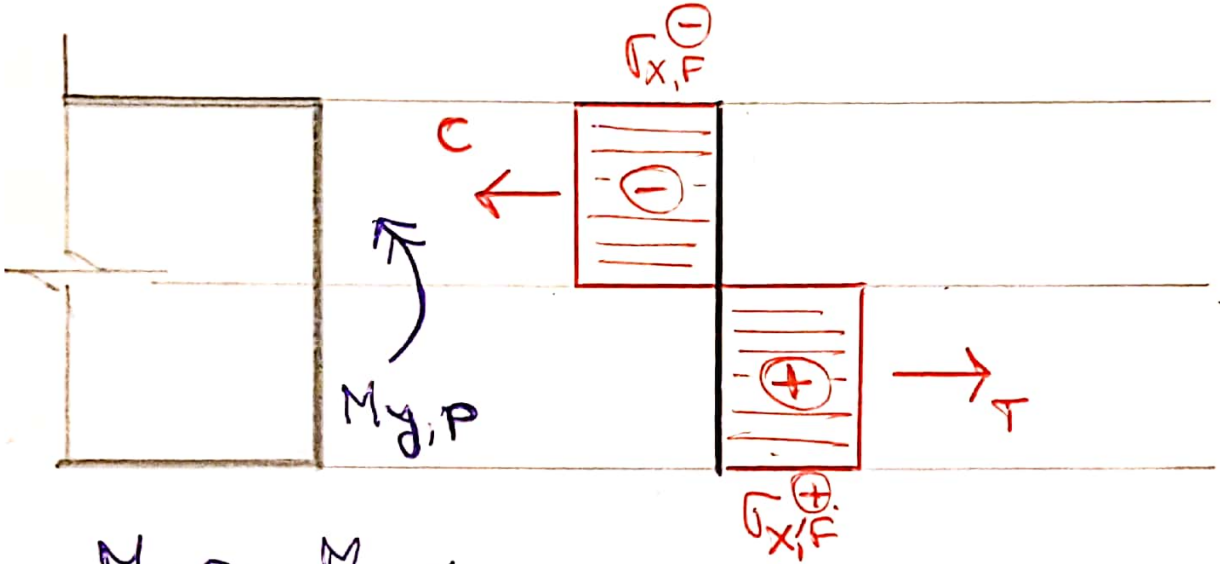
$z_e = h$

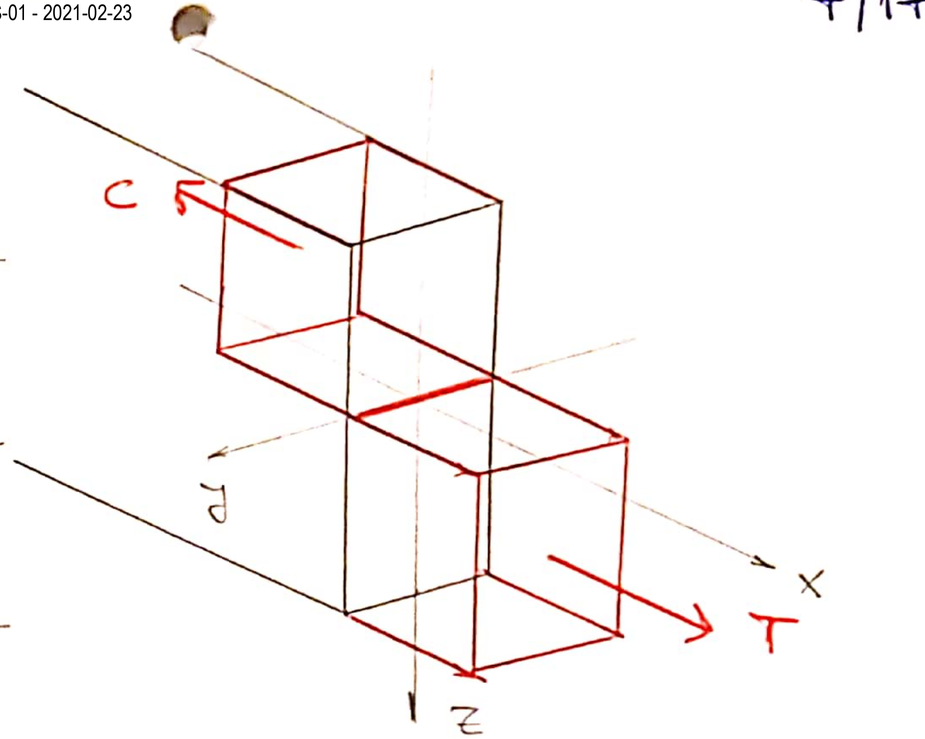
De (6) $M_y = \frac{3}{2} M_{y,e}\left[1 - \frac{1}{3}\right]$

$$M_y = \frac{\cancel{3}}{\cancel{2}} M_{y,e} \cdot \frac{\cancel{2}}{\cancel{3}}$$

$$\boxed{M_y = M_{y,e}}$$

SECCIÓN TOTALMENTE PLASTIFICADA:





$M_{y,P} = M_{y,u}$

$C = T = \sigma_{x,F} \cdot b \cdot h.$; $z = h.$; $M_{y,P} = M_{y,u} = C \cdot z = T \cdot z$

$$\boxed{M_{y,P} = M_{y,u} = \sigma_{x,F} \cdot bh \cdot h = bh^2 \sigma_{x,F}} \quad (7).$$

EN PLASTIFICACIÓN TOTAL $Z_E = 0.$

DE (4) $M_y = bh^2 \sigma_{x,F}$

DE (5) $M_y = bh^2 \sigma_{x,F}$

DE (6) $M_y = \frac{3}{2} M_{y,E}$

$$\boxed{M_{y,P} = \frac{3}{2} M_{y,E}} \quad (8).$$

FACTOR DE FORMA:

DE (2) $M_{y,E} = \frac{2}{3} b h^2 \sigma_{x,F}$

DE (7) $M_{y,P} = b h^2 \sigma_{x,F}$.

$$\frac{M_P}{M_E} = \frac{b h^2 \sigma_{x,F}}{\frac{2}{3} b h^2 \sigma_{x,F}} = \frac{3}{2} = k$$

FACTOR DE FORMA.

$$\boxed{\frac{M_E}{\sigma_F}} = \frac{\frac{2}{3} b h^2 \cancel{\sigma_{x,F}}}{\cancel{\sigma_{x,F}}} = \frac{2}{3} b h^2 = S_y$$

P/ SECCIÓN RECTANGULAR.

$$\boxed{\frac{M_P}{\sigma_F}} = \frac{b h^2 \cancel{\sigma_{x,F}}}{\cancel{\sigma_{x,F}}} = b h^2 = Z_y$$

Z_y: MÓDULO PLÁSTICO DE LA SECCIÓN.

$$\boxed{\begin{aligned} M_P &= Z_y \cdot \sigma_F \\ M_E &= S_y \cdot \sigma_F \end{aligned}} \quad (9).$$

P/ SECCIÓN RECTANGULAR

$$\frac{Z_y}{S_y} = \frac{b h^2}{\frac{2}{3} b h^2} = \frac{3}{2} = \frac{M_P}{M_E}.$$

P/ SECCIÓN RECTANGULAR $k = 1{,}50$.

P/ " CUADRADA $k = 2{,}00$

P/ IPN 200 $S_y = 214$

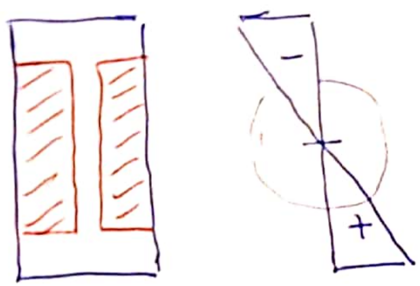
$Z_y = 250$

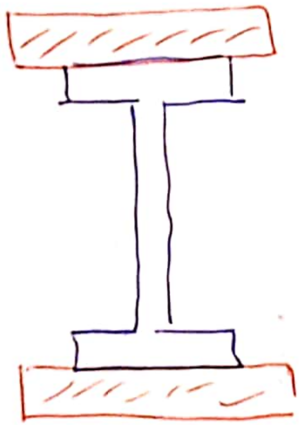
$k = 1{,}168$

- P/ DOBLE 'T' 1,08 a 1,20.

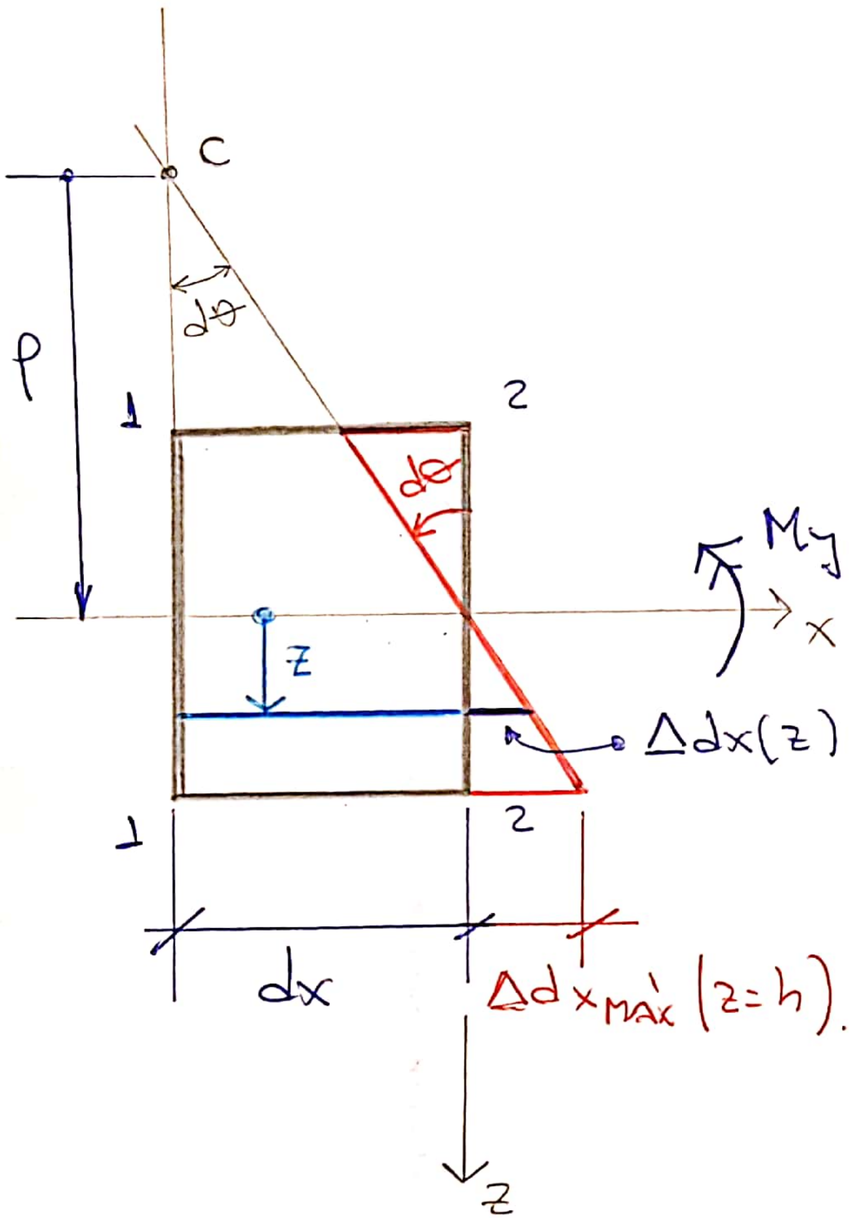
UDS ELEGIRÍAN UNA SECCIÓN CON UN FACTOR DE FORMA + GRANDE O MÁS CHICO ???

↳ k CUANTO + CHICO + SE APROVECHA LA SECCIÓN.





DEFORMACIONES:



- Si $dx = 1 \rightarrow \frac{d\theta}{\underbrace{dx}_{=1}} = \chi$: curvatura de flexión.

$$\boxed{d\theta = \chi} \quad (10)$$

- $\varepsilon_x = \frac{\Delta dx}{\underbrace{dx}_{=1}} \rightarrow \boxed{\varepsilon_x = \Delta dx.} \quad (11a)$

- $\Delta dx(z) = z \cdot d\theta \rightarrow \boxed{\varepsilon_x = z \cdot \chi.} \quad (11b)$

- $\rho \cdot d\theta = dx \rightarrow \rho \cdot \chi = 1.$

$$\boxed{\rho = \frac{1}{\chi} \; ; \; \chi = \frac{1}{\rho}} \quad (12)$$

- $\boxed{\varepsilon_x = z \, \frac{1}{\rho}} \quad (13)$

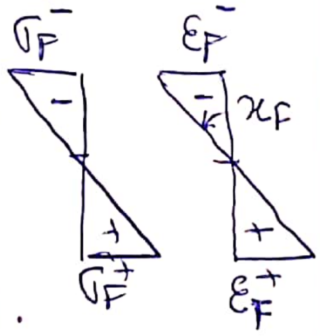
APLIQUEMOS ESTAS EXPRESIONES A NUESTRO PROBLEMA:

• Si $M_y = M_{y,e}$:

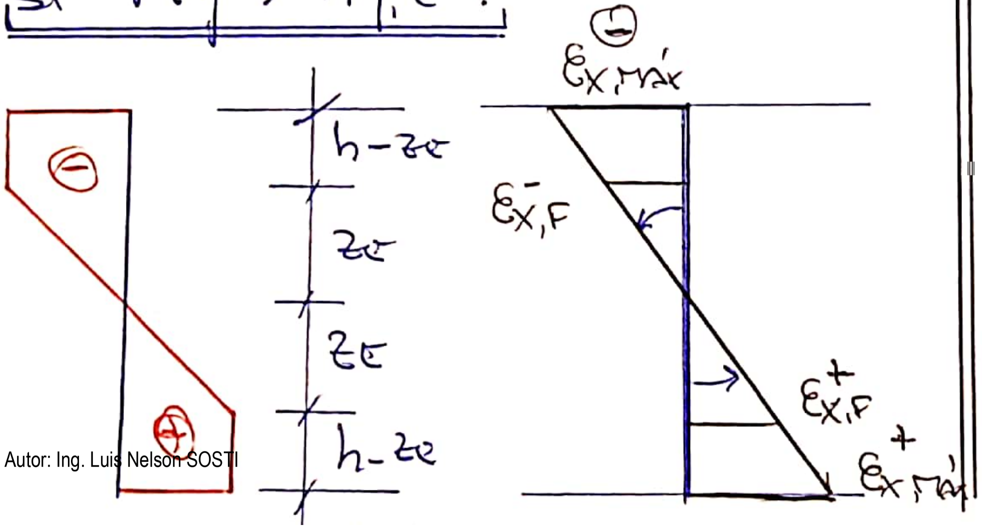
$z_e = h$. $\qquad \varepsilon_{x,F} = h \cdot x_E = h \cdot x_F$

$$\boxed{x_F = \frac{\varepsilon_{x,F}}{h}} \quad (14a)$$

$$\boxed{\rho_F = \frac{h}{\varepsilon_{x,F}}} \quad (14b).$$



• Si $M_y > M_{y,e}$:



$\varepsilon_{x,F} = z_e \cdot x \rightarrow$ $\boxed{z_e = \dfrac{\varepsilon_{x,F}}{x}}$

$\varepsilon_{x,F} = z_e \dfrac{1}{\rho} \rightarrow$ $\boxed{z_e = \varepsilon_{x,F} \cdot \rho}$ (15).

(14a) $\rightarrow$ $h = \dfrac{\varepsilon_{x,F}}{x_F}$

(14b) $\rightarrow$ $h = \varepsilon_{x,F} \cdot \rho_F$

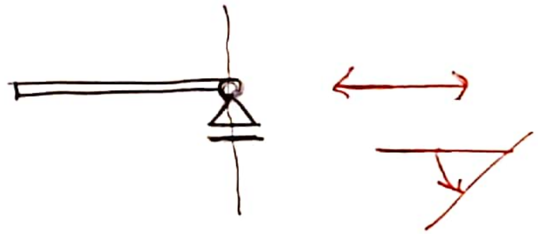
$\left.\dfrac{(15)}{(14)}\right] \rightarrow \dfrac{z_E}{h} = \dfrac{\varepsilon_{x,F}/x}{\varepsilon_{x,F}/x_F} = \dfrac{x_F}{x}$ (16a)

$\rightarrow \dfrac{z_e}{h} = \dfrac{\varepsilon_{x,F} \cdot \rho}{\varepsilon_{x,F}\, \rho_F} = \dfrac{\rho}{\rho_F}$ (16b).

(5) $M_y = b\,h^2\,\sigma_{x,F}\left[1 - \frac{1}{3}\frac{z_e^2}{h^2}\right] = \boxed{b\,h^2\sigma_{x,F}\left[1 - \frac{1}{3}\frac{x_F^2}{x^2}\right]}$ (17a)

(6) $M_y = \frac{3}{2}M_{y,e}\left[1 - \frac{1}{3}\frac{z_e^2}{h^2}\right] = \cancel{b\,h^2\sigma_x}\ \frac{3}{2}M_{y,e}\left[1 - \frac{1}{3}\frac{x_F^2}{x^2}\right]$ (17b)

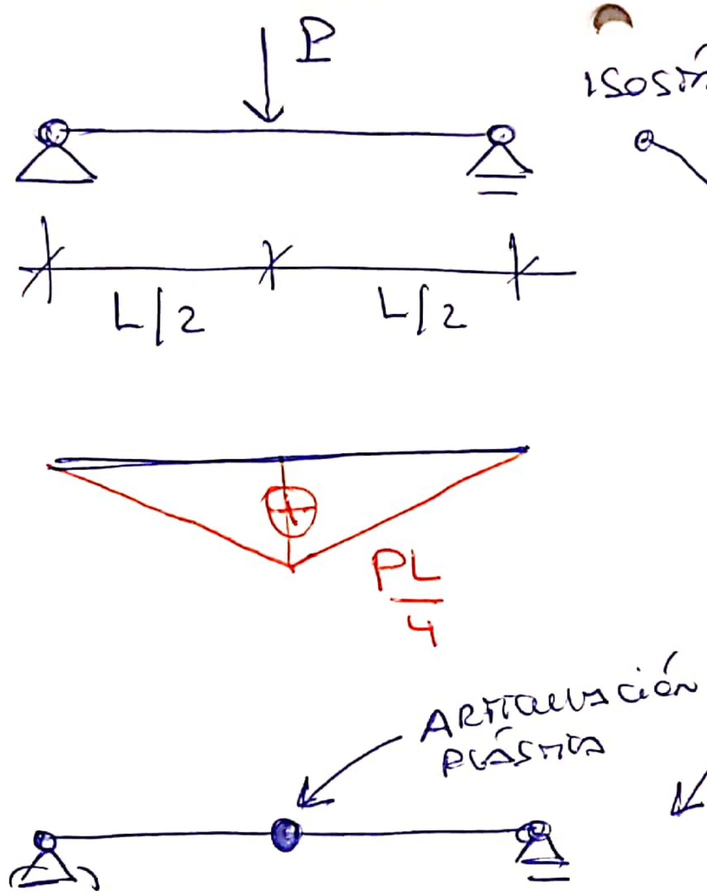
$$\boxed{\begin{array}{ll} M_y = b\,h^2\sigma_{x,F}\left[1 - \frac{1}{3}\frac{\rho^2}{\rho_F^2}\right] & (18a) \\ M_y = \frac{3}{2}M_{y,e}\left[1 - \frac{1}{3}\frac{\rho^2}{\rho_F^2}\right] & (18b) \end{array}}$$



(I) si $M_y = M_{y,e} \rightarrow z_e = h \rightarrow x = x_F \wedge \rho = \rho_F$.

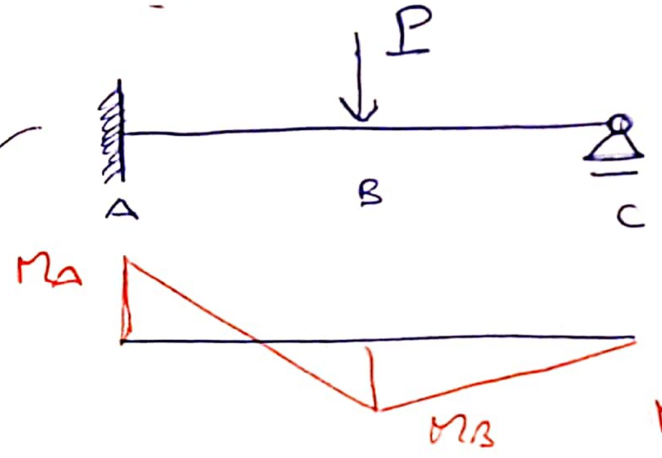
(II) si $M_y = M_{y,p} \rightarrow z_e = 0 \quad \underbrace{\frac{z_e}{h}}_{=0} = \frac{x_F}{x} \rightarrow x \rightarrow \infty \wedge$

$\underbrace{\frac{z_e}{h}}_{=0} = \frac{\rho}{\rho_F} \rightarrow \rho \rightarrow 0.$



ISOSTÁTICO

HIPOSTÁTICO.

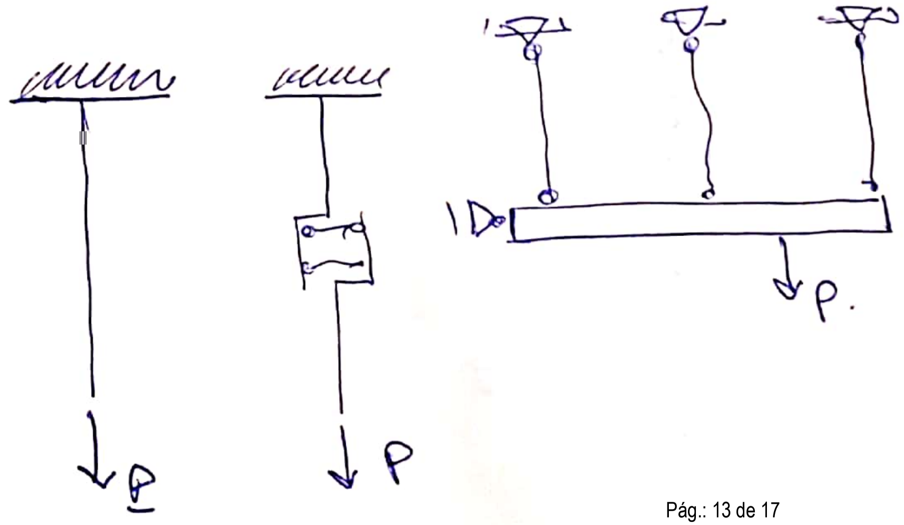


GH = 1

MA > MB.

CUANDO EN 'A' SE FORMA UNA ARTICULACIÓN PLÁSTICA:

GH = 0



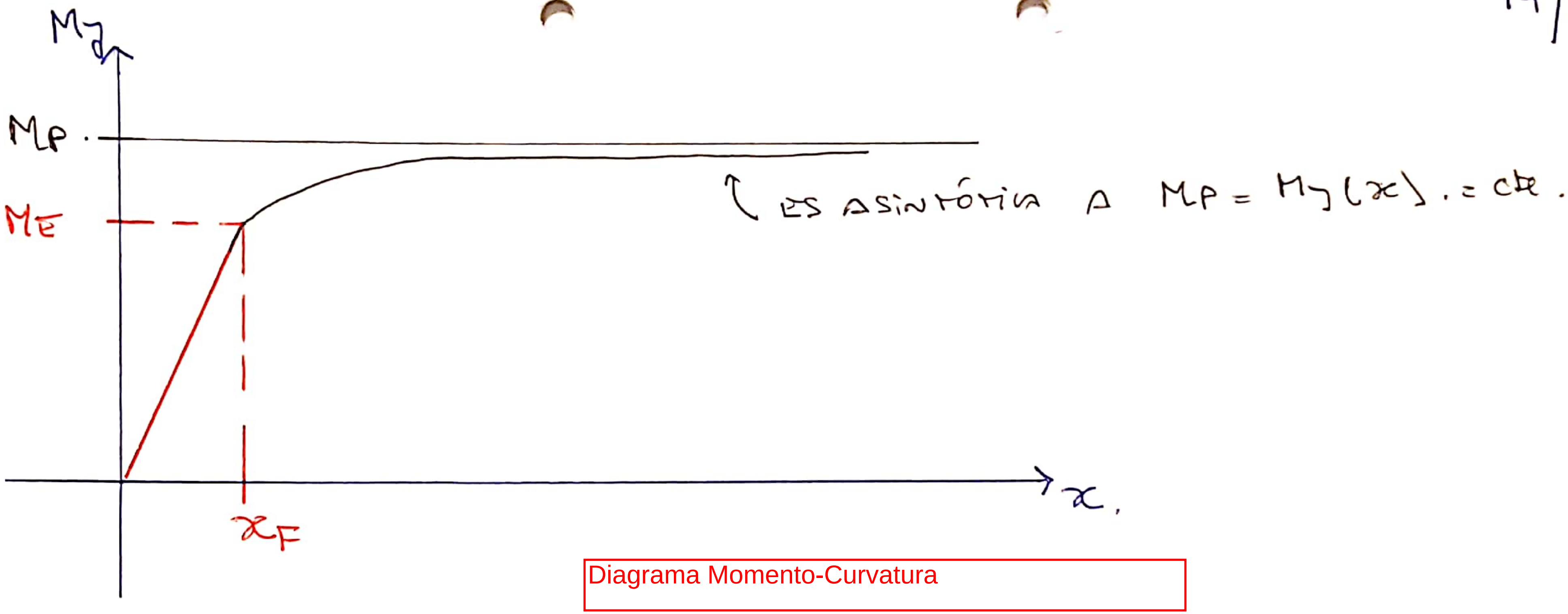
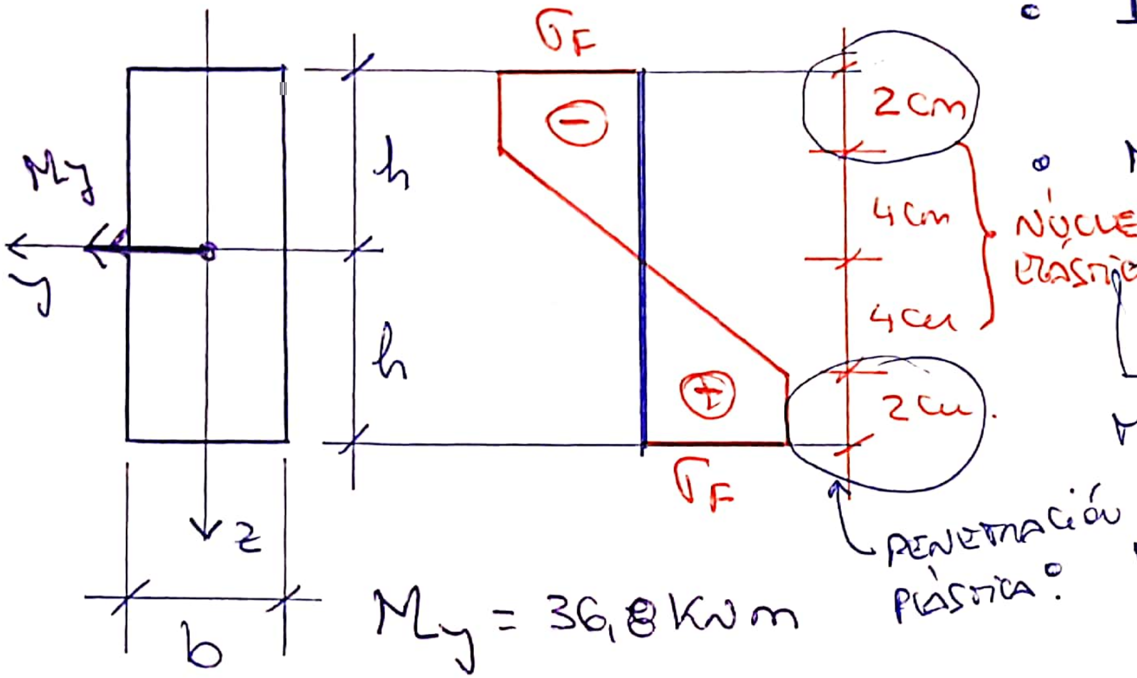


Diagrama Momento-Curvatura

• Si $M_y \le M_{y,E} \rightarrow \chi = \dfrac{M_y}{E I_y} \rightarrow M_y = E I_y \cdot \chi$

$$\frac{1}{\rho} = \frac{M_y}{E I_y} \rightarrow M_y = \frac{E I_y}{\rho}.$$

EJEMPLO:



$M_y = 36{,}8\ KNm$

DATOS:

$b = 50\,mm$

$h = 60\,mm$

$H = 120\,mm$.

$\sigma_F^+ = \sigma_F^- = 24\ KN/cm^2$

$E = 20000\ KN/cm^2$

$M_y = 36{,}8\ KNm$.

- $I_y = \dfrac{b(2h)^3}{12} = \dfrac{\cancel{8}^{\,2}}{\cancel{12}_{\,3}} bh^3 = \dfrac{2}{3} bh^3$.

- $M_{y,E} = \dfrac{I_y}{z_{max}} \cdot \sigma_F = \dfrac{2}{3}\dfrac{bh^3}{h}\sigma_F$

$$\boxed{M_{y,E} = \frac{2}{3} bh^2 \sigma_F}$$

$M_{y,E} = \dfrac{2}{3} \cdot 5 \cdot 6^2\ 24 = 28{,}80\ KNm$.

PENETRACIÓN PLÁSTICA: $M_{y,P} = \dfrac{3}{2} M_E = 43{,}20\ KNm$

- $M_E \leq M_y \leq M_P$

- $M_y = \dfrac{3}{2} M_E \left[1 - \dfrac{1}{3}\dfrac{z_E^2}{h^2}\right]$

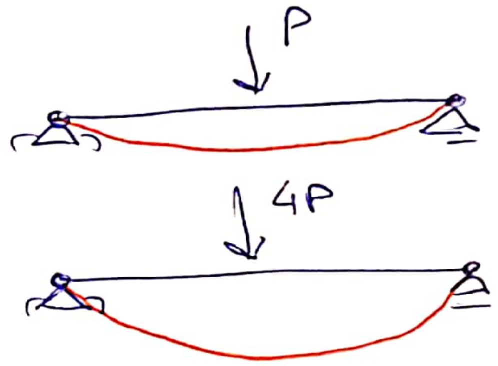
$\dfrac{1}{3}\dfrac{z_E^2}{h^2} = 1 - \dfrac{2}{3}\dfrac{M_y}{M_E}$

$z_E = \sqrt{\left(1 - \dfrac{2}{3}\dfrac{M_y}{M_E}\right) 3h^2}$

$$\boxed{z_E = 4\,cm}$$

CURVATURA - RADIO DE CURVATURA - DIAGRAMA DE DEFORMACIONES:

$$\frac{z_E}{h} = \frac{\chi_F}{\chi} \rightarrow \boxed{\chi = \chi_F \frac{h}{z_E}}$$



$$\chi_F = \frac{\varepsilon_{x,F}}{h} \qquad \varepsilon_{x,F} = \frac{\sigma_F}{E} = \frac{24}{20000} = 1{,}2 \cdot 10^{-3}$$

$$\boxed{\chi_F} = \frac{1{,}2 \cdot 10^{-3}}{6\,cm} = \boxed{2 \cdot 10^{-4} \frac{1}{cm.}}$$

Si $M_y \uparrow \rightarrow \chi \uparrow$

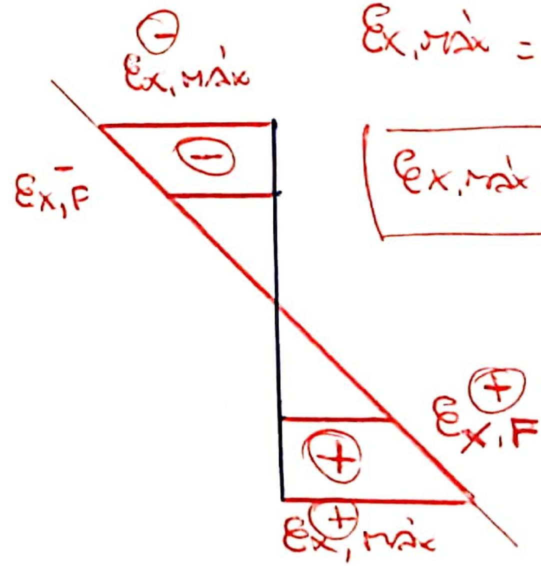
$\quad \rightarrow \rho \downarrow$

$$\chi_F = \frac{M_E}{E\, I_y} = \text{VERIFICAR.}$$

$$\chi = 2 \cdot 10^{-4} \frac{1}{cm} \cdot \frac{6\,cm}{4\,cm} = 3 \cdot 10^{-4} \frac{1}{cm.}$$

$$\rho = \frac{1}{\chi} = 3333^{\overline{33}}\ cm.$$

$$\rho_F = \frac{1}{\chi_F} = 5000\ cm.$$



$$\varepsilon_{x,máx} = \varepsilon_{x,F} \cdot \frac{h}{z_E}$$

$$\boxed{\varepsilon_{x,máx} = 1{,}8 \cdot 10^{-3}}$$

DESCARGA.